

# Workbook



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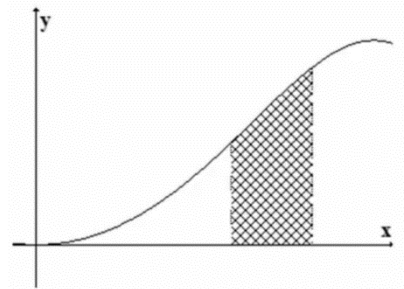
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# Applications of the Definite Integral - Volume and Surface Area

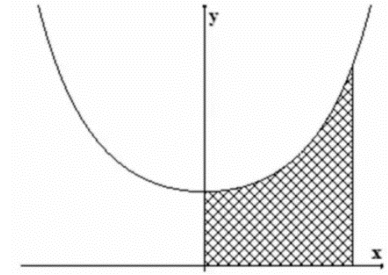
## Volume – Solids of Revolution

### Questions:

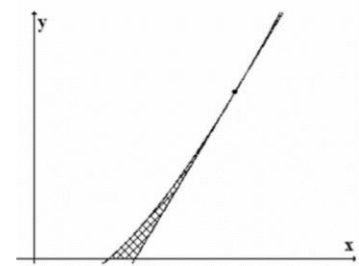
- 1) An area, bounded by the graphs of the function  $y = x^2$  and  $y = 2x$ , revolves around the  $x$ -axis.  
Compute the volume of the obtained solid in two ways:
- The disc method (due to Cavalieri).
  - The cylindrical shell method.
- 2) An area, bounded by the graphs of the function  $y = x^2$  and  $y = 2x$ , revolves around the  $y$ -axis.  
Compute the volume of the obtained solid in two ways:
- The disc method (due to Cavalieri).
  - The cylindrical shell method.
- 3) An area, bounded by the graph of  $f(x) = 1 - x^3$  and the axes, is shown in the figure.  
Compute the volume of the solids obtained by revolving the area about each of the following lines:
- The  $x$ -axis.
  - The line  $y = -1$ .
  - The line  $y = 2$ .
  - The  $y$ -axis.
  - The line  $x = -1$ .
  - The line  $x = 2$ .
- 4) The area bounded by the graph of  $y = \sin(x^2)$  and the lines  $x = \sqrt{\frac{\pi}{6}}$ ,  $x = \sqrt{\frac{\pi}{3}}$  and  $y = 0$  is shown in the figure. What is the volume of the solid obtained by revolving it around the  $y$ -axis?



- 5) The area bounded by the graph of  $y = e^{x^2}$  and the line  $x=0$ ,  $x=1$ , and  $y=0$  is shown in the figure. What is the volume of the solid obtained by revolving it around the  $y$ -axis?



- 6) The shaded area in the figure is bounded by the graph of  $y = e^{x^2}$ , the tangent at  $(e, e)$  and the  $x$ -axis. What is the volume of the solid obtained by revolving this area about the  $x$ -axis?



- 7) State and prove the formula for computing the volume of a cylinder.
- 8) State and prove the formula for computing the volume of a ball.
- 9) State and prove the formula for computing the volume of a cone.

**Answer Key:**

- 1)  $\frac{64\pi}{15}$       2)  $\frac{8\pi}{3}$       3) a.  $\frac{9\pi}{14}$ , b.  $\frac{15\pi}{7}$ , c.  $\frac{33\pi}{14}$ , d.  $\frac{3\pi}{5}$ , e.  $\frac{21\pi}{10}$ , f.  $\frac{12\pi}{5}$
- 4)  $\frac{\pi}{2}(\sqrt{3}-1)$       5)  $\pi(e-1)$       6)  $\frac{\pi}{54}(e^3-4)$
- 7)  $V = \pi r^2 h$       8)  $V = \frac{4}{3}\pi r^3$       9)  $V = \frac{1}{3}\pi r^2 h$

## Surface Area – Solid of Revolution

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### Questions:

- 1) Write the formulae for computing the surface area of the solid obtained by rotating a curve respectively about:
  - a. The  $x$ -axis
  - b. The  $y$ -axis
- 2) Determine the surface area of the solid obtained by rotating the graph of  $y = \sqrt{4 - x^2}$ ,  $-1 \leq x \leq 1$  about the  $x$ -axis.
- 3) State and prove the formula for computing the surface area of a cone.
- 4) Determine the surface area of the solid obtained by rotating the graph of  $x = \sqrt{9 - y^2}$ ,  $-2 \leq x \leq 2$  about the  $y$ -axis.

### Answer Key:

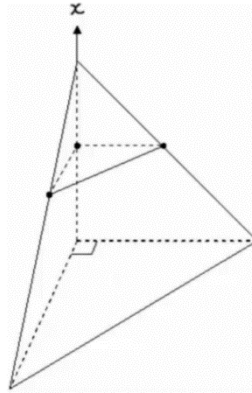
- 1) a.  $S = 2\pi \int_a^b f(x) \sqrt{1 + (f'(x))^2} dx$       b.  $S = 2\pi \int_c^d g(y) \sqrt{1 + (g'(y))^2} dy$
- 2)  $8\pi$
- 3)  $S = \pi r l = \pi r \sqrt{h^2 + r^2}$ .
- 4)  $24\pi$

## Volume by Integrating Cross Sections

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**Questions:**

- 1) Find the formula for computing the volume of a right pyramid with height  $h$  and whose base is a square with side of length  $a$ .
  
- 2) Find a formula for computing the volume of a right pyramid with height  $c$  and whose base is a right triangle with legs of lengths  $a$  and  $b$  respectively.  
[For simplicity, you may assume that the apex is directly above the right-angle of the base]



**Answer Key:**

- 1)  $V = \frac{a^2 h}{3}$
  
- 2)  $V = \frac{abc}{6}$