

# Binomial Expansion

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### Questions

1) Calculate the binomial coefficients:

a.  $\binom{5}{3}$

b.  $\binom{8}{3}$

c.  $\binom{10}{1}$

d.  $\binom{n}{n-1}$

2) Prove that for all  $n$ ,  $\binom{n}{n} = 1$  and  $\binom{n}{0} = 1$ .

3) Prove that for all  $n$ , and for all  $0 \leq k \leq n$ ,  $\binom{n}{k} = \frac{n(n-1)\dots(k+1)}{(n-k)!} = \frac{n(n-1)\dots(n-k+1)}{k!}$ .

4) Write out the terms of  $(x+y)^6$ .

You may use the results of the previous problems.

5) Write the binomial expansion of  $(3a-4b)^5$ .

6) Write the first four terms in the binomial expansion of  $(x+y)^{16}$ .

7) Write the first three terms in the binomial expansion of  $(3u^2+2v^3)^{10}$ .

8) Prove that for all  $n$ , and for all  $0 \leq k \leq n$ ,  $\binom{n}{n-k} = \binom{n}{k}$ .

9) Prove that for all  $n \geq 1$  and for all  $1 \leq k \leq n$ ,  $\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$ .

- 10) Show that the binomial coefficients can be arranged in the following form, where each entry, except the 1's, is the sum of the two entries above it and to the right and left.
- |  |       |
|--|-------|
|  | 1     |
|  | 11    |
|  | 121   |
|  | 1331  |
|  | 14641 |
|  | ..... |

11) Use mathematical induction to prove the binomial theorem for  $n > 0$ .

12) Write the seventh term in the expansion of  $\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right)^{12}$ .

13) Use the binomial theorem to approximate  $(0.99)^{18}$  to three decimal places.

**Answer Key**

- 1) a. 10                      b. 56                      c. 10                      d.  $n$   
 2) Solution in the recording.  
 3) Solution in the recording.  
 4)  $x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$   
 5)  $243a^5b^1 - 1620a^4b + 4320a^3b^2 - 5760a^2b^3 + 3840ab^4 - 1024b^5$   
 6)  $x^{16} + 16x^{15}y + 120x^{14}y^2 + 560x^{13}y^3 + \dots$   
 7)  $3^{10}u^{20} + 20 \cdot 3^9u^{18}v^3 + 180 \cdot 3^8u^{16}v^6 + \dots = 59,049u^{20} + 393,660u^{18}v^3 + 1,180,980u^{16}v^6 + \dots$   
 8) Solution in the recording.  
 9) Solution in the recording.  
 10) Solution in the recording.  
 11) Solution in the recording.  
 12) Solution in the recording.  
 13) 0.834 or 0.835.