proprep™

Workbook

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Polar Coordinates

Introduction

Questions:

1) Given the point with polar coordinates $(r,\theta)=\left(3,\frac{\pi}{5}\right)$, find three other sets of coordinates for the same point with different angles θ in the range $-2\pi\le\theta\le2\pi$.

Conversion between Polar and Cartesian Coordinates

Questions:

2) Convert the following sets of Cartesian coordinates to polar coordinates:

a. $(1,\sqrt{3})$ b. $(-2,-2)$ c. $(0,-5)$ d. $(-3,4)$

3) Convert the following sets of polar coordinates to Cartesian coordinates:

a. $\left(\sqrt{8},\frac{3\pi}{4}\right)$ b. $\left(-4,\frac{2\pi}{3}\right)$ c. $\left(0,\frac{\pi}{2}\right)$ d. $\left(6,-\frac{\pi}{3}\right)$

Conversion between Polar and Cartesian Coordinates

Questions:

4) Convert the following equations in x,y to polar form in r,θ:

a. $2x-5x^3=1+xy$ b. $x^2+y^2=6y$ c. $x=3$ d. $y=-4$.

5) Convert the following polar equations in r,θ to Cartesian form in x,y:

a. $r=-8\cos\theta$

b. $6r^3\sin\theta=4-\cos\theta$

c. $r=2$

d. $\theta=\dfrac{\pi}{4}$

6) Sketch the graph of $r=2+4\sin\theta$.

Tangent Lines in Polar Coordinates

Questions:

7) Find the equation(s) of the tangent line(s) to $r=\sin(4\theta)\cos(\theta)$ at $\theta=\dfrac{\pi}{6}$.

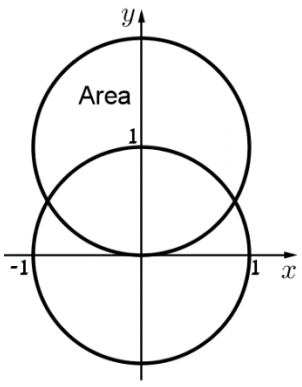
8) Find the equation(s) of the tangent line(s) to $r=1-2\sin\theta$ at the origin.

Area in Polar Coordinates

Questions:

9) Find the area inside the graph of $r=6+4\cos\theta$ and to the left of the y-axis.

10) Find the area between the circles $r=1$ and $r=2\sin\theta$ as in the figure:

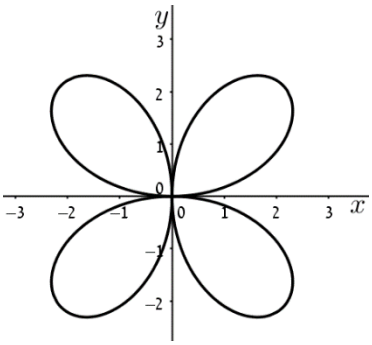


Arc Length in Polar Coordinates

Questions:

11) Find the length of the circumference of the cardioid $r = 1 + \cos\theta$.

12) Find the arc length of one petal of the rose $r = 6 + 4\cos\theta$.
[No need to evaluate the integral]



Surface Area in Polar Coordinates

Questions:

13) Find the surface area obtained by revolving the curve $r = \cos\theta$, $0 \le \theta \le \frac{\pi}{2}$ about the x-axis

14) Find the surface area obtained by revolving the curve $r = 4 + 4\sin\theta$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ about the y-axis.

Answer Key:

1) $\left(3,-\frac{9\pi}{5}\right)$ $\left(-3,\frac{6\pi}{5}\right)$ $\left(-3,-\frac{4\pi}{5}\right)$

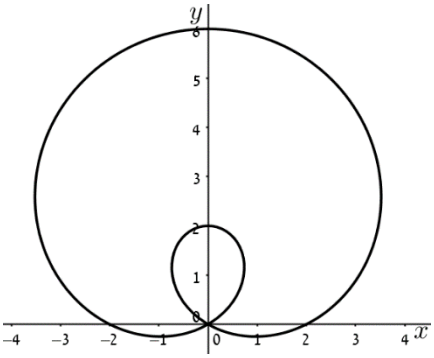
2) a. $\left(2,\frac{\pi}{3}\right)$ b. $\left(2\sqrt{2},\frac{5\pi}{4}\right)$ c. $\left(5,\frac{3\pi}{2}\right)$ d. $(5,2.214)$

3) a. $(-2,2)$ b. $\left(2,-2\sqrt{3}\right)$ c. $(0,0)$ d. $\left(3,-3\sqrt{3}\right)$

4) a. $1+r^2\cos\theta\sin\theta$ b. $r=6\sin\theta$ c. $r\cos\theta=3$ d. $r\sin\theta=-4$

5) a. $(x+4)^2+y^2=16$ b. $4\sqrt{x^2+y^2}-x$ c. $x^2+y^2=4$ d. $y=x$

6) see figure:



7) $y=\frac{1}{3\sqrt{3}}x+\frac{1}{4}$

8) $y\left(\theta=\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}x$, $y\left(\theta=\frac{5\pi}{6}\right)=-\frac{\sqrt{3}}{3}x$

9) $A=22\pi-48$

10) $A=\frac{\pi}{3}+\frac{\sqrt{3}}{2}$

11) $L=8$

12) $L=3\int_0^{\frac{\pi}{2}}\sqrt{\sin^2 2\theta+4\cos^2 2\theta}\,d\theta$

13) $S=\pi$

14) $\frac{512\pi}{5}$