

Workbook



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Questions:

In the following exercises, assume that all given derivatives exist.

- 1) Given $\ln(x^2 - y^2)$, $x = 2u - v$, $y = u^2 + v^3$. Compute: z_u , z_v .
- 2) Given $v = 4t + k^2$, $u = t^2 + 4m$, $z = e^{u-v}$. Compute: $\frac{\partial z}{\partial t}$, $\frac{\partial z}{\partial m}$, $\frac{\partial z}{\partial k}$.
- 3) Given $z = f(x^2 - y^2)$. Prove $y \cdot z_x + x \cdot z_y = 0$.
- 4) Given $z = f(xy)$. Prove $x \cdot z_x - y \cdot z_y = 0$.
- 5) Given $z = f\left(\frac{x}{y}\right)$. Prove $x \cdot z_x + y \cdot z_y = 0$.
- 6) Given $z = f(x - y, y - x)$. Prove $z_x + z_y = 0$.
- 7) Given $w = f(x - y, y - z, z - x)$. Prove $w_x + w_y + w_z = 0$.
- 8) Given $u = \sin x + f(\sin y - \sin x)$. Prove $u_x \cos y + u_y \cos x = \cos x \cos y$.
- 9) Given $z = y \cdot f(x^2 - y^2)$. Prove $\frac{1}{x} z_x + \frac{1}{y} z_y = \frac{z}{y^2}$.
- 10) Given $z = xy + xf\left(\frac{y}{x}\right)$. Prove $x \cdot z_x + y \cdot z_y = xy + z$.
- 11) Given $u(x, y, z) = x^2 \cdot f\left(\frac{y}{x}, \frac{z}{x}\right)$. Prove $xu_x + yu_y + zu_z = 2u$.
- 12) Given $h(x, y) = f(y + ax) + g(y - ax)$. Prove $h_{xx} = a^2 \cdot h_{yy}$.
- 13) Given $u(x, y) = f(e^x \sin y) - g(e^x \sin y)$. Prove:
 - a. $u_{xx} + u_{yy} = \frac{u_{xx} - u_x}{\sin^2 y}$.
 - b. $u_{xy} = u_{yx}$.
 - c. Compute: $u_{xy}(1, \pi)$, given that $f'(0) = 2$, $g'(0) = 1$.

14) Given $u = f(x, y)$, $x = r \cos \theta$, $y = r \sin \theta$. Prove:

a. $(u_x)^2 + (u_y)^2 = (u_r)^2 + \frac{1}{r^2}(u_\theta)^2$.

b. $u_{rr} = f_{xx} \cos^2 \theta + 2f_{xy} \cos \theta \sin \theta + f_{yy} \sin^2 \theta$.

15) Given $z = h(u, v)$ and that $u = f(x, y)$, $v = g(x, y)$ satisfy the Cauchy-Riemann equations, i.e. $u_x = v_y$, $u_y = -v_x$. Prove:

a. u, v satisfy the Laplace equation, i.e. $u_{xx} + u_{yy} = 0$, $v_{xx} + v_{yy} = 0$.

b. $h_{xx} + h_{yy} = ((u_x)^2 + (v_x)^2)(h_{uu} + h_{vv})$.

16) Given $u = f(x, y)$, $x = r \cosh s$, $y = r \sinh s$. Prove: $(u_x)^2 - (u_y)^2 = (u_r)^2 - \frac{1}{r^2}(u_s)^2$.

Answer Key:

1) $z_u = \frac{2 \cdot 2(2u - v)}{(2u - v)^2 - (u^2 + v^3)^2} - \frac{2 \cdot 2(u^2 + v^3)u}{(2u - v)^2 - (u^2 + v^3)^2}$, $z_v = \frac{2x}{x^2 - y^2} \cdot (-1) + \frac{-2y}{x^2 - y^2} 3v^2$

2) $\frac{\partial z}{\partial t} = 2e^{t^2 - 4t + 4m - k^2} (t - 2)$, $\frac{\partial z}{\partial m} = e^{u-v} \cdot 4$, $\frac{\partial z}{\partial k} = -e^{u-v} 2k$

3-12) Refer to the videos.

13) a+b. Refer to the videos. c. $-e$

14-16) Refer to the videos.