

# Exponents, Exponential Equations

## Exponents

### Questions:

1) Simplify each of the following and write the answers with only positive exponents:

$$\begin{array}{lll} \text{a. } a^3 b^2 a^4 b^5 & \text{b. } \frac{x^7 y^5}{y^2 x^3} & \text{c. } (x^2)^3 (y^2)^4 x^2 y^3 \\ \text{d. } (2xy^2)^3 (2x)^4 & \text{e. } \left(\frac{a^2}{b^3}\right)^3 \cdot \left(\frac{b}{a^2}\right)^2 & \text{f. } \left(\frac{4x^2 y^3}{x^4 y^5}\right)^2 \end{array}$$

2) Simplify each of the following and write the answers with only positive exponents:

$$\begin{array}{lll} \text{a. } a^{-3} b^{-2} a^4 b^5 & \text{b. } \frac{x^7 y^{-5}}{y^2 x^{-3}} & \text{c. } (x^{-2})^3 (y^{-1})^4 x^{-2} y^3 \\ \text{d. } (2c^{-1} d^{-2})^3 (2c)^4 & \text{e. } \left(\frac{m^{-2}}{n^3}\right)^3 \cdot \left(\frac{n^{-3}}{m^2}\right)^{-2} & \text{f. } \left(\frac{4x^2 y^{-3}}{x^{-4} y^5}\right)^{-2} \end{array}$$

3) Simplify each of the following and write the answers with only positive exponents:

$$\begin{array}{lll} \text{a. } \left(\frac{a^{-2}}{4v^{-\frac{1}{2}}}\right)^{\frac{1}{2}} & \text{b. } \left(\frac{x^{-\frac{1}{2}} y^{-5}}{y^{-25} x^{-3}}\right)^{\frac{1}{5}} & \text{c. } \left(\frac{a^{-2} b^{\frac{1}{2}}}{c^3 d^{-2}}\right)^{\frac{1}{6}} \\ \text{d. } \frac{(2c^{-1} d^{-2})^{\frac{1}{3}}}{(2c^{-4})^{\frac{1}{4}}} & & \end{array}$$

4) Evaluate each of the following:

a.  $\sqrt[4]{16}$

b.  $\sqrt[3]{125}$

c.  $\sqrt[3]{64^2}$

d.  $\sqrt[5]{32^4}$

e.  $\sqrt[3]{-8}$

f.  $\sqrt[5]{-32}$

g.  $\sqrt[3]{-27}$

h.  $\sqrt[5]{-243}$

5) Simplify each of the following and write the answers with only positive exponents:

a.  $9^{\frac{1}{2}}$

b.  $16^{\frac{1}{4}}$

c.  $81^{\frac{1}{3}}$

d.  $-8^{\frac{1}{3}}$

e.  $32^{\frac{1}{5}}$

f.  $-125^{\frac{1}{3}}$

g.  $16^{\frac{3}{4}}$

h.  $27^{\frac{2}{3}}$

i.  $\left(\frac{243}{32}\right)^{\frac{4}{5}}$

6) Write each of the following radicals in exponent form:

a.  $\sqrt[4]{16}$

b.  $\sqrt[10]{5x}$

c.  $\sqrt{x^2 + y^2}$

d.  $\sqrt[3]{(a+b)^5}$

7) Simplify each of the following. Assume that  $x$ ,  $y$ , and  $z$  are positive:

a.  $\sqrt{x^6}$

b.  $\sqrt[9]{y^6}$

c.  $\sqrt{18x^3y^7}$

d.  $\sqrt[5]{x^3}$

e.  $\sqrt[3]{x^2y^3z^4}$

f.  $\sqrt[5]{x^{15}y^5z^{20}}$

g.  $\sqrt[3]{x^3y^6}$

h.  $\sqrt[5]{x^{15}y^5}\sqrt[5]{z^{20}}$

8) Multiply each of the following. Assume that  $x$  is positive:

a.  $(\sqrt{x}+2)(\sqrt{x}-2)$

b.  $(\sqrt{x}+2)(\sqrt{x}-5)$

c.  $(4\sqrt{x}+\sqrt{y})(2\sqrt{x}-5\sqrt{y})$

9) Rationalize the denominator for each of the following. Assume that  $x$  is positive:

a.  $\frac{5}{\sqrt{x}}$

b.  $\frac{2}{\sqrt[3]{x}}$

c.  $\sqrt[4]{\frac{2}{x^3}}$

d.  $\frac{5}{\sqrt{x-1}}$

e.  $\frac{2}{3\sqrt{x}-4}$

f.  $\frac{5}{2\sqrt{x}+\sqrt{5}}$

Answer Key:

- 1) a.  $a^7b^7$       b.  $x^4y^3$       c.  $x^8y^{11}$       d.  $128x^7y^6$       e.  $\frac{a^2}{b^7}$   
 f.  $\frac{16}{x^4y^4}$
- 2) a.  $ab^3$       b.  $\frac{x^{10}}{y^7}$       c.  $\frac{1}{x^8y}$       d.  $\frac{128c}{d^6}$       e.  $\frac{1}{m^2n^3}$   
 f.  $\frac{y^{16}}{16x^{12}}$
- 3) a.  $\frac{\sqrt[4]{v}}{2a}$       b.  $\sqrt{xy^4}$       c.  $\frac{\sqrt[12]{b^3}\sqrt[3]{d}}{\sqrt[3]{a}\sqrt{c}}$       d.  $\frac{\sqrt[12]{2^3}\sqrt[3]{c^2}}{\sqrt[3]{d^2}}$
- 4) a. 2      b. 5      c. 16      d. 16      e. -2      f. -2  
 g. -3      h. -3
- 5) a. 3      b. 2      c.  $3^3\sqrt{3}$       d. -2      e. 2      f. -5  
 g. 8      h. 9      i.  $5\frac{1}{16}$
- 6) a. 2      b.  $(5x)^{\frac{1}{10}}$       c.  $(x^2 + y^2)^{\frac{1}{2}}$       d.  $(a + b)^{\frac{5}{3}}$
- 7) a.  $x^3$       b.  $\sqrt[3]{y^2}$       c.  $3\sqrt{2x}\sqrt{xy^3}\sqrt{y}$       d.  $x^{\frac{3}{5}}$       e.  $\sqrt[3]{x^2yz}\sqrt[3]{z}$   
 f.  $x^3yz^4$       g.  $xy^2$       h.  $x^3yz^4$
- 8) a.  $x - 4$       b.  $x - 3\sqrt{x} - 10$       c.  $8x - 18\sqrt{x}\sqrt{y} - 5y$
- 9) a.  $\frac{5\sqrt{x}}{x}$       b.  $\frac{2\sqrt[3]{x^2}}{x}$       c.  $\frac{\sqrt[4]{2}\sqrt[4]{x}}{x}$       d.  $\frac{5(\sqrt{x} + 1)}{x - 1}$   
 e.  $\frac{2(3\sqrt{x} + 4)}{9x - 16}$       f.  $\frac{5(2\sqrt{x} - \sqrt{5})}{4x - 5}$

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**Exponential Equations - Same Base**

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**Questions:**

1) Solve the following equations:

a.  $2^{3x} = 64$

b.  $5^{3x-6} = 125$

c.  $9^{x-1} = 27^{x+2}$

d.  $49^{x-2} = 7^{3x-4}$

2) Solve the following equations:

a.  $8^{4x-2} = \left(\frac{1}{2}\right)^{3x-3}$

b.  $10^{6x-2} = 0.001^{1.5x-2}$

3) Solve the following equations:

a.  $\left(\frac{25}{9}\right)^{3-x} = \left(\frac{27}{125}\right)^{x-2}$

b.  $\left(\frac{81}{16}\right)^{-x} = \left(\frac{2}{3}\right)^{5-x^2}$

4) Solve the following equations:

a.  $\frac{8}{2^x} \cdot 0.25^{2x^2+1} = \frac{1}{16}$

b.  $\left(\frac{5}{3}\right)^{1-x} \cdot \left(\frac{3}{5}\right)^{-\frac{1}{2}} = \left(\frac{9}{25}\right)^{2x}$

5) Solve the following equations:

a.  $\left(\frac{1}{\sqrt{2}}\right)^{3-x} = 16^{-x}$

b.  $(\sqrt{3})^{3-x} = 81^{0.5x+1}$

6) Solve the following equations:

a.  $\left(\frac{1}{25\sqrt{5}}\right)^{-x} = 0.2^{\frac{1}{2}x-2}$

b.  $\sqrt[3]{4^x} = \sqrt[6]{0.125}$

7) Solve the following equations:

a.  $\sqrt{\frac{1}{32}} \cdot \left(\frac{1}{2}\right)^{6x-x^2} = 16\sqrt{2}$

b.  $\frac{\sqrt[3]{36^{0.5x-1}}}{36} = \sqrt{6}$

8) Solve the following equations:

a.  $27^{-1} \cdot (3^x)^x = \frac{1}{9^{-x}}$

b.  $0.5^{2x-4} = \left(\frac{1}{4^{x-3}}\right)^{\frac{8}{x}}$

9) Solve the following equations:

a.  $\sqrt[x]{4^{x^2+x}} = \frac{16^x}{64}$

b.  $\sqrt[x]{\sqrt[x]{81}} = 243\left(\frac{1}{3^x}\right)^x$

10) Solve the following equations:

a.  $\left(\left(4^x\right)^x\right)^x \cdot 32^{x^2} = 8^x$

b.  $3^x \cdot \frac{1}{27^{\sqrt{x}}} = 9^5$

11) Solve the following equations:

a.  $192 \cdot 3^{2x} - 27 \cdot 4^{2x+1} = 0$

b.  $16 \cdot 3^{x^2+x} - 9 \cdot 4^{x^2+x} = 0$

12) Solve the following equations:

a.  $2 \cdot 4^x + 5 \cdot 4^x = 112$

b.  $3 \cdot 5^x - 5^x = \frac{2}{25}$

13) Solve the following equations:

a.  $2^{x+1} + 2^x = 48$

b.  $\frac{\sqrt[x+1]{36^{0.5x-1}}}{36} = \sqrt{6}$

14) Solve the following equations:

a.  $2^{x+3} - 2^x = 28$

b.  $\frac{1}{2}4^{x+2} + 4^{x+1} = \frac{3}{16}$

15) Solve the following equations:

a.  $4 \cdot 5^{x-1} + 5^x = 0.36$

b.  $3 \cdot 2^x - 5 \cdot 2^{x-2} - 2^{x-1} = 40$

16) Solve the following equations:

a.  $5 \cdot 6^{x-1} - 3^x \cdot 2^{x-1} = 72$

b.  $2^x \cdot 5^{x+2} - 2^{x+2} \cdot 5^{x+1} = 50$

### Answer Key:

1) a.  $x = 2$

b.  $x = 3$

c.  $x = -8$

d.  $x = 0$

2) a.  $x = \frac{3}{5}$

b.  $x = \frac{16}{21}$

3) a.  $x = 0$

b.  $x = 1, -5$

4) a.  $x = 1, -1\frac{1}{4}$

b.  $x = -\frac{1}{2}$

5) a.  $x = \frac{1}{3}$

b.  $x = -1$

6) a.  $x = \frac{2}{3}$

b.  $x = -\frac{3}{4}$

7) a.  $x = 7, -1$

b.  $x = -3$

8) a.  $x = 3, -1$

b.  $x = 6, 4$

9) a.  $x = 4$

b.  $x = 2, 1$

10) a.  $x = 0, \frac{1}{2}, -3$

b.  $x = 25$

11) a.  $x = 1$

b.  $x = 1, -2$

12) a.  $x = 2$

b.  $x = -2$

13) a.  $x = 4$

b. No solution.

14) a.  $x = 2$

b.  $x = -3$

15) a.  $x = -1$

b.  $x = 5$

16) a.  $x = 2$

b.  $x = 1$

## Exponential Equations - Substitution

### Questions:

1) Solve the following equations:

a.  $2 \cdot 2^{2x} - 9 \cdot 2^x + 4 = 0$

b.  $25^x - 6 \cdot 5^x + 5 = 0$

2) Solve the following equations:

a.  $3 \cdot 81^{0.5x} + 5 \cdot 3^x - 2 = 0$

b.  $36^{x+0.5} - 6^{x+1} = 36^{0.5x} - 6^0$

3) Solve the following equations:

a.  $100^{-x+0.5} + \frac{9}{10^x} = 1$

b.  $\left(6\frac{1}{4}\right)^x - 2\left(\frac{5}{2}\right)^{x+1} + \frac{25}{4} = 0$

4) Solve the following equations:

a.  $2^{x+2} + 2^{-x+2} = 10$

b.  $4^{x+1.5} - 2 \cdot 4^{2-x} = 60$

5) Solve the following equations:

a.  $27^x - 3^{2x+1} = 9 \cdot 9^x - 3 \cdot 3^{x+2}$

b.  $5 \cdot 6^x - 3 \cdot 4^x - 2 \cdot 9^x = 0$

6) Solve the following equations:

a.  $\frac{1}{3^x+1} - \frac{2}{9^x-1} = 0$

b.  $\frac{124}{4^{x+1}-4} + \frac{8}{4^{x-1}} = 2$

7) Solve the following equations:

a.  $\frac{9}{2-5^x} - \frac{2}{5^{x+1}} = 3$

b.  $4 - \frac{2}{9^{x-1} - \frac{1}{9}} = -\frac{5}{9^{x-\frac{1}{2}}}$

8) Solve the following equations:

a.  $1 - \frac{48}{3^{x+2} - 36} = \frac{3^{-x+1}}{4 - 3^x}$

b.  $\frac{5 \cdot 2^x - 10}{4^x - 2^x - 2} = 1$

9) Solve the following equations:

a.  $49^{2\sqrt{x}} - 8 \cdot 7^{2\sqrt{x}} + 7 = 0$

b.  $\sqrt{2^{2x+2}} + \frac{1}{4^{0.5x-1}} = 9$

10) Solve the following equations:

a.  $e^{2x} - 2e^x + 1 = 0$

b.  $e^{2x} - e^{x+1} - e^x + e = 0$

### Answer Key:

1) a.  $x = 2, -1$

b.  $x = 1, 0$

2) a.  $x = -1$

b.  $x = 0, -1$

3) a.  $x = 1$

b.  $x = 1$

4) a.  $x = \pm 1$

b.  $x = 1.5$

5) a.  $x = 1, 2$

b.  $x = 0, 1$

6) a.  $x = 1$

b.  $x = 2\frac{1}{2}, -\frac{1}{2}$

7) a.  $x = -1$

b.  $x = \frac{1}{2}$

8) a.  $x = 2, -1$

b.  $x = 2, 1$

9) a.  $x = \frac{1}{4}, 0$

b.  $x = 2, -1$

10) a.  $x = 0$

b.  $x = 1, 0$



## Scientific Notation

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### Questions:

- 1) Convert the following into scientific notation and then perform the computations.

Express your answer in scientific notation:  $\frac{(24,000,000)(0.009)}{0.00015}$ .

- 2) The speed of light is approximately  $1.86 \times 10^5$  miles per second.  
If the planet Mars is  $1.416 \times 10^8$  miles from the sun, how long does it take light from the sun to reach Mars? Express your answer using scientific notation.
- 3) The mass of Pluto is approximately  $1.3 \times 10^{22}$  kilograms.  
If one ton is 888.9 kilograms, what is the weight of the Pluto in tons?  
Express your answer using scientific notation.

### Answer Key:

- 1)  $1.44 \cdot 10^9$
- 2)  $7.613 \cdot 10^2$  secs.
- 3)  $1.46 \cdot 10^{19}$  tons.