

# Inequalities

## Linear Inequalities

### Questions:

1) Solve the following inequalities:

a.  $45x - 26 > 109$

b.  $7x - 11 > 3x + 5$

c.  $6x > 2(3x - 1)$

d.  $5x + 15 < 3(x - 12) + 3x$

e.  $2(x - 5) \geq \frac{1}{2}(4x + 6)$

2) Solve the following inequalities:

a.  $(x - 2)^2 + 4 < (x + 2)^2 + 20$

b.  $\frac{8x - 4}{2} < \frac{9(x + 1)}{3}$

c.  $4(6x - 8) < 8(3x - 4)$

d.  $\frac{x - 6}{3} - \frac{x - 4}{4} \geq 12 - x$

e.  $\frac{7 - x}{10} - \frac{3x - 1}{5} + \frac{x + 4}{3} < 7$

3) Find the solution of the following inequalities:

a.  $5x - 15 < 2(x - 10) + 3x$

b.  $2(x - 5) \geq \frac{1}{2}(4x + 6)$

c.  $6\left(\frac{x}{3} + 1\right) \geq 2x + 3$

d.  $63\left(\frac{2x - 6}{2} + 7 - x\right) > 0$

4) Solve the following inequalities:

a.  $6x - 8 < 2x + 12$

and

$7x - 6 > 1$

b.  $2(x + 4) - 3 > -7$

and

$5x + 6 \leq 9 - (3 - x)$

c.  $10x < 4(x - 3)$

and

$3x - (x - 8) > -13 - x$

d.  $6x - 15 - 2x < 20 - 3x$

and

$3x + 131 \geq 3x + 131$

e.  $4x - 2 > 4x + 3$

and

$5x - 3(4 + x) \leq 5 - x$

5) Solve the following inequalities:

a.  $3 < x + 1 < 5$

b.  $6x - 38 \leq x - 3 \leq 5x + 7$

c.  $6(x - 1) + 3 < 7x + 14 < 4(2x - 1) + 12$

d.  $6 < \frac{2x + 10}{3} \leq \frac{7x - 20}{5}$

e.  $-1 \leq \frac{2x - 6}{4} < \frac{x + 2}{3}$

6) Solve the following inequalities:

a.  $3x - 6 > 18$

or

$5x + 4 \leq 14$

b.  $10x + 8 \geq 6x + 20$

or

$4x + 8 < x - 10$

c.  $5x + 9 > 30 - 2x$

or

$6 - x < x + 4$

d.  $4x + 8 > x - 10$

or

$x - 10 < 9x + 14$

e.  $4x - 6 > 14 - x$

or

$5x - 11 < 19 + 2x$

**Answer Key:**

1) a.  $x > 3$       b.  $x > 4$       c. All  $x$       .      d.  $x > 51$       e. No solution.

2) a.  $x > -2$       b.  $x < 5$       c. No solution.      d.  $x \geq 12$       e.  $x > -13$

3) a. No solution.      b. No solution.      c. All  $x$       d. All  $x$ .

4) a.  $1 < x < 5$       b.  $-6 < x \leq 0$       c.  $-7 < x < -2$   
 d. All  $x$       e. No such  $x$ .

5) a.  $2 < x < 4$       b.  $-2.5 \leq x \leq 7$       c.  $x > 6$       d.  $x \geq 10$   
 e.  $1 \leq x \leq 13$ .

6) a.  $x > 8$  or  $x \leq 2$       b.  $x \geq 6$  or  $x < -6$       c.  $x > -5$       d.  $x > -6$   
 e. All  $x$ .

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## Polynomial Inequalities

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### Questions:

1) Solve the following inequalities:

a.  $x^2 < 144$

b.  $-3x^2 + 12x > 0$

c.  $x^2 - 10x + 25 > 0$

d.  $x^2 - 12x > -32$

2) Solve the following inequalities:

a.  $2x^2 + 2x + 24 \geq 0$

b.  $x^2 + x + 24 \geq 0$

c.  $-x^2 + 13x + 30 < 0$

d.  $-2x^2 + 7x + 4 < 0$

3) Solve the following inequalities:

a.  $(x+2)(x+5) < 0$

b.  $(x+2)(x+4) < 35$

c.  $(x-3)(x-7) \geq 8x - 56$

d.  $(x-3)^2 > (x-1)(x+6) - x^2 - 3x$

4) Solve the following sets of inequalities:

a.  $x^2 - 7x + 6 \leq 0$

and

$3x - 1 < 11$

b.  $x^2 - 8x + 15 > 0$

and

$x^2 - 36 < 0$

c.  $x^2 - 19x + 90 \geq 0$

or

$x^2 - x - 2 \leq 0$

d.  $x^2 - 3x - 4 < 0$

or

$x^2 - 1 > 0$

5) Solve the following inequalities:

a.  $(x-1)(x-2)(x-3) > 0$

b.  $x(x^2 + x + 1) > 0$

c.  $(-2x^2 - 3x + 2)(x+1) \leq 0$

d.  $(x^2 + 6)(x+3) > 0$

e.  $(x^2 + 8x + 20)(3x - 5) \leq 0$

6) Solve the following inequalities:

a.  $x^3 - 25x \geq 0$

b.  $x^4 > x^2$

c.  $x^3 - 6x^2 + 9x \leq 0$

d.  $4x^3 - 4x^2 + x \leq 0$

e.  $3x^4 + 4x^3 > -x^2$

7) Solve the following sets of inequalities:

a.  $3x+1 > 0$  and  $(x-2)(2x-1)(3x-2) < 0$

b.  $x^2 < 2x$  or  $(x-3)(x-6)(x-9) > 0$

**Answer Key:**

1) a.  $-12 < x < 12$       b.  $0 < x < 4$       c.  $x < 5$  or  $x > 5$       d.  $x < 4$  or  $x > 8$

2) a. All  $x$ .      b. All  $x$ .      c.  $x < -2$  or  $x > 15$       d.  $x < -\frac{1}{2}$  or  $x > 4$

3) a.  $-5 < x < -2$       b.  $-9 < x < 3$       c.  $x \leq 7$  or  $x \geq 11$       d.  $x < 3$  or  $x > 5$

4) a.  $1 \leq x < 4$       b.  $-6 < x < 3$  or  $5 < x < 6$       c.  $x \leq 9$  or  $x \geq 10$   
d.  $x < -1$  or  $x > -1$

5) a.  $1 < x < 2$  or  $x > 3$       b.  $x > 0$       c.  $-2 \leq x \leq -1$  or  $x \geq \frac{1}{2}$

d.  $x > -3$       e.  $x \leq \frac{1}{2}$

6) a.  $-5 \leq x \leq 0$  or  $x \leq 5$       b.  $x < -1$  or  $x > 1$       c.  $x \leq 0$  or  $x = 3$

d.  $x \leq 0$  or  $x = \frac{1}{2}$       e.  $x < -1$  or  $-\frac{1}{3} < x < 0$  or  $x > 0$

7) a.  $-\frac{1}{3} < x < \frac{1}{2}$  or  $\frac{2}{3} < x < 2$       b.  $0 < x < 2$  or  $3 < x < 6$  or  $x > 9$

## Rational Inequalities

### Questions:

1) Solve the following inequalities:

a.  $\frac{2x-1}{x-5} \leq 0$

b.  $\frac{1}{x^2-16} > 0$

c.  $\frac{1}{-3(x-1)} < 0$

d.  $\frac{x-1}{x^2-9} > 0$

e.  $\frac{5-2x}{(x-8)^2} \leq 0$

2) Solve the following inequalities:

a.  $\frac{(x-4)(x+2)}{x-1} < 0$

b.  $\frac{1}{x^2-8x+12} \geq 0$

c.  $\frac{x-3}{2x^2-10x+12} > 0$

d.  $\frac{(x-5)(3x+1)}{(2-x)(x+7)} < 0$

e.  $\frac{(x-6)^2(x+1)}{x-2} > 0$

3) Solve the following inequalities:

a.  $\frac{x-1}{3x+2} \geq -3$

b.  $\frac{x-3}{x-2} < 2$

c.  $\frac{x-3}{x^2-3x+2} < 3$

d.  $\frac{2x+7}{x^2-6x+8} \geq 1$

e.  $\frac{7x^2+1x}{3x-4} \geq 2x-10$

4) Solve the following inequalities:

a.  $\frac{25}{x^2} - \frac{x+7}{x} \geq \frac{3}{x} - 2$

b.  $\frac{x-10}{x} + \frac{2}{3} < \frac{x-6}{x-2}$

5) Solve the following inequalities:

a.  $\frac{2x+7}{5-x} < 7 - \frac{6x}{x+3}$

b.  $\frac{3}{3x-2} - \frac{1}{4x+1} \geq \frac{2x}{9x-6}$

6) Solve the following inequalities:

a.  $\frac{x+3}{x} + \frac{x-4}{5} \leq \frac{x^2-4x+4}{5x-20}$

b.  $\frac{x}{x^2-4} + \frac{1}{x+2} < \frac{1}{x-2}$

7) Solve the following inequalities:

a.  $\frac{4}{2x+4} + \frac{3}{3x-6} < \frac{4}{x^2-4}$

b.  $\frac{x}{x+5} - \frac{1}{x^2+x-20} > \frac{1}{x-4}$

8) Solve the following inequalities:

a.  $\frac{27}{x^2-6x+9} + \frac{10x}{3x-9} < 3$

b.  $\frac{x-7}{x-1} - \frac{3x}{4x+28} \geq \frac{3}{x^2+6x-7}$

**Answer Key:**

1) a.  $\frac{1}{2} \leq x < 5$

b.  $x < -4$  or  $x > 4$     c.  $x > 1$

d.  $-3 < x < 1$  or  $x > 3$

e.  $2\frac{1}{2} \leq x < 8$  or  $x > 8$

2) a.  $x < -2$  or  $1 < x < 4$

b.  $x < 2$  or  $x > 6$     c.  $x > 2$  and  $x \neq 3$

alternative:  $2 < x < 3$  or  $x > 3$

d.  $x < -7$  or  $-\frac{1}{3} < x < 2$  or  $x > 5$

e.  $x < -1$  or  $(x > 2$  and  $x \neq 6)$

3) a.  $x < -\frac{2}{3}$  or  $x \geq -\frac{1}{2}$

b.  $x < 1$  or  $x > 2$     c.  $x < 1$  or  $x > 2$

d.  $1 \leq x < 2$  or  $4 < x \leq 7$

e.  $-40 \leq x \leq 1$  or  $x \geq \frac{4}{3}$

4) a.  $x \neq 0$

b.  $0 < x < 2$  or  $5 < x < 6$

5) a.  $x < -12$  or  $-3 < x < 2\frac{1}{3}$  or  $x > 5$

b.  $-\frac{1}{2} \leq x < -\frac{1}{4}$  or  $\frac{2}{3} < x \leq 2\frac{1}{2}$

6) a.  $-12 \leq x < 0$  or  $4 < x \leq 5$

b.  $x < -2$  or  $2 < x < 4$

7) a.  $x < -2$

b.  $x < -5$  or  $-1 < x < 4$  or  $x > 6$

8) a.  $x < -24$  or  $0 < x < 3$  or  $x > 3$

b.  $x \leq -16$  or  $-7 < x < 1$  or  $x > 13$

**Absolute Value Inequalities**

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**Questions:**

1) Solve the following sets of inequalities:

a.  $|x-4| < 5$

b.  $|x+3| < 7$

c.  $|3x-6| < 18$

d.  $|5x+16| < 41$

2) Solve the following sets of inequalities:

a.  $|14-x| < 7$

b.  $|12-x| < x$

c.  $|x^2-x| < 6$

d.  $|x^2+2x| < 8$

3) Solve the following sets of inequalities:

a.  $|2x+1| < 7$       and       $|x+1| > 3$

b.  $|x+2| < x$  and       $|x+3| > 1$

c.  $|x^2+1| > 2$       and       $|x^2-2x| < 3$

**Answer Key:**

- 1) a.  $-1 < x < 9$       b.  $-10 < x < 4$       c.  $-4 < x < 8$       d.  $-11\frac{2}{5} < x < 5$
- 2) a.  $7 < x < 21$       b.  $x > 6$       c.  $-2 < x < 3$       d.  $-4 < x < 2$
- 3) a.  $2 < x < 3$       b. No such  $x$ .      c.  $1 < x < 3$