Polynomial and Rational Functions

Polynomial Division

Questions:

1) Use long division to perform the following division problems:

a. Divide $4x^3+5x^2-x-11$ by $4x^2+x-6$.

b. Divide $3x^4+10x^3+17x^2+10x+24$ by $3x^2+14x+8$.

2) Use long division to evaluate the following problems:

a. $\dfrac{x^5-8x^4+15x^3+20x^2-76x+48}{x^2-x-6}$

b. $\dfrac{x^5-8x^4+15x^3+20x^2-76x+48}{x^3-2x^2-5x+6}$

3) Use long division to solve the following division problems:

a. Divide $3x^3+3x^2+3x+1$ by $x+6$.

b. Divide $x^3+12x^2+10x+8$ by $4x^2+3x+1$.

4) Use long division to solvethe following division problems:

a. Divide $x^4+4x^3+3x^2-4x-3$ by $x^2+10x-9$.

b. Divide $3x^4-12x^3+8x^2-10x+2$ by x^3+x^2+7x+1.

5) Use long division to solve the following division problems:

a. Divide $x^3+x^2-10x+8$ by x^2+3x-4

b. Divide $2x^4-x^3-14x^2+19x-6$ by $2x^2-3x+1$

6) Use long division to evaluate the following:

a. $\dfrac{2x^6-13x^5-31x^3+31x^4+8x-4+7x^2}{(2x+1)(x-2)}$

b. $\dfrac{2x^7+25x^4+120x^3+270x+270x^2+81}{(x-1)^2}$

7) Use long division to evaluate the following:

a. $\dfrac{x^4-ax^3+x-a}{x-a}$

b. $\dfrac{x^3+(1-b)x^2-(2+b)x+2b}{x-2+x^2}$

Answer Key:

1) a. $x+1+\dfrac{4x-5}{4x^2+x-6}$ b. $x^2-\dfrac{4}{3}x+9\dfrac{2}{9}-\dfrac{108\dfrac{4}{9}x+49\dfrac{7}{9}}{3x^2+14x+8}$

2) a. $x^3-7x^2+14x-8$ b. x^2-6x+8

3) a. $3x^2-15x+93-\dfrac{557}{x+6}$ b. $4x^2+3x+1+\dfrac{21x+83}{16(4x^2+3x+1)}$

4) a. $x^2-6x+72+\dfrac{-778x+645}{x^2+10x-9}$ b. $3x-15+\dfrac{2x^2+92x+17}{x^3+x^2-7x+1}$

5) a. $x-2$ b. x^2+x-6

6) a. $x^4-5x^3+9x^2-7x+2$ b. $2x^5+4x^4+6x^3+33x^2+80x-43$

7) a. x^3+1 b. $x-b$

Rationals

Questions

1) In sections a-c, reduce the rational expression to the lowest term possible:

a. $\dfrac{x^2+4x}{2x^2-5x}$ b. $\dfrac{x-1}{x^2+2x-3}$ c. $\dfrac{x^2-2x-8}{x^2-4}$

2) In sections a–d, perform the indicated operation,
and reduce the rational expression to the lowest term possible:

a. $\dfrac{x^2+x-2}{x^2-x-6}\cdot\dfrac{x^2+2x-15}{x^2+3x-4}$ b. $\dfrac{x^2-9}{x^2+4x-21}:\dfrac{x^2+2x-3}{x^2+9x+14}$

c. $\dfrac{x^2-4x+3}{x^2+4x-5}:\dfrac{x^2+6x-27}{x^2+14x+45}$ d. $\dfrac{\dfrac{3}{x^2-4}}{\dfrac{x+5}{x^2+8x-20}}$

3) In sections a–f, perform the indicated operations:

a. $\dfrac{1}{x-4}+\dfrac{3}{x+4}-\dfrac{5}{x^2-4}$ b. $\dfrac{3}{x}+\dfrac{1}{x^2}+\dfrac{3}{x+1}$

c. $\dfrac{x}{x^2+11x+30}+\dfrac{x-1}{x+5}$ d. $\dfrac{1}{x^2-6x-7}+\dfrac{x+1}{x-7}+\dfrac{x^2}{x+1}$

e. $\dfrac{3}{x-4}+\dfrac{5}{x+1}-\dfrac{2}{x}$ f. $\dfrac{x+1}{(2x+3)^3}+\dfrac{x}{(2x+3)^2}$

Answer Key

1) a. $\frac{x^2+4x}{2x^2-5x}$ b. $\frac{x-1}{x^2+2x-3}$ c. $\frac{x^2-2x-8}{x^2-4}$

2) a. $\frac{x^2+x-2}{x^2-x-6}\cdot\frac{x^2+2x-15}{x^2+3x-4}$ b. $\frac{x^2-9}{x^2+4x-21}:\frac{x^2+2x-3}{x^2+9x+14}$

c. $\frac{x^2-4x+3}{x^2+4x-5}:\frac{x^2+6x-27}{x^2+14x+45}$ d. $\frac{\frac{3}{x^2-4}}{\frac{x+5}{x^2+8x-20}}$

3) a. $\frac{1}{x-4}+\frac{3}{x+4}-\frac{5}{x^2-4}$ b. $\frac{3}{x}+\frac{1}{x^2}+\frac{3}{x+1}$ c. $\frac{x}{x^2+11x+30}+\frac{x-1}{x+5}$

d. $\frac{1}{x^2-6x-7}+\frac{x+1}{x-7}+\frac{x^2}{x+1}$ e. $\frac{3}{x-4}+\frac{5}{x+1}-\frac{2}{x}$ f. $\frac{x+1}{(2x+3)^3}+\frac{x}{(2x+3)^2}$

Roots (Zeros) of Polynomials

Questions:

1) Solve the following equations:

a. $x^3-2x^2+x-2=0$

b. $x^3+2x^2-5x-6=0$

c. $x^3-2x^2-5x+6=0$

d. $x^4-x^3-7x^2+x+6=0$

e. $x^4-x^3-11x^2+9x+18=0$

f. $x^3-7x^2+14x-8=0$

g. $x^3+x^2-17x+15=0$

2) Solve the following equations:

a. $2x^3+3x^2-8x+3=0$

b. $2x^3+x^2-2x-1=0$

c. $4x^3+5x^2-7x-2=0$

3) Solve the following equations (knowledge about the derivative is needed):

a. $x^3+x^2-5x+3=0$

b. $x^3-3x-2=0$

c. $x^5-3x^4-5x^3+27x^2-32x+12=0$

d. $x^3-3x^2+3x-1=0$

4) Answer the following questions:

a. Solve the equation $x^3-x^2-2x+2=0$.

b. Find all the zeroes of the polynomial $p(x)=x^3-x^2-2x+2$.

5) Answer the following questions:

a. Solve the equation $x^3-4x^2+5x-2=0$.

b. Find all the zeroes of the polynomial $f(x)=x^3-4x^2+5x-2$.

6) Solve the equations:

a. $x^3-x^2-x-2=0$

b. $x^4-x^3-5x^2+3x+6=0$

7) For each polynomial, list all its zeros and give their multiplicities:

a. $p(x)=x^4+3x^3+3x^2-x-6$

b. $q(x)=x^5-5x^4+9x^3-9x^2+8x-4$

Answer Key:

1) a. $x=2 \;\; z=2,\pm i$ b. $x=-1,2,-3$ c. $x=1,-2,3$ d. $x=\pm 1,-2,3$

e. $x=-1,2,\pm 3$ f. $x=1,2,4$ g. $x=1,3,-5$

2) a. $x=1,3,-\frac{1}{2}$ b. $x=-1,-\frac{1}{2}$ c. $x=-\frac{1}{4},-2$

3) a. $x=1,-3$ b. $x=-1,2$ c. $x=1,2,-3$ d. $x=1$

4) a. $x=1,\pm\sqrt{2}$ b. $x=1,\pm\sqrt{2}$

5) a. $x=1,2$ b. $x=1,2$

6) a. $x=2$, $z=-\frac{1}{2}+\frac{\sqrt{3}}{2}i,-\frac{1}{2}-\frac{\sqrt{3}}{2}i$ b. $x=-1,2,\pm\sqrt{3}$

7) a. $x=1,-2$, $z=-1\pm\sqrt{2}i$ b. $x=1,2$, $z=\pm i$

Partial Fractions

Questions:

Determine the partial fractions decomposition of the following expressions:

1) $\frac{1}{x^2-4}$

2) $\frac{5-x}{x^2+5x}$

3) $\frac{x}{x^2+5x+6}$

4) $\frac{8x-1}{2x^2-3x-2}$

5) $\frac{x+4}{(x-1)^2}$

6) $\frac{6-x}{x^2+8x+16}$

7) $\frac{x^2+x-1}{x^3-x}$

8) $\frac{10x}{x^4-13x^2+36}$

9) $\frac{8x}{(x-2)^2(x+2)}$

10) $\frac{5-x}{x^3+x^2}$

11) $\frac{9x+36}{x^3+6x^2+9x}$

12) $\frac{1}{(x^2-2x+1)(x^2-4x+4)}$

13) $\frac{x+4}{(x-1)^3}$

14) $\frac{6x^2-4x+1}{(x-1)^3}$

15) $\frac{2x^2+2x+1}{(x^2+1)(x+2)}$

16) $\frac{2x^2+x-1}{(x^2+1)(x-3)}$

17) $\frac{3}{(x^2+1)(x^2+4)}$

18) $\frac{1}{x(x^2+1)^2}$

Answer Key:

1) $\frac{1/4}{x-2}-\frac{1/4}{x+2}$

2) $\frac{1}{x}-\frac{2}{x+5}$

3) $\frac{3}{x+3}-\frac{2}{x+2}$

4) $\frac{2}{2x+1}+\frac{3}{x-2}$

5) $\frac{1}{x-1}+\frac{5}{(x-1)^2}$

6) $-\frac{1}{x+4}+\frac{10}{(x+4)^2}$

7) $\frac{1}{x}+\frac{1/2}{x-1}-\frac{1/2}{x+1}$

8) $\frac{1}{x+3}+\frac{1}{x-3}-\frac{1}{x+2}-\frac{1}{x-2}$

9) $\frac{1}{x-2}+\frac{4}{(x-2)^2}-\frac{1}{x+2}$

10) $-\frac{6}{x}+\frac{5}{x^2}+\frac{6}{x+1}$

11) $\frac{4}{x}-\frac{4}{x+3}-\frac{3}{(x+3)^2}$

12) $\frac{2}{x-1}+\frac{1}{(x-1)^2}-\frac{2}{x-2}+\frac{1}{(x-2)^2}$

13) $\frac{1}{(x-1)^2}+\frac{5}{(x-1)^3}$

14) $\frac{6}{x-1}+\frac{8}{(x-1)^2}+\frac{3}{(x-1)^3}$

15) $\frac{x}{x^2+1}+\frac{1}{x+2}$

16) $\frac{1}{x^2+1}+\frac{2}{x-3}$

17) $\frac{1}{x^2+1}-\frac{1}{x^2+4}$

18) $\frac{1}{x}-\frac{x}{x^2+1}-\frac{x}{(x^2+1)^2}$

End Behavior of Polynomial Functions

Questions

1) Fill in the following table:

$p(x)$	$\lim_{x\to\infty} p(x)$	$\lim_{x\to-\infty} p(x)$
$2x-3$		
$5-2x$		
7		
$3x^2-5x-7$		
$3x^2-5x^4-7$		
$3x^3-5x-7$		
$3x^2-5x-7x^5$		

2) Match each polynomial function with the correct graph:

a. $y=ax^3+bx^2+cx-2,\ a<0$

b. $y=ax^2+bx-2,\ a>0$

c. $y=ax^3+bx^2+cx-2,\ a>0$

d. $y=ax-3,\ a<0$

e. $y=ax^2+bx-2,\ a<0$

f. $y=ax-3,\ a>0$

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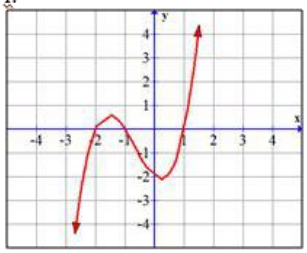

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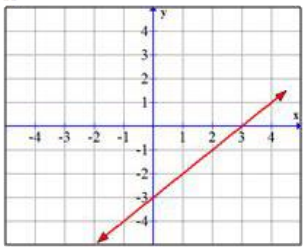

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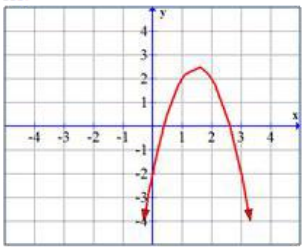

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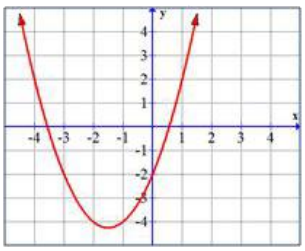

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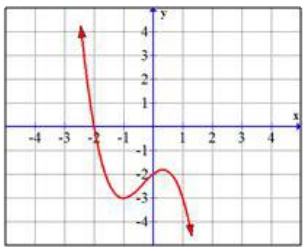

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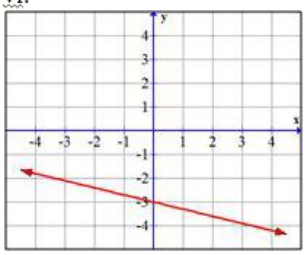


3) Match each polynomial function with the correct graph:

a. $y=-2x+3$

b. $y=x^2-4x$

c. $y=-2x^2-5x$

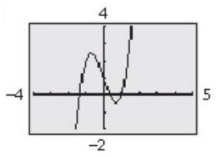
d. $y=2x^3-3x+1$

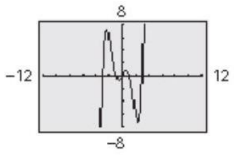
e. $y=-\frac{1}{4}x^4+3x^2$

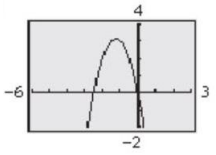
f. $y=-\frac{1}{3}x^3+x^2-\frac{4}{3}$

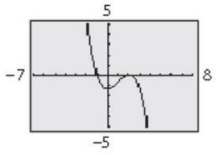
g. $y=x^4+2x^3$

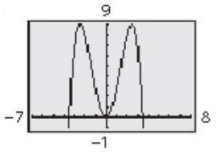
h. $y=\frac{1}{5}x^5-2x^3+\frac{9}{5}x$

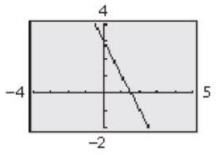
1. 

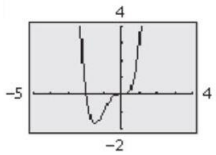
2. 

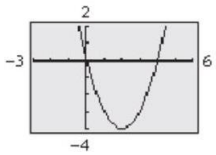
3. 

4. 

5. 

6. 

7. 

8. 

4) Match each polynomial function with the correct graph:

a. $y=x-5$

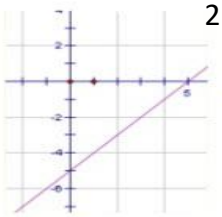
b. $y=x^2-4x$

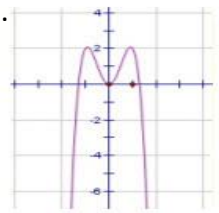
c. $y=-3x^4+5x^2$

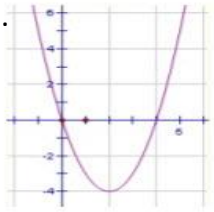
d. $y=-2x^5+x^4-2x^3+5x-2$

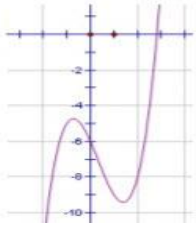
e. $y=-\frac{1}{2}x^2+3x-5$

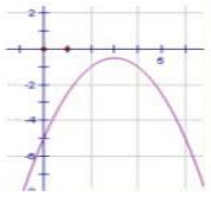
f. $y=x^3-x^2+3x-6$

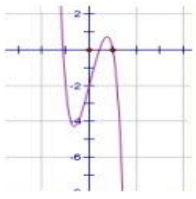
1. 

2. 

3. 

4. 

5. 

6. 

5) Match each polynomial function with the correct graph:

a. $y = x^3 - x$

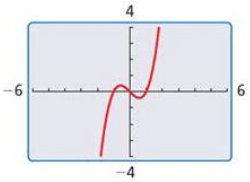
b. $y = -x^3 + x$

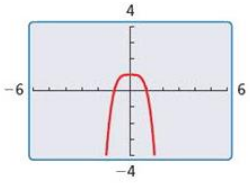
c. $y = -x^4 + 1$

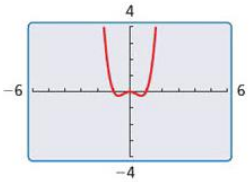
d. $y = x^4$

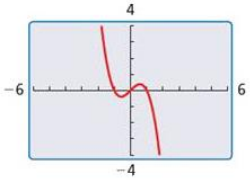
e. $y = x^3$

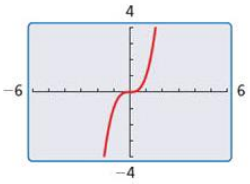
f. $y = x^4 - x^2$

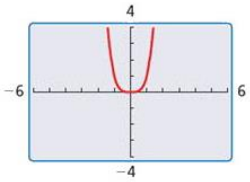
1. 

2. 

3. 

4. 

5. 

6. 

Answer Key

1)

$p(x)$	$\lim_{x\to\infty} p(x)$	$\lim_{x\to-\infty} p(x)$
$2x-3$	∞	$-\infty$
$5-2x$	$-\infty$	∞
7	7	7
$3x^2-5x-7$	∞	∞
$3x^2-5x^4-7$	$-\infty$	$-\infty$
$3x^3-5x-7$	∞	$-\infty$
$3x^2-5x-7x^5$	$-\infty$	∞

2) a. v b. iv c. i d. vi e. iii f. ii

3) a. 6 b. 8 c. 3 d. 1 e. 5 f. 4

4) a. 1 b. 3 c. 2 d. 6 e. 5 f. 4

5) a. 1 b. 4 c. 2 d. 6 e. 5 f. 3

Graphing Rational Functions

Questions

1) Find any vertical asymptotes for the graph of:

a. $f(x)=\dfrac{2x}{x^2-9}$ b. $f(x)=\dfrac{x}{x^2+1}$

c. $f(x)=\dfrac{x+4}{x^2+2x-3}$ d. $f(x)=\dfrac{5}{x^3-27}$

2) Find any vertical asymptotes for the graph of $f(x)=\dfrac{x^2+2x}{x^2-4}$.

Hint: is the rational expression in its lowest terms?

3) Find any horizontal asymptotes for the graph of:

a. $f(x)=\dfrac{4x^2}{2x^2-3}$ b. $f(x)=\dfrac{x^2}{x+4}$

c. $f(x)=\dfrac{3x}{x^2-x-2}$ d. $f(x)=\dfrac{2x^2-3x+4}{5x^2+6x-7}$

4) Find any oblique asymptotes for the graph of:

a. $f(x)=\dfrac{x^2}{x-3}$ b. $f(x)=\dfrac{x^3}{2x-3}$

c. $f(x)=\dfrac{x^2-3x-4}{2x-3}$ d. $f(x)=\dfrac{x^3+x^2}{x^2+x-2}$

5) Sketch a graph of $f(x)=\dfrac{3}{x-3}$.

6) Sketch a graph of $f(x)=1-\dfrac{1}{x^2}$.

7) Sketch a graph of $f(x)=\dfrac{x-4}{x+2}$.

8) Sketch a graph of $f(x)=\dfrac{3x}{9-x^2}$.

9) Sketch a graph of $f(x)=\dfrac{x^2}{x^2-1}$.

10) Sketch a graph of $f(x)=\dfrac{x^3}{18-2x^2}$.

11) Sketch a graph of $f(x)=\dfrac{x^2}{x^2+x-2}$.

12) Sketch a graph of $f(x)=\dfrac{4-x^2}{x^2+1}$.

13) Analyze and sketch a graph of $f(x)=\dfrac{x^2-9}{2x^2-9x+9}$.

Note: the numerator and denominator here have factors in common.

Answer Key

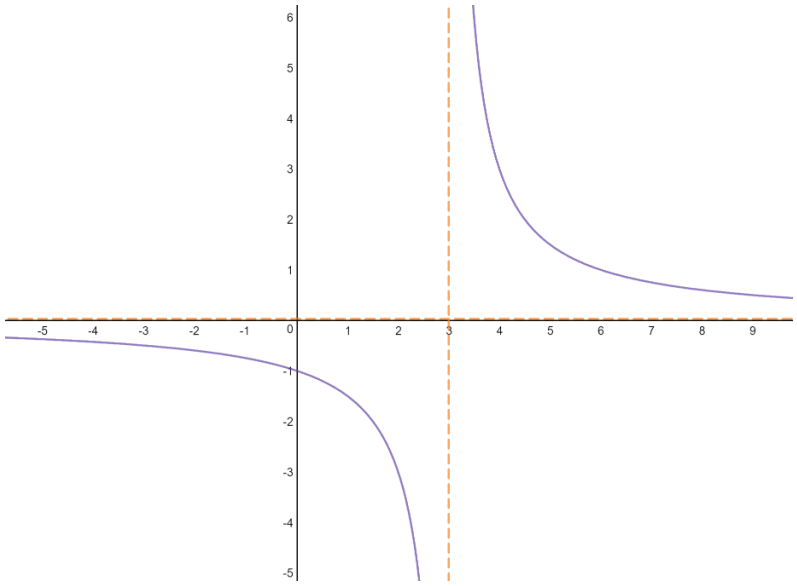
1) a. $x=\pm 3$ b. There are no vertical asymptotes. c. $x=1,\ x=-3$ d. $x=3$

2) $x=2$

3) a. $y=2$ b. There are no horizontal asymptotes. c. $y=0$ (the *x*-axis) d. $y=\frac{2}{5}$

4) a. $y=x+3$ b. There are no oblique asymptotes. c. $y=\frac{1}{2}x-\frac{3}{4}$ d. $y=x$

5)



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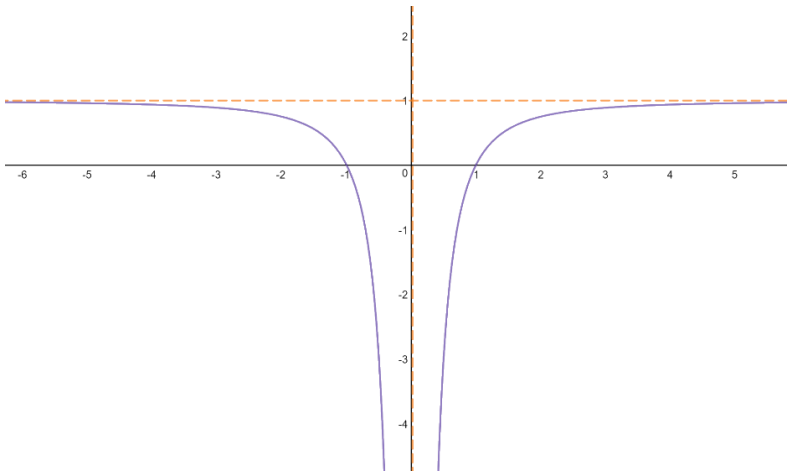

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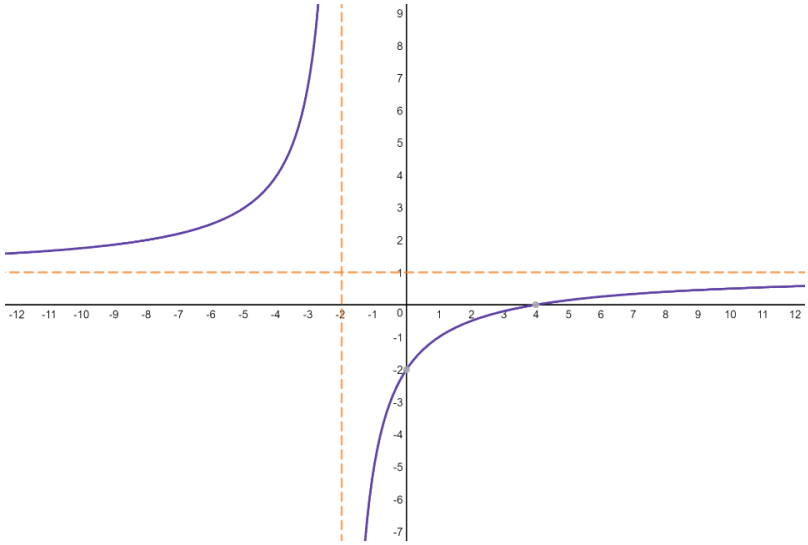

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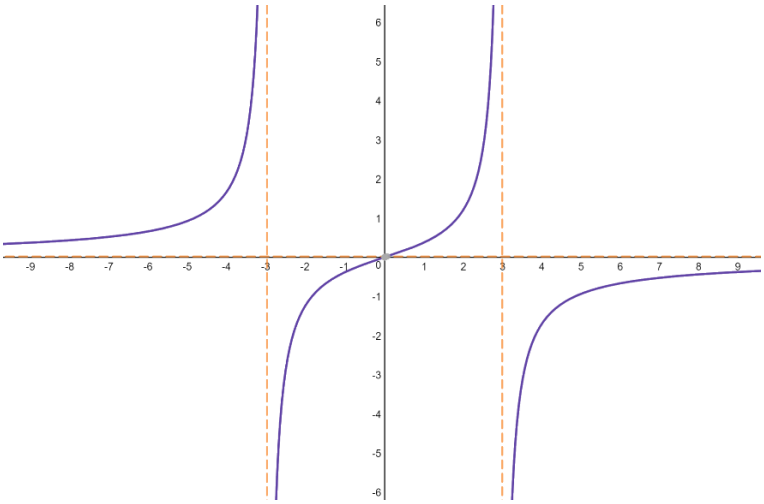

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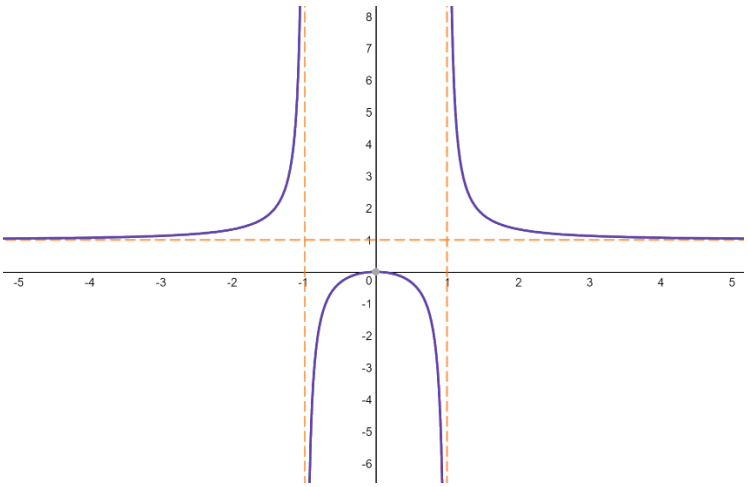

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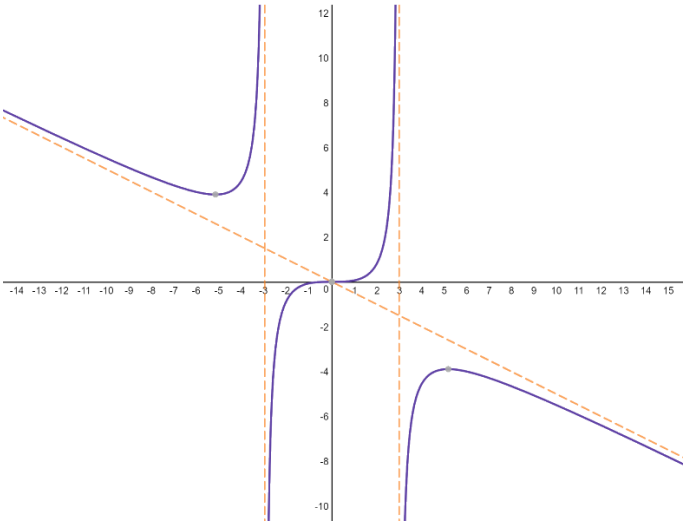

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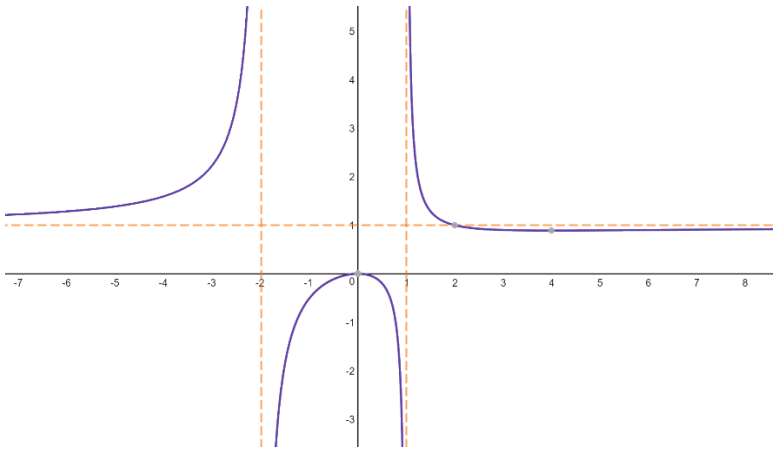

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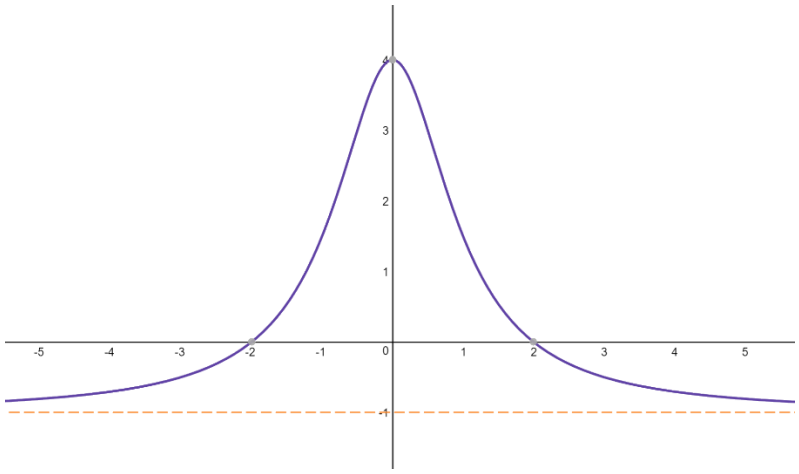

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