

Trigonometry

Trigonometric Functions

Questions

- 1) Answer the following questions:
 - a. Evaluate the six trigonometric functions of 0 .
 - b. Evaluate the six trigonometric functions of $\frac{\pi}{4}$.
 - c. Evaluate the six trigonometric functions of $\frac{\pi}{2}$.
 - d. Evaluate the six trigonometric functions of $\frac{3\pi}{4}$.
 - e. Evaluate the six trigonometric functions of $-\frac{\pi}{4}$.

- 2) Answer the following questions:
 - a. Given $\sin \theta = \frac{2}{5}$ and θ in quadrant I, evaluate the other five trigonometric functions of θ .
 - b. Given $\cos \alpha = -\frac{1}{2}$ with α in quadrant II, evaluate the other five trigonometric functions of α .
 - c. Given $\tan t = -\frac{3}{5}$ and t in quadrant IV, evaluate the other five trigonometric functions of t .

- 3) If $P(t)$ is a point on the unit circle with coordinates $\left(-\frac{8}{17}, -\frac{15}{17}\right)$, evaluate the six trigonometric functions of t .

Answer Key

1) Answers:

a. $\sin(0) = 0$, $\cos(0) = 1$, $\tan(0) = 0$, $\csc(0) = \cot(0) = \text{Undefined}$, $\sec(0) = 1$

b. $\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, $\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, $\tan\left(\frac{\pi}{4}\right) = 1$, $\csc\left(\frac{\pi}{4}\right) = \sqrt{2}$, $\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$, $\cot\left(\frac{\pi}{4}\right) = 1$

c. $\sin\left(\frac{\pi}{2}\right) = 1$, $\cos\left(\frac{\pi}{2}\right) = 0$, $\tan\left(\frac{\pi}{2}\right) = \text{Un.}$, $\csc\left(\frac{\pi}{2}\right) = 1$, $\sec\left(\frac{\pi}{2}\right) = \text{Un.}$, $\cot\left(\frac{\pi}{2}\right) = 0$

d. $\sin\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}$, $\cos\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}}$, $\tan\left(\frac{3\pi}{4}\right) = -1$, $\csc\left(\frac{3\pi}{4}\right) = \sqrt{2}$, $\sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$, $\cot\left(\frac{3\pi}{4}\right) = -1$

e. $\sin\left(\frac{-\pi}{4}\right) = -\frac{1}{\sqrt{2}}$, $\cos\left(\frac{-\pi}{4}\right) = \frac{1}{\sqrt{2}}$, $\tan\left(\frac{-\pi}{4}\right) = -1$, $\csc\left(\frac{-\pi}{4}\right) = -\sqrt{2}$, $\sec\left(\frac{-\pi}{4}\right) = \sqrt{2}$, $\cot\left(\frac{-\pi}{4}\right) = -1$

2) a. $\cos \theta = \frac{\sqrt{21}}{5}$, $\tan \theta = \frac{2}{\sqrt{21}}$, $\csc \theta = \frac{5}{2}$, $\sec \theta = \frac{5}{\sqrt{21}}$, $\cot \theta = \frac{\sqrt{21}}{2}$

b. $\sin \alpha = \frac{\sqrt{3}}{2}$, $\tan \alpha = -\sqrt{3}$, $\csc \alpha = \frac{2}{\sqrt{3}}$, $\sec \alpha = -2$, $\cot \alpha = -\frac{1}{\sqrt{3}}$

c. $\sin t = -\frac{3}{\sqrt{34}}$, $\cos t = \frac{5}{\sqrt{34}}$, $\csc t = -\frac{\sqrt{34}}{3}$, $\sec t = \frac{\sqrt{34}}{5}$, $\cot t = -\frac{5}{3}$

3) $\sin \theta = -\frac{15}{17}$, $\cos \theta = -\frac{8}{17}$, $\tan \theta = \frac{15}{8}$, $\csc \theta = -\frac{17}{15}$, $\sec \theta = -\frac{17}{8}$, $\cot \theta = \frac{8}{13}$

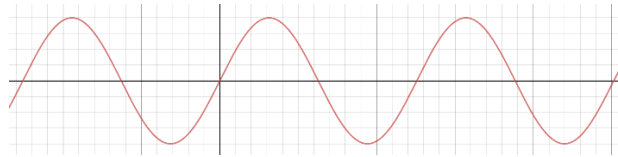
Graphs of Trigonometric Functions

Questions

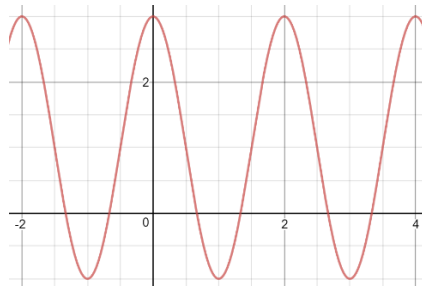
- 1) Sketch the graph of $y = 4\sin\left(\frac{1}{2}x\right)$.
- 2) Sketch the graph of $u = 2\cos\pi t + 1$.
- 3) Sketch the graph of $y = 2\sin(4x - \pi)$.
- 4) Sketch the graph of $u = -\frac{2}{3}\cos\left(3t - \frac{\pi}{4}\right) + 1$.
- 5) Sketch the graph of $y = |\sin x|$.
- 6) Sketch the graph of $y = \tan\left(x + \frac{\pi}{3}\right)$.
- 7) State the amplitude and period of $u = \cos\pi t$ and sketch the graph.
- 8) State the amplitude and period of $y = 2\sin x - 3$ and sketch the graph.

Answer Key

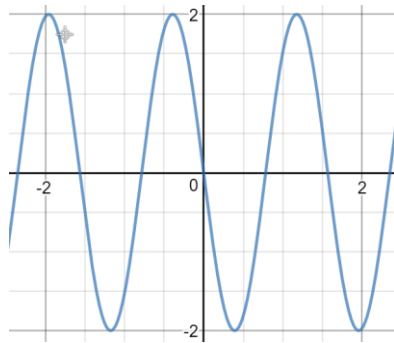
1)



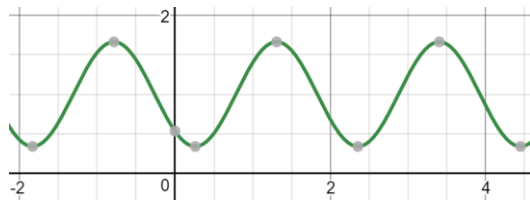
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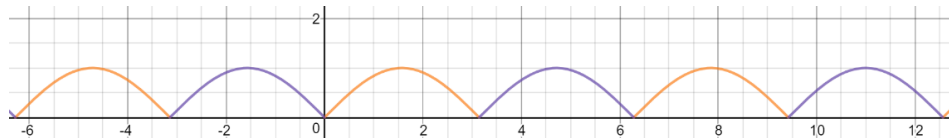
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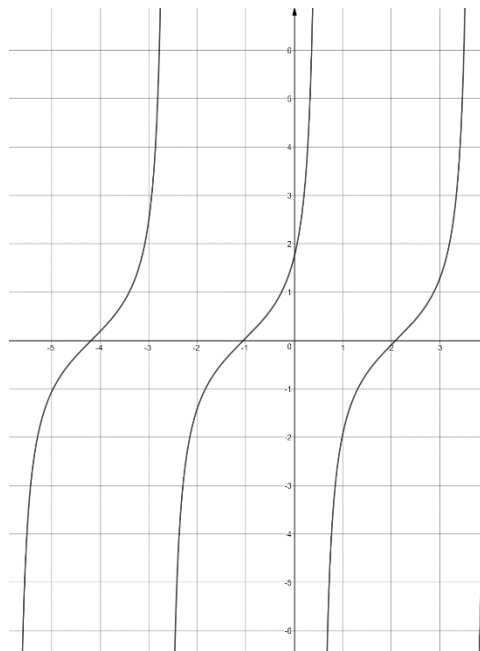
4)



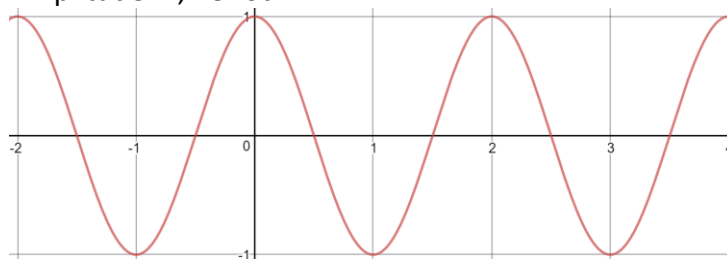
5)



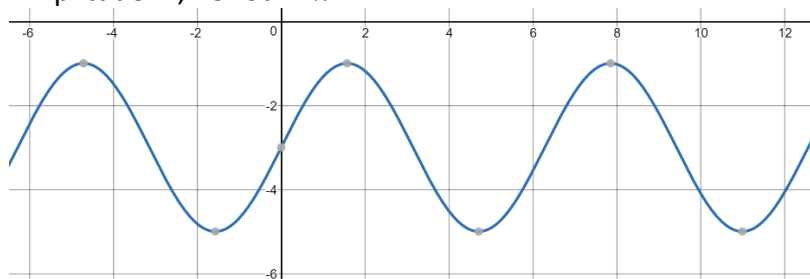
6)



7) Amplitude: 1, Period: 2π .



8) Amplitude: 2, Period: 2π .



Angles

Questions

- 1) List all angles coterminal with:
 - a. 50° ;
 - b. $\frac{3\pi}{4}$ radians.

- 2) Find the six trigonometric functions of:
 - a. 720° ;
 - b. -180° .

- 3) Find an angle supplementary to α , if:
 - a. $\alpha = \frac{\pi}{6}$
 - b. $\alpha = 42^\circ 25'$

- 4) Convert 4 radians to degrees, minutes, and seconds.

- 5) Convert $563^\circ 24' 54''$ to radians.

- 6) Given an angle θ in standard position with $(12, -35)$ on its terminal side, compute the six trigonometric functions of θ .

- 7) Compute the six trigonometric functions of:
 - a. 45°
 - b. 30°
 - c. 60°

Answer Key

- 1) a. $50^\circ + n \cdot 360$ b. $\frac{3\pi}{4} + 2\pi \cdot n$
- 2) a. $\sin(0) = 0$, $\cos(0) = 1$, $\tan(0) = 0$, $\csc(0) = \cot(0) = \text{Undefined}$, $\sec(0) = 1$
b. $\sin(\pi) = 0$, $\cos(\pi) = -1$, $\tan(\pi) = 0$, $\csc(\pi) = \cot(\pi) = \text{Undefined}$, $\sec(\pi) = -1$
- 3) a. $\frac{5\pi}{6}$ b. $137^\circ 35'$
- 4) Degrees: 229.183118 , minutes: 10.9870 , seconds: 59.2 .
- 5) 9.8334
- 6) $\sin \theta = \frac{-35}{37}$, $\cos \theta = \frac{12}{37}$, $\tan \theta = \frac{-35}{12}$, $\csc \theta = -\frac{37}{35}$, $\sec \theta = \frac{37}{12}$, $\cot \theta = -\frac{12}{35}$
- 7) a. $\sin 45^\circ = \frac{1}{\sqrt{2}}$, $\cos 45^\circ = \frac{1}{\sqrt{2}}$, $\tan 45^\circ = 1$, $\csc 45^\circ = \sqrt{2}$, $\sec 45^\circ = \sqrt{2}$, $\cot 45^\circ = 1$
b. $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\tan 30^\circ = \frac{1}{\sqrt{3}}$, $\csc 30^\circ = 2$, $\sec 30^\circ = \frac{2}{\sqrt{3}}$, $\cot 30^\circ = \sqrt{3}$
c. $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\cos 60^\circ = \frac{1}{2}$, $\tan 60^\circ = \sqrt{3}$, $\csc 60^\circ = \frac{2}{\sqrt{3}}$, $\sec 60^\circ = 2$, $\cot 60^\circ = \frac{1}{\sqrt{3}}$

Trig Identities

Questions

- 1) Verify that $\sec t - \cos t = \tan t \sin t$, is an identity.
- 2) Verify that $\cos^4 \alpha - \sin^4 \alpha = \cos^2 \alpha - \sin^2 \alpha$, is an identity.
- 3) Verify that $\frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta} = 2 \sec^2 \theta$, is an identity.
- 4) Verify that $\frac{1 + \sin x}{\cos x} = \frac{\cos x}{1 - \sin x}$, is an identity.
- 5) Verify that $\frac{\cot u + \cot v}{\cot u \cot v - 1} = \frac{\sin u \cos v + \cos u \sin v}{\cos u \cos v - \sin u \sin v}$, is an identity.
- 6) Verify that $\frac{1}{\cos \alpha} - \cos \alpha = \frac{\sin^2 \alpha}{\cos \alpha}$, is an identity.
- 7) Verify that $\frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 + \cos x} = 2 \csc x$, is an identity.
- 8) Verify that $(\csc \varphi + \cot \varphi)^2 = \frac{1 + \cos \varphi}{1 - \cos \varphi}$, is an identity.
- 9) Verify that $\frac{\cos \beta}{\csc \beta - \sin \beta} = \tan \beta$, is an identity.
- 10) Verify that $\csc^2 x - (1 + \cot x)^2 = -2 \cot x$, is an identity.
- 11) Verify that $\frac{1}{\tan u} + \tan u = \frac{1}{\sin u \cos u}$, is an identity.
- 12) Verify that $\sin x(1 - 2 \cos^2 x + \cos^4 x) = \sin^5 x$, is an identity.
- 13) Show that $\frac{\cos^3 t - \sin^3 t}{\cos t - \sin t} = 1 + \cos t \sin t$, is an identity.

14) Show that $\frac{\cot x - \tan x}{\cot x + \tan x} = \cos^2 x - \sin^2 x$, is an identity.

15) Show that $\ln(\sec \alpha) = -\ln(\cos \alpha)$, is an identity.

Answer Key

Solutions in the recordings.

Advanced Trigonometric Identities

Questions

- 1) Name the quadrant in which the angle θ lies in each case:
- $\tan \theta < 0$, $\cos \theta < 0$
 - $\tan \theta > 0$, $\cos \theta < 0$
 - $\sin \theta < 0$, $\cos \theta < 0$
 - $\cos \theta > 0$, $\sin \theta < 0$
 - $\cos \theta < 0$, $\cot \theta > 0$
 - $\cot \theta < 0$, $\sin \theta < 0$
 - $\sin \theta < 0$, $\tan \theta < 0$
- 2) A point on the terminal side of an angle θ is given.
Find the exact value of the specified trigonometric function(s) of θ .
- $(3,1)$, find $\sin \theta$, $\cos \theta$.
 - $(-2,-1)$, find $\tan \theta$.
 - $(4,-5)$, find $\cos \theta$, $\sec \theta$.
 - $(-3,5)$, find $\cot \theta$, $\operatorname{cosec} \theta$.
- 3) Find the exact value of the specified trigonometric function(s) of θ .
- $\sin \theta = \frac{7}{25}$, θ is in quadrant I. Find $\cos \theta$ and $\tan \theta$.
 - $\cos \theta = \frac{4}{7}$, θ is in quadrant II. Find $\sin \theta$ and $\cot \theta$.
 - $\tan \theta = -\frac{5}{13}$, θ is in quadrant IV. Find $\cos \theta$ and $\sec \theta$.
 - $\sec \theta = -\frac{7}{15}$, θ is in quadrant III. Find $\cos \theta$ and $\tan \theta$.

4) Find the exact value of the expressions. Do not use a calculator.

- $\sin 45^\circ + \cos 45^\circ$
- $3 \sin 60^\circ - 6 \cos 30^\circ$
- $\frac{(\sin 30^\circ + 1)^2}{(\cos 30^\circ - 1)^2}$
- $\sin 90^\circ \cdot \cos 45^\circ + \sin 30^\circ \cdot \cos 90^\circ$

5) Find the exact value of the expressions. Do not use a calculator.

- $\tan 45^\circ \cdot \tan 30^\circ - \sin 45^\circ \cdot \sin 30^\circ$
- $\sqrt{2} \cos 45^\circ - \sqrt{3} \tan 60^\circ + \sqrt{18} \cot 60^\circ$
- $\frac{\sec 60^\circ - \csc 45^\circ}{\sin 60^\circ - \cos 45^\circ}$
- $\cot 60^\circ \csc 60^\circ - \sec 30^\circ \sin 30^\circ$

6) Using the sum and difference formulas, find the exact value of the expressions. Do not use a calculator.

- $\sin 15^\circ$
- $\cos 75^\circ$
- $\tan 15^\circ$ (Hint: use $\tan 15^\circ = \frac{\sin 15^\circ}{\cos 15^\circ}$).
- $\sec 75^\circ$

7) Find the exact value of the expressions. Do not use a calculator.

- $\sin \frac{\pi}{6} + \cos \frac{\pi}{4} + \tan \frac{\pi}{3} + \cot \frac{\pi}{2}$
- $\left(2 \cos \frac{\pi}{3} - \sin \frac{\pi}{3}\right) \left(\cos \frac{\pi}{3} + 2 \sin \frac{\pi}{3}\right)$
- $\frac{1}{\left(\sec \frac{\pi}{4} + \csc \frac{\pi}{4}\right)^2} + \frac{1}{\left(\tan \frac{\pi}{4} + \cot \frac{\pi}{4}\right)^2}$
- $\left(\sin \frac{\pi}{4} + \csc \frac{\pi}{2}\right)^2 - \left(\sec \frac{\pi}{4} + \cos \frac{\pi}{2}\right)^2$

8) Find the exact value of the expressions. Do not use a calculator.

- a. $\cos 135^\circ + \tan 210^\circ$
- b. $\sin 270^\circ - \cot 135^\circ$
- c. $\cos 120^\circ \cdot \sin 120^\circ - \tan 120^\circ \cdot \cot 120^\circ$
- d. $\sec 240^\circ - \cos 315^\circ + \cot 300^\circ$

9) Find the exact value under the given conditions:

- a. $\sin \alpha = \frac{4}{9}$, $0 < \alpha < \frac{\pi}{2}$; $\sin \beta = -\frac{3}{5}$, $\frac{\pi}{2} < \beta < \pi$, find: $\sin(\alpha + \beta)$ and $\cos(\alpha + \beta)$.
- b. $\sin \alpha = -\frac{4}{9}$, $\pi < \alpha < \frac{3\pi}{2}$; $\cos \beta = \frac{5}{12}$, $0 < \beta < \frac{\pi}{2}$, find: $\sin(\alpha - \beta)$ and $\cos(\alpha - \beta)$.
- c. $\cos \alpha = \frac{1}{10}$, $0 < \alpha < \frac{\pi}{2}$; $\sin \beta = -\frac{1}{5}$, $\frac{3\pi}{2} < \beta < 2\pi$, find: $\tan(\alpha - \beta)$.
 [Hint: use $\tan(\alpha - \beta) = \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)}$]
- d. $\cos \alpha = \frac{1}{3}$, $0 < \alpha < \frac{\pi}{2}$; $\cos \beta = -\frac{1}{3}$, $\frac{\pi}{2} < \beta < \pi$, find: $\sin(\alpha + \beta)$ and $\cos(\alpha - \beta)$.

10) Use the information about the given angle θ , $0 \leq \theta \leq 2\pi$ to find the exact value of the indicated trigonometric function:

- a. $\sin \theta = \frac{3}{5}$, $0 \leq \theta \leq \frac{\pi}{2}$, find: $\sin 2\theta$.
- b. $\cos \theta = \frac{3}{5}$, $0 \leq \theta \leq \frac{\pi}{2}$, find: $\cos 2\theta$.
- c. $\tan \theta = \frac{4}{3}$, $\pi < \theta < \frac{3\pi}{2}$, find: $\cos 2\theta$.
- d. $\sec \theta = -3$, $\frac{\pi}{2} < \theta < \pi$, find: $\sin 2\theta$.

11) Given an angle θ which satisfy: $\sin 2\theta = 0.8$ where: $\frac{\pi}{2} < \theta < \pi$. Find $\csc \theta$.

Answer Key

- 1) a. II b. III c. III d. IV e. III f. IV g. IV
- 2) a. $\sin \theta = \frac{1}{\sqrt{10}}$, $\cos \theta = \frac{3}{\sqrt{10}}$ b. $\frac{1}{2}$ c. $\cos \theta = \frac{4}{\sqrt{41}}$, $\sec \theta = \frac{\sqrt{41}}{4}$
 d. $\cot \theta = -\frac{3}{5}$, $\csc \theta = \sqrt{\frac{14}{5}}$
- 3) a. $\cos \theta = \frac{24}{25}$, $\tan \theta = \frac{7}{24}$ b. $\sin \theta = \frac{\sqrt{33}}{6}$, $\cot \theta = \frac{4}{\sqrt{33}}$
 c. $\cos \theta = \frac{13}{\sqrt{194}}$, $\sec \theta = \frac{\sqrt{194}}{13}$ d. Error.
- 4) a. $\sqrt{2}$ b. $\frac{-3\sqrt{3}}{2}$ c. $\frac{9}{7-4\sqrt{3}}$ d. $\frac{\sqrt{2}}{2}$
- 5) a. $\frac{\sqrt{3}}{3} - \frac{\sqrt{2}}{4}$ b. $\sqrt{6} - 2$ c. $4\sqrt{3} + 4\sqrt{2} - 2\sqrt{6} - 4$ d. $\frac{2\sqrt{3}}{9}$
- 6) a. $\frac{\sqrt{6} - \sqrt{2}}{4}$ b. $\frac{\sqrt{6} - \sqrt{2}}{4}$ c. $\frac{\sqrt{6} + \sqrt{2}}{4}$ d. $\frac{4}{\sqrt{6} - \sqrt{2}}$
- 7) a. $\frac{1 + \sqrt{2}}{2} + \sqrt{3}$ b. $-1 + \frac{3\sqrt{3}}{4}$ c. $\frac{3}{8}$ d. $-\frac{1}{2} + \sqrt{2}$
- 8) a. $-\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{3}$ b. 0 c. $-\frac{\sqrt{3}}{4} - 1$ d. $-\left(2 + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{3}\right)$
- 9) a. $\sin(\alpha + \beta) = \frac{3\sqrt{65} - 16}{45}$, $\cos(\alpha + \beta) = \frac{12 - 4\sqrt{65}}{45}$
 b. $\sin(\alpha - \beta) = \frac{\sqrt{65}\sqrt{119} - 20}{108}$, $\cos(\alpha - \beta) = \frac{5\sqrt{65} - 4\sqrt{119}}{108}$
 c. $\tan(\alpha - \beta) = \frac{\sqrt{99}\sqrt{24} + 1}{\sqrt{24} - \sqrt{99}}$ d. $\sin(\alpha + \beta) = 0$, $\cos(\alpha - \beta) = \frac{7}{9}$
- 10) a. $\frac{24}{25}$ b. $\frac{-7}{25}$ c. $\frac{-7}{25}$ d. $\frac{-4\sqrt{2}}{9}$
- 11) $\csc \theta = \frac{\sqrt{5}}{2}$

Trigonometric Equations

Questions:

1) Solve the following equations:

a. $\sin x = \sin 30$

b. $\sin x = \frac{\sqrt{3}}{2}$

2) Solve the following equations:

a. $\sin 3x = \frac{\sqrt{2}}{2}$

b. $2 \sin 2x = 1$

3) Solve the following equations:

a. $\sin(2x + 30) = \sin x$

b. $\sin 2x - \sin 4x = 0$

4) Solve the following equations:

a. $\sin 3x + \sin x = 0$

b. $-\sin(x - 30) = \sin(60 + x)$

5) Solve the following equations:

a. $\sin x = 0$

b. $\sin x = 1$

6) Solve the following equations:

a. $\cos x = \sin 30$

b. $\cos x = \frac{\sqrt{3}}{2}$

7) Solve the following equations:

a. $\cos 3x = \frac{\sqrt{2}}{2}$

b. $2 \cos 2x = 1$

8) Solve the following equations:

a. $\cos(2x + 30) = \cos x$

b. $\cos 2x - \cos 4x = 0$

9) Solve the following equations:

a. $\cos 3x + \cos x = 0$

b. $-\cos(x - 30) = \cos(60 + x)$

10) Solve the following equations:

a. $\cos x = 0$

b. $\cos x = 1$

11) Solve the following equations:

a. $\sin 3x = \cos x$

b. $\sin(2x + 60) = -\cos x$

12) Solve the following equations:

a. $\sin^2 x = \frac{1}{4}$

b. $\cos^2 2x = \frac{3}{4}$

13) Solve the following equations:

a. $2\sin x \cos x = -\cos^2 x + \sin^2 x$

b. $\cos^4 x - \sin^4 x = 2\sin 2x \cos 2x$

14) Solve the following equations:

a. $\sin 3x = \frac{1}{2} \quad 0 < x < 180$

b. $\cos 3x = \frac{\sqrt{3}}{2} \quad -90 < x < 90$

15) Solve the following equations:

a. $\sin x + \cos x = 0 \quad 0 \leq x \leq 360$

b. $\sin^2 2x = \frac{1}{4} \quad 0 < x < 90$

16) Solve the following equations:

a. $\sin x + \sin 4x = 0 \quad 0 < x < 180$

b. $\sin x - \cos 3x = 0 \quad 0 < x < 270$

17) Solve the following equations:

a. $\tan x = \sqrt{3}$

b. $\tan 3x = 1$

18) Solve the following equations:

a. $\tan x = -\frac{1}{\sqrt{3}}$

b. $\frac{1}{\sqrt{3}} \tan 4x = 1 \quad -90 < x < 90$

19) Solve the following equations:

a. $\tan 5x = 2 + \sqrt{3}$

b. $\tan(-3x) = -\frac{3}{\sqrt{3}}$

20) Solve the following equations:

a. $\tan^2 4x = \frac{1}{3} \quad 0 < x < 180$

b. $\tan^2 3x = 4$

21) Solve the following equations:

a. $\tan(x-30) = \tan(3x)$

b. $\tan(5x+15) = \tan(95) \quad 0 < x < 90$

22) Solve the following equations:

a. $2\cos^2 x + 3\cos x - 2 = 0$

b. $2\sin^2 x + 3\sin x + 1 = 0$

23) Solve the following equations:

a. $4\cos^2 2x + 3\cos 2x = 1$

b. $12\sin^2 2x - 13\sin 2x + 3 = 0$

24) Solve the following equations:

a. $2\cos^2 4x + 3\cos 4x = -1$

b. $2\sqrt{3}\sin^2 3x + \sin 3x = 2\sqrt{3}$

25) Solve the following equations:

a. $\sqrt{2}\sin^2 x + \cos x = 0$

b. $-4\cos^2 x + 3\sin x + 3 = 0$

26) Solve the following equations:

a. $\cos 2x - 3\cos x + 2 = 0$

b. $6\sin^2 x + \cos 2x - 2\sin x = 2$

27) Solve the following equations:

a. $\sin^2 x + \sin^2 2x = 1$

b. $4\sin^4 x + 7\cos 2x = 1$

28) Solve the following equations:

a. $\tan^2 x - 4\tan x + 3 = 0$

b. $\sqrt{3}\tan^2 x - 4\tan x + \sqrt{3} = 0$

29) Solve the following equations:

a. $\tan^2 x + 3\cot^2 x = 4$

b. $9\cot^2 2x + 4\sin^2 2x = 6$

30) Solve the following equations:

a. $\cos x + \sin x \cos x = 0$

b. $\cos 2x - 2\sin x \cos 2x = 0$

31) Solve the following equations:

a. $\sqrt{2}\sin x - \sin 2x = 0$

b. $2\sin^3 x - \sin x = 0$

32) Solve the following equations:

a. $2\sin x + \sin 2x = 0$

b. $\cos^2 x - \sin x - 1 = 0$

33) Solve the following equations:

a. $2\sin x \cos x - \sin x - 2\cos x + 1 = 0$

b. $2\sin x \cos x + \cos x - 2\sin x - 1 = 0$

34) Solve the following equations:

a. $2\sin 2x - 2\sin x - 2\cos x + 1 = 0$, $0 < x < 90$

b. $\sin 2x + 2\sin x - 2\cos x - 2 = 0$, $0 < x < \pi$

35) Solve the following equations:

a. $\sin 8x + \sin 4x = \sin 10x + \sin 2x$

b. $\sin x + \sin 3x = \cos x + \cos 3x$

36) Solve the following equations:

a. $\sin 4x = \sin 2x - 2\sin x$

b. $\sin 5x + \sin 6x + \sin 7x = 0$

Answer Key

- 1) a. $x_1 = 30^\circ + 360^\circ k$, $x_2 = 150^\circ + 360^\circ k$ b. $x_1 = 60^\circ + 360^\circ k$, $x_2 = 120^\circ + 360^\circ k$
- 2) a. $x_1 = 15^\circ + 120^\circ k$, $x_2 = 45^\circ + 120^\circ k$ b. $x_1 = 15^\circ + 180^\circ k$, $x_2 = 75^\circ + 180^\circ k$
- 3) a. $x_1 = -30^\circ + 360^\circ k$, $x_2 = 50^\circ + 120^\circ k$ b. $x_1 = 30^\circ + 60^\circ k$, $x_2 = 180^\circ k$
- 4) a. $x_1 = 90^\circ + 180^\circ k$, $x_2 = 90^\circ k$ b.
- 5) a. $x = 180^\circ k$ b. $x = 90^\circ + 360^\circ k$
- 6) a. $x_{1,2} = \pm 60^\circ + 360^\circ k$ b. $x_{1,2} = \pm 30^\circ + 360^\circ k$
- 7) a. $x_{1,2} = \pm 15^\circ + 120^\circ k$ b. $x_{1,2} = \pm 30^\circ + 180^\circ k$
- 8) a. $x_1 = -10^\circ + 120^\circ k$, $x_2 = -30^\circ + 360^\circ k$ b. $x_1 = -180^\circ k$, $x_2 = 60^\circ k$
- 9) a. $x_1 = -90^\circ + 180^\circ k$, $x_2 = 45^\circ + 90^\circ k$ b.
- 10) a. $x = 90^\circ + 180^\circ k$ b. $x = 360^\circ k$
- 11) a. $x_1 = 45^\circ + 180^\circ k$, $x_2 = 22\frac{1}{2}^\circ + 90^\circ k$ b. $x_1 = 70^\circ + 120^\circ k$, $x_2 = -150^\circ + 360^\circ k$
- 12) a. $x_1 = 30^\circ + 360^\circ k$, $x_2 = 150^\circ + 360^\circ k$, $x_3 = -30^\circ + 360^\circ k$, $x_4 = 210^\circ + 360^\circ k$
b. $x_{1,2} = \pm 15^\circ + 180^\circ k$, $x_{3,4} = \pm 75^\circ + 180^\circ k$
- 13) a. $x = 67\frac{1}{2}^\circ + 90^\circ k$ b. $x_1 = 45^\circ - 180^\circ k$, $x_2 = 15^\circ + 60^\circ k$
- 14) a. $x = 10, 50, 130, 170$ b. $x = \pm 10$
- 15) a. $x = 135, 315$ b. $x = 15, 75$
- 16) a. $x = 60, 72, 144$ b. $x = 22\frac{1}{2}, 112\frac{1}{2}, 202\frac{1}{4}$
- 17) a. $x = 60^\circ + 180^\circ k$ b. $x = 15^\circ + 60^\circ k$
- 18) a. $x = -30^\circ + 180^\circ k$ b. $x = -75, -30, 15, 60$
- 19) a. $x = 15^\circ + 36^\circ k$ b. $x = 20^\circ + 60^\circ k$
- 20) a. $x_1 = 7\frac{1}{2}^\circ + 45k$ ($k = 0, 1, 2, 3$), $x_2 = -7\frac{1}{2}^\circ + 45^\circ k$ ($k = 1, 2, 3, 4$)
b. $x_{1,2} = \pm 21.145^\circ + 60^\circ k$
- 21) a. $x = -15^\circ + 90^\circ k$ b. $x = 16, 52, 88$
- 22) a. $x_{1,2} = \pm 60^\circ + 360^\circ k$
b. $x_1 = -30^\circ + 360^\circ k$, $x_2 = 210^\circ + 360^\circ k$, $x_3 = -90^\circ + 360^\circ k$
- 23) a. $x_{1,2} = \pm 37.76^\circ + 180^\circ k$, $x_3 = 90^\circ + 180^\circ k$
b. $x_1 = 24.3^\circ + 180^\circ k$, $x_2 = 65.7^\circ + 180^\circ k$, $x_3 = 9.74^\circ + 180^\circ k$, $x_4 = 80.26^\circ + 180^\circ k$

- 24) a. $x_{1,2} = \pm 30^\circ + 90^\circ k$, $x_3 = 45^\circ + 90^\circ k$ b. $x_1 = 20^\circ + 120^\circ k$, $x_2 = 40^\circ + 120^\circ k$
- 25) a. $x_{1,2} = \pm 135^\circ + 360^\circ k$
 b. $x_1 = 14.5^\circ + 360^\circ k$, $x_2 = 165.5^\circ + 360^\circ k$, $x_3 = -90^\circ + 360^\circ k$
- 26) a. $x_1 = 360^\circ k$, $x_{2,3} = \pm 60^\circ + 360^\circ k$
 b. $x_1 = 54^\circ + 360^\circ k$, $x_2 = 126^\circ + 360^\circ k$, $x_3 = -18^\circ + 360^\circ k$, $x_4 = 198^\circ + 360^\circ k$
- 27) a. $x = 90^\circ + 360^\circ k$
 b. $x_1 = 45^\circ + 360^\circ k$, $x_2 = 135^\circ + 360^\circ k$, $x_3 = -45^\circ + 360^\circ k$, $x_4 = 225^\circ + 360^\circ k$
- 28) a. $x_1 = 71.565^\circ + 180^\circ k$, $x_2 = 45^\circ + 180^\circ k$ b. $x_1 = 60^\circ + 180^\circ k$, $x_2 = 30^\circ + 180^\circ k$
- 29) a. $x_{1,2} = \pm 60^\circ + 180^\circ k$, $x_{3,4} = \pm 45^\circ + 180^\circ k$
 b. $x_1 = 30^\circ + 180^\circ k$, $x_2 = 60^\circ + 180^\circ k$, $x_3 = -30^\circ + 180^\circ k$, $x_4 = 120^\circ + 180^\circ k$
- 30) a. $x_{1,2} = \pm 90^\circ + 360^\circ k$ b. $x_{1,2} = \pm 45^\circ + 180^\circ k$
- 31) a. $x_1 = 180^\circ k$, $x_{2,3} = \pm 45^\circ + 360^\circ k$
 b. $x_1 = 180^\circ k$, $x_2 = 45^\circ + 360^\circ k$, $x_3 = 135^\circ + 360^\circ k$, $x_4 = -45^\circ + 360^\circ k$, $x_5 = 225^\circ + 360^\circ k$
- 32) a. $x = 180^\circ k$ b. $x_1 = 180^\circ k$, $x_2 = -90^\circ + 360^\circ k$
- 33) a. $x_1 = 90^\circ + 360^\circ k$, $x_{2,3} = \pm 60^\circ + 360^\circ k$
 b. $x_1 = 360^\circ k$, $x_2 = -30^\circ + 360^\circ k$, $x_3 = 210^\circ + 360^\circ k$
- 34) a. $x = 30, 60$ b. $x = 90$
- 35) a. $x = 30^\circ k$ b. $x_1 = 90^\circ + 360^\circ k$, $x_2 = 22\frac{1}{2} + 360^\circ k$
- 36) a. $x_1 = 180^\circ k$, $x_2 = 60^\circ + 120^\circ k$ b. $x_1 = 30^\circ k$, $x_{2,3} = \pm 120^\circ + 360^\circ k$

Inverse Trigonometric Functions

Questions

1) Evaluate:

a. $\cos\left(\cos^{-1}\frac{1}{3}\right)$

b. $\cos\left(\cos^{-1}\frac{1}{2}\right)$

c. $\cos\left(\cos^{-1}\frac{3}{2}\right)$

2) Evaluate:

a. $\arctan\left(\tan\left(-\frac{1}{3}\right)\right)$

b. $\arctan\left(\tan\frac{\pi}{3}\right)$

c. $\arctan\left(\tan\frac{5\pi}{6}\right)$

3) Evaluate:

a. $\sin\left(\arccos\frac{3}{5}\right)$

b. $\cos\left(\arcsin\left(-\frac{3}{4}\right)\right)$

c. $\tan\left(\operatorname{arcsec}\left(-\frac{7}{3}\right)\right)$

d. $\tan(\arcsin(-2))$

4) Evaluate:

a. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

b. $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

c. $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

d. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

5) Evaluate:

a. $\cos\left(\cos^{-1}\left(\frac{2}{3}\right)\right)$

b. $\sin\left(\sin^{-1}\left(-\frac{1}{4}\right)\right)$

c. $\tan(\tan^{-1}1)$

d. $\csc\left(\csc^{-1}\frac{1}{5}\right)$

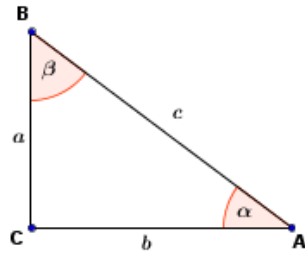
Answer Key

- 1) a. $\frac{1}{3}$ b. $\frac{1}{2}$ c. Undefined.
- 2) a. $-\frac{1}{3}$ b. $\frac{\pi}{3}$ c. $-\frac{\pi}{6}$
- 3) a. $\frac{4}{5}$ b. $\frac{\sqrt{7}}{4}$ c. $\frac{2\sqrt{10}}{3}$ d. Undefined.
- 4) a. $-\frac{\pi}{3}$ b. $\frac{2\pi}{3}$ c. $-\frac{\pi}{6}$ d. $\frac{5\pi}{6}$
- 5) a. $\frac{2}{3}$ b. $-\frac{1}{4}$ c. 1 d. Undefined.

Triangles

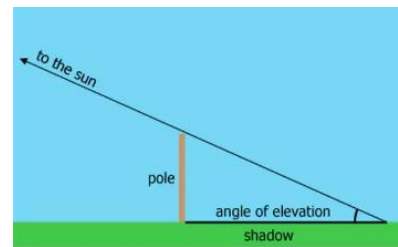
Questions

- 1) Given a right triangle $\triangle ABC$ (see figure), with $\alpha = 34.5^\circ$ and $a = 67.8$, solve the triangle.



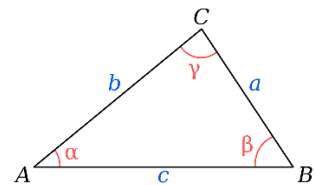
- 2) Given a right triangle $\triangle ABC$ (see figure), with $c = 7.65$ and $a = 5.43$, solve the triangle. Express angles in degrees (as a decimal) and then convert to degrees & minutes.

- 3) A pole casts a shadow of length 12 meters, when the angle of elevation of the sun is 28° (see figure). Find the height of the pole (assume the ground is level).



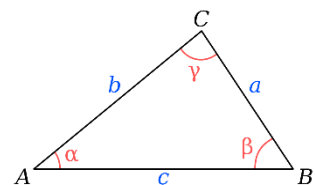
- 4) A ship leaves a port and sails at an average speed of 70 kilometers per hour at a bearing of $S20^\circ E$. After 4 hours how far south and how far east is it from the port?
- 5) The angle of elevation of the top of a tower is 32.1° from a point on the ground (assumed to be level). The angle of elevation is 54.3° from another point on the ground, 50 meters closer to the tower. Find the height of the tower.

- 6) Given a triangle $\triangle ABC$ (see figure) with $c = 78.9$, $\alpha = 26^\circ$, and $\beta = 110^\circ$. Solve the triangle.

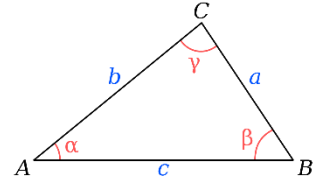


- 7) Observation Point #2 is situated 5 miles due east of Observation Point #1. A truck is spotted on a heading of $148^\circ 24'$ from Point #1 and of $216^\circ 45'$ from Point #2. Find the distance of the truck from each Observation Point.

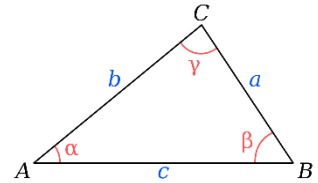
- 8) Given a triangle $\triangle ABC$ (see figure) with $a = 32.4$, $b = 40.3$, and $\alpha = 24.6^\circ$. Solve the triangle.



- 9) Given a triangle $\triangle ABC$ (see figure) with $a = 34.56$, $c = 81.23$, and $\beta = 16.78^\circ$. Solve the triangle.



- 10) Given a triangle $\triangle ABC$ (see figure) with $a = 2.83$, $b = 4.53$, and $c = 2.37$. Solve the triangle.



- 11) A man leaves his house for work on a straight road and drives at 52 miles per hour (average). 5 minutes later his wife leaves the house (which is on an intersection) on another straight road at 45 miles per hour. The angle between the two roads is 108° . How far apart are they 16 minutes after the man leaves?

Answer Key

- 1) $\beta = 55.5^\circ$, $b = 98.649$, $c = 119.702$
- 2) $\alpha = 45^\circ 13' \rightarrow 13.14'$ & $45^\circ 13'$, $\beta = 44^\circ 47'$, $b \approx 5.38$
- 3) 6.380 meters.
- 4) South: 247.246km, East: 95.765km.
- 5) 57.106 meters.
- 6) $\gamma = 44^\circ$, $a = 49.79$, $b = 106.73$
- 7) #1 = 4.310 miles, #2 = 4.582 miles.
- 8) $\beta = 31.2^\circ$, $c = 64.3$, $\gamma = 124.2^\circ$
- 9) $\alpha = 11.7^\circ$ $\gamma = 151.52^\circ$, $b = 49.17$
- 10) $\beta = 120.83^\circ$ $\gamma = 26.73^\circ$, $\alpha = 32.44^\circ$
- 11) 89.366 miles.