

Statistics



$$\{\sqrt{x}\}^2$$



Table of Contents

Confidence Intervals for Proportions	2
Constructing confidence intervals	2
Determining the Sample Size in Estimating a Proportion	5

Confidence Intervals for Proportions

Constructing Confidence Intervals

Our goal is to estimate a population proportion, denoted by p . We do this by using the sample proportion and building a confidence interval around that sample proportion.

The standard error of the sample proportion is distributed normally, $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, and we use this to estimate the population standard error.

We create a confidence interval for p using the sample proportion, like this: $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

Conditions:

- 1) The sample needs to be a random sample.
- 2) There must be at least 10 successes and 10 failures so that the sampling distribution of the sample proportion will be approximately normal.
- 3) The individual trials must be independent.

Example (solution in the recording):

In order to estimate the unemployment rate in New York, 200 adults are sampled, and it is found that 24 are unemployed.

- a. Construct a 95% confidence interval for the state unemployment rate.
- b. What is the standard error of the sample proportion?

Questions:

- 1) 200 apartments in Orlando were randomly sampled, 48 of which have security systems.
 - a. Construct a 95% confidence interval for the proportion of apartments in Orlando with security systems. Explain what it means.
 - b. Suppose that there are 80,000 apartments in Orlando. Construct a 95% confidence interval for the number of apartments with security systems.

- 2) 300 employees were randomly selected at a large national company to be polled, and 180 of them prefer a flex work schedule.
 - a. Construct a 95% confidence interval for the proportion of all employees that prefer a flex work schedule.
 - b. How would the interval length change if the confidence level were lower?
 - c. How would the length of the interval change if the sample size were increased?

- 3) The following confidence interval for the proportion of drivers who need vision correction was derived from a random sample of 400 drivers: $0.08 < p < 0.18$
 - a. How many drivers in the sample need vision correction?
 - b. What level of confidence was used to create this confidence interval?

- 4) During election season, 840 people were asked whether they planned to vote for Candidate A and 546 people answered in the affirmative. The survey reported an error estimate of $\pm 3\%$.
What's the confidence level of this maximum error?

- 5) In a sample of 300 women aged 35-40 in Italy, 140 were married, 80 were divorced, 60 were single, and 20 were widows.
 - a. Find a 90% confidence interval for the proportion of divorced women among women aged 35-40 in Italy.
 - b. Find a 99% confidence interval for the probability that a 35-40 year old woman in Italy is unmarried (divorced, single or widowed).

- 6) A random sample is taken from a certain population.
The rate of success in the samples was 10%, and a 95% confidence interval was constructed. The length of the interval is 8.3156%.
What was the sample size?

Answer Key:

- 1) a. (0.18081, 0.29919); b. (14,465, 23,935)
- 2) a. (0.54456, 0.65544); b. The interval will be shorter, because Z^* will be smaller.
c. The Length of the confidence interval will be smaller, because the standard error of the sample proportion will be smaller.
- 3) a. 52; b. 99.7%
- 4) 93.17%
- 5) a. (0.22467, 0.30866); b. (0.45914, 0.60753)
- 6) 200

Determining the Sample Size in Estimating a Proportion

Theory

This chapter discusses how to determine the sample size when estimating a proportion in a given population. The researcher determines the confidence level in advance and the maximum statistical error and uses this to plan for the sample size.

We use $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, and the fact that the maximum value of $\hat{p}(1-\hat{p})$ will occur when $\hat{p} = 0.5$.

Example (solution in the recording):

We want to estimate the unemployment rate in Gainesville at a 90% level of confidence and with an estimation error of at most 4%.

What sample size is needed?

Questions:

- 1) Every month, the White House estimates the percent of support for the president. What size sample is necessary to obtain a 95% level of certainty that the estimate will not deviate from the actual support rate by more than 3%?
- 2) The Department of Labor wants to know the percentage of households with a broadband internet connection. How many households should be sampled, if we want a 90% level of certainty for a confidence interval no more than 8% long?
- 3) A television station wishes to estimate its prime time viewing rate. The goal is a 95% level of certainty that the maximum difference between the estimator and the real rating will not be greater than 4%. What sample size is necessary?
- 4) A public health organization is examining the chances of a vaccinated person getting influenza. It wants a 98% confidence level and for its estimation error to not exceed 3%.
 - a. How many vaccinated people should be sampled?
 - b. Suppose they sampled using the sample size from part a and found that 15% of the sample got the flu. Construct a 98% confidence interval for the probability that a vaccinated person gets the flu.
 - c. What is the maximum estimation error for part b? Why is it less than 3%?

Answer Key:

1) 1068

2) 423

3) 601

4) a. 1503; b. (0.12892, 0.17181) c. 0.02181