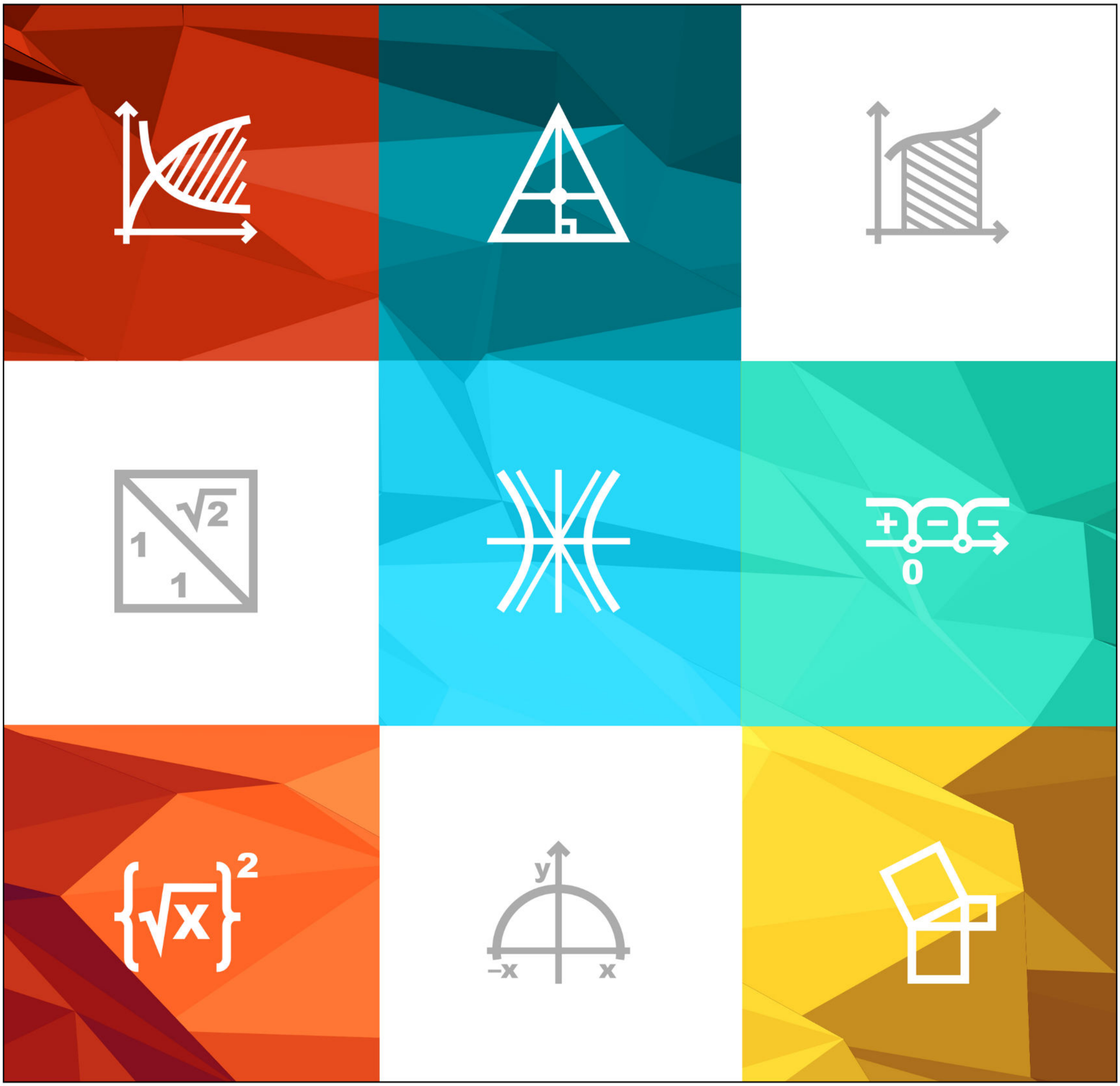
proprep™

# Workbook



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# Conservative Vector Fields

## Conservative Vector Fields

### Questions:

**1)** Determine if $\mathbf{F}$ is a conservative field. If so, find a function $\phi$, so that $\nabla\phi = \mathbf{F}$.

a. $\mathbf{F}(x,y) = \langle 6x+5y, 5x+4y \rangle$

b. $\mathbf{F}(x,y) = \langle 2x\cos y - y\cos x, -x^2\sin y - \sin x \rangle$

c. $\mathbf{F}(x,y,z) = z^2\mathbf{i} + e^{-y}\mathbf{j} + 2xz\mathbf{k}$

d. $\mathbf{F}(x,y,z) = \langle z, 2yz, y^2 \rangle$

**2)** Given the integral $\int_{(1,2)}^{(3,4)} \left(6xy^2 - y^3\right)dx + \left(6x^2y - 3xy^2\right)dy$

a. Prove that the integral is independent of the path joining $(1,2)$ to $(3,4)$.

b. Compute the integral in two different ways.

**3)** Compute the integral $\int_{(1,4)}^{(3,1)} 2xy^3dx + \left(1+3x^2y^2\right)dy$.

**4)** Compute the integral $\int_{(1,0)}^{(2,1)} \left(2xy - y^4 + 3\right)dx + \left(x^2 - 4xy^3\right)dy$.

**5)** Given the force field $\mathbf{F}(x,y) = e^y\mathbf{i} + xe^y\mathbf{j}$, find the work done by it on a particle which moves from $(1,0)$ to $(-1,0)$ along the curve $y = \sqrt{1-x^2}$.

**6)** Compute the integral $\int_{(1,-1,1)}^{(2,1,-1)} \left(2xz^3 + 6y\right)dx + \left(6x - 2yz\right)dy + \left(3x^2z^2 - y^2\right)dz$.
Give a physical meaning to the result.

**7)** Given a vector field $\mathbf{F} = \frac{x^2 + y^2 - y}{x^2 + y^2} \cdot \mathbf{i} + \frac{x}{x^2 + y^2} \cdot \mathbf{j}$, and the following three closed curves:

$L_1 : \ x^2 + y^2 = 1$ with positive orientation (counterclockwise).

$L_2 : \ \frac{x^2}{16} + \frac{y^2}{9} = 1$ with negative orientation (clockwise).

$L_3 : \ (x-10)^2 + (y-7)^2 = 1$ with positive orientation (counterclockwise).

Compute:

a. $\oint_{L_1} \mathbf{F} dr$    b. $\oint_{L_2} \mathbf{F} dr$    c. $\oint_{L_3} \mathbf{F} dr$

**8)** Given a vector field $\mathbf{F} = \frac{-y}{x^2 + y^2} \cdot \mathbf{i} + \frac{x}{x^2 + y^2} \cdot \mathbf{j}$ and the following two paths from $(2,0)$ to $(-2,0)$: $L_1 : x^2 + y^2 = 4 \ , \ y \geq 0$    $L_2 : \ x^2 + y^2 = 4 \ , \ y \leq 0$

a. Compute $\int_{L_1} \mathbf{F} dr$ , $\int_{L_2} \mathbf{F} dr$ .

b. Prove that $\mathbf{F}$ is conservative in the half-annulus $D = \left\{ (x, y) \,|\, 1 \leq x^2 + y^2 \leq 9, \ \ y \geq 0 \right\}$.

## Remark on Notation:

A vector field has various notations in the technical literature:

$$\mathbf{F}(x, y, z) = f(x, y, z)\mathbf{i} + g(x, y, z)\mathbf{j} + h(x, y, z)\mathbf{k}$$
$$\mathbf{F}(x, y, z) = \left( f(x, y, z), g(x, y, z), h(x, y, z) \right)$$
$$\mathbf{F}(x, y, z) = f(x, y, z)\hat{x} + g(x, y, z)\hat{y} + h(x, y, z)\hat{z}$$
$$\mathbf{A} = A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k}$$

## Answer Key:

**1)** a. $\phi(x, y) = 3x^2 + 5xy + 2y^2$    b. The field is not conservative

c. $\phi(x, y) = x^2 \cos y - y \sin x$    d. $\phi(x, y, z) = xz^2 - e^{-y}$

e. $\phi(x, y, z) = xyz + z^3$    f. The field is not conservative

**2)** a. proved    b. 236    **3)** -58    **4)** 5    **5)** -2

**6)** 15 = Work done by force field to move particle from $(1, -1, 1)$ to $(2, 1, -1)$ along curve

**7)** a. $2\pi$    b. $-2\pi$    c. $0$

**8)** a. $L_1 : \pi$ , $L_2 : -\pi$    b. Refer to the videos