proprep™

Workbook

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Continuity of a Function

Definition of Continuity

Questions:

1) Consider the function: $f(x)=\begin{cases} x & x\geq 1 \\ x^2 & x<1 \end{cases}$.

Is the function $f(x)$ continuous at $x=1$? Sketch the graph of $f(x)$.

2) Consider the function: $f(x)=\begin{cases} x+1 & x\leq 2 \\ 5-x & x>2 \end{cases}$.

Is the function $f(x)$ continuous at $x=2$? Sketch the graph of $f(x)$.

3) Is the function $f(x)=\begin{cases} \dfrac{\sin 4x}{x} & x>0 \\ 4+e^{\frac{1}{x}} & x<0 \end{cases}$ continuous at $x=0$?

4) Is the function $f(x)=\begin{cases} \dfrac{\sin x}{x} & x>0 \\ 2 & x=0 \\ 1+e^{\frac{1}{x}} & x<0 \end{cases}$ continuous at $x=0$?

5) Is the function $f(x)=\begin{cases} \sin x & x<0 \\ x^2 & 0\leq x<1 \\ 2-x & 1\leq x<2 \\ x-3 & x\geq 2 \end{cases}$ continuous at $x=0,1,2$?

6) Is the function $f(x)=\begin{cases}\dfrac{1}{x} & x\le 1\\ |x-2| & 1<x<2\\ 1 & x=2\\ x-2 & x>2\end{cases}$ continuous at $x=1,2$?

7) Find the value of k for which $f(x)=\begin{cases}kx^2+x-2 & x<2\\ 5kx-6 & x>2\end{cases}$ is continuous everywhere.

8) Find k which makes the function $f(x)=\begin{cases}\dfrac{x^2+2x-3}{x-1} & x\neq 1\\ k & x=1\end{cases}$ continuous for every x.

9) Find k which makes the function $f(x)=\begin{cases}\dfrac{\sqrt{x^2+5}-3}{x-2} & x\neq 2\\ k & x=2\end{cases}$ continuous for every x.

10) Find k which makes the function $f(x)=\begin{cases}2x-k & x\le 0\\ 5kx-6 & x>0\end{cases}$ continuous for every x.

11) Find the values of constants a and b for which function $f(x)=\begin{cases}ax+b & x\le 0\\ \dfrac{\sin x}{2x} & 0<x<\pi\\ a\cos x & x\ge\pi\end{cases}$

is continuous in its domain.

Answer Key:

1) Continuous at $x=1$

2) Continuous at $x=2$

3) Not defined at $x=0$

4) Not continuous

5) Continuous at $x=0$, Continuous at $x=1$, Not continuous at $x=2$

6) Continuous at $x=1$, Not continuous at $x=2$

7) $k=1$

8) $k=4$

9) $k=\frac{2}{3}$

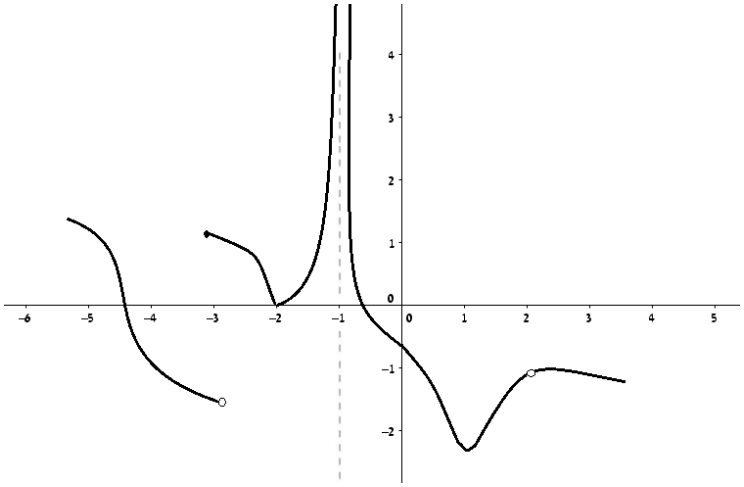
10) $k=-1$

11) $a=0,\ \ b=\frac{1}{2}$

Points of Discontinuity

Questions:

1) Classify any points of discontinuity of f over the graphed interval.



2) Find and classify any points of discontinuity of the functions below:

a. $f(x)=\dfrac{x+1}{x-1}$ b. $f(x)=\dfrac{x^2+x-2}{x^2-1}$

3) Find and classify any points of discontinuity of the functions below:

a. $f(x)=\begin{cases} x^2 & x<1 \\ 0 & x=1 \\ 2-x & x>1 \end{cases}$ b. $f(x)=\begin{cases} x^2 & x<1 \\ 0 & x=1 \\ -x^2+2x+1 & x>1 \end{cases}$ c. $f(x)=\begin{cases} x^2 & x<1 \\ 0 & x=1 \\ \dfrac{1}{x-1} & x>1 \end{cases}$

Answer Key:

1) $x=-3$, $x=-1$, $x=2$.

2) a. $x=1$ undefined b. $f(1)$ undefined

3) a. $x=1$ removable discontinuity b. Jump discontinuity at $x=1$
c. Essential discontinuity at $x=1$

The Intermediate Value Theorem

Questions:

1) Use the intermediate value theorem to show that the equation $\cos x = x$ must have at least one solution.

2) Use the intermediate value theorem to show that the equation $x^3 + 4x = 1$ must have at least one solution.

3) Use the intermediate value theorem to show that the equation $\ln x = -x^2$ must have at least one solution.

4) Use the intermediate value theorem to show that the equation $x^3 + bx^2 + cx + d = 0$ must have at least one solution.

5) Use the intermediate value theorem to show that the equation $4x^3 + 5x = \frac{1}{x}$ must have at least two solutions.

6) Use the intermediate value theorem to show that the equation $e^x = 5x$ must have at least one solution.

Answer Key:

Refer to the videos.