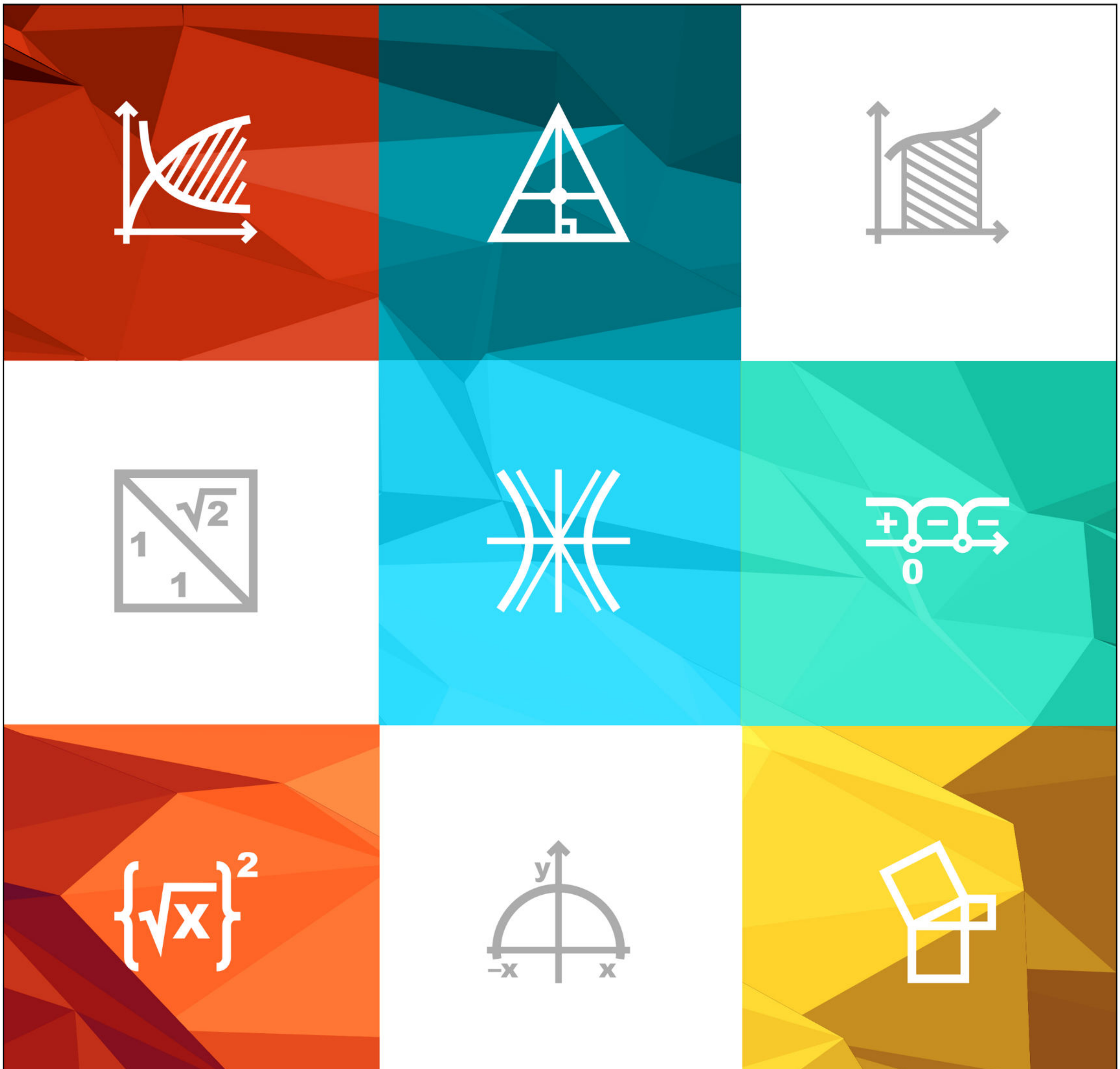
proprep™

Workbook



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Divergence Theorem

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Questions:

In each of the exercises 1-3 verify the Divergence Theorem $\left(\iiint_R div\mathbf{F}dV = \iint_S \mathbf{F}\cdot\mathbf{n}dS \right)$:

$\mathbf{n}$ is the outward unit normal to S (See remark on notation below).

1) Where $\mathbf{F} = (2x - z)\mathbf{i} + x^2 y\mathbf{j} - xz^2\mathbf{k}$ and S is the surface of the cube R determined by the planes: $x = 0,\ x = 1,\ y = 0,\ y = 1,\ z = 0,\ z = 1$.

2) Where $\mathbf{F} = x\mathbf{i} - 2y\mathbf{j} + 3z\mathbf{k}$ and S is the surface of the unit ball R given by $x^2 + y^2 + z^2 \le 1$.

3) Where $\mathbf{F} = (2xy + z)\mathbf{i} + y^2\mathbf{j} - (x + 3y)\mathbf{k}$ and S is the surface of the pyramid R determined by the planes: $2x + 2y + z = 6,\ x = 0,\ y = 0,\ z = 0$.

4) Let S be the surface of the body bounded by the cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = 2$. Compute the flux of the vector field $\mathbf{F} = x^3\mathbf{i} + y^3\mathbf{j} + z^2\mathbf{k}$ through S. I.e. compute $\iint_S \mathbf{F}\cdot\mathbf{n}dS$ where $\mathbf{n}$ is the outward unit normal to S.

5) Compute $\iint_S \mathbf{F}\cdot\mathbf{n}dS$ where $\mathbf{n}$ is the outward unit normal to S, $\mathbf{F} = (z^2 - x)\mathbf{i} - xy\mathbf{j} + 3z\mathbf{k}$, and S is the surface of the body bounded by: $x = 0,\ x = 3,\ z = 4 - y^2,\ z = 0$.

6) Compute $\iint_S \mathbf{F}\cdot\mathbf{n}dS$ where $\mathbf{n}$ is the outward unit normal to S

$\mathbf{F} = xz^2\mathbf{i} + (x^2 y - z^3)xy\mathbf{j} + (2xy + y^2 z)\mathbf{k}$, and S is the surface of the body bounded by:

$z = \sqrt{a^2 - x^2 - y^2},\ z = 0$.

7) Let S be the open surface $0 \le y \le 4$, $x^2+z^2=16$ (a cylinder without the bases). Compute the **flux** of the vector field $\mathbf{F}=z^2\mathbf{i}+5y\mathbf{j}+x^5\mathbf{k}$ through S . I.e. compute $\iint_S \mathbf{F}\cdot\mathbf{n}dS$ where $\mathbf{n}$ is the outward unit normal to S .

8) Compute $\iint_S \mathbf{F}\cdot\mathbf{n}dS$ where $\mathbf{n}$ is the outward unit normal to S ,

$\mathbf{F}=\left(\frac{x^2y}{1+y^2}+6yz^2\right)\mathbf{i}+2x\arctan y\mathbf{j}-\frac{2xz(1+y)+1+y^2}{1+y^2}\mathbf{k}$, and S is the open surface $z=4-x^2-y^2,\quad z\ge 0$.

Remark on Notation:

The Divergence Theorem states that given a vector field $\mathbf{F}=f(x,y,z)\mathbf{i}+g(x,y,z)\mathbf{j}+h(x,y,z)\mathbf{k}$, the equality $\iiint_R div\mathbf{F}dV=\iint_S \mathbf{F}\cdot\mathbf{n}dS$ holds.

There are various other formulations of the Divergence Theorem, such as:

$$\iiint_R \nabla\cdot\mathbf{F}dV=\iint_S \mathbf{F}\cdot\mathbf{n}dS$$

$$\iiint_R (f_x+g_y+h_z)dV=\iint_S \mathbf{F}\cdot\mathbf{n}dS$$

$$\iiint_R (f_x+g_y+h_z)dV=\iint_S fdydz+gdzdx+hdxdy$$

Answer Key:

1) The common value is $\frac{11}{6}$

2) The common value is $\frac{8}{3}\pi$

3) The common value is 27

4) 279π

5) 16

6) $\frac{2\pi a^5}{5}$

7) 0

8) -4π