

# Double Integrals - Applications

## Double Integrals, Applications

### Questions:

- 1) Compute the **areas** of the **regions** bounded by the given curves:
  - a.  $x + y = 2, x^2 - 4y = 4$
  - b.  $xy = a^2, x + y = 2.5a$  ( $a > 0$ )
  - c.  $x^2 + y^2 = 2x, y = 0, y = x\sqrt{3}$
  - d.  $x + y = 3, y^2 = 4x$
  
- 2) Compute the **volumes** of the **solids** bounded by the given surfaces:
  - a.  $y = 0, x = 0, x + y = 1, z = 0, z = 1 + x + y$
  - b.  $y = x^2, y = 1, z = x^2 + y^2, z = 0$
  - c.  $y = 2/x, y = 2x, y = 0.5x, z = x^2 + y, z = 0$  ( $x \geq 0$ )
  - d.  $2y^2 = x, \frac{x}{4} + \frac{y}{2} + \frac{z}{4} = 1, z = 0$
  - e.  $z = y, x^2 + 0.25y^2 = 1$  ( $z \geq 0$ )
  - f.  $y = 0, x = 0, z = 6, z = x + y$
  
- 3) A flat triangular board with vertices  $(0,0), (1,0)$  and  $(0,1)$  has a density function  $\delta(x, y) = xy$ .
  - a. Compute the **mass** of the board
  - b. Compute its center of mass.
  
- 4) A flat board with rectangular shape  $R = \left\{ (x, y) \mid -\frac{b}{2} \leq y \leq \frac{b}{2}, -\frac{a}{2} \leq x \leq \frac{a}{2} \right\}$  has a constant density function (the board is homogeneous).  
Compute the moment of inertia of the board about the z-axis. Express your answer in terms of the mass  $M$  of the board.
  
- 5) Find the surface area of the part of the cylinder  $x^2 + z^2 = 4$  which lies above the rectangle  $R = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq 4\}$  in the  $xy$  plane.

**Final answers:**

- 1) a.  $21\frac{1}{3}$       b.  $a^2\left(\frac{15}{8} - 2\ln 2\right)$       c.  $\frac{\pi}{3} + \frac{\sqrt{3}}{4}$       d.  $21\frac{1}{3}$
- 2) a.  $\frac{5}{6}$       b.  $\frac{88}{105}$       c.  $\frac{17}{6}$       d.  $16\frac{1}{5}$
- e.  $\frac{8}{3}$       f. 36
- 3) a.  $\frac{1}{24}$       b.  $\left(\frac{2}{5}, \frac{2}{5}\right)$
- 4)  $\frac{M(a^2 + b^2)}{12}$
- 5)  $\frac{1}{6}\pi(5\sqrt{5} - 1)$