

# Workbook



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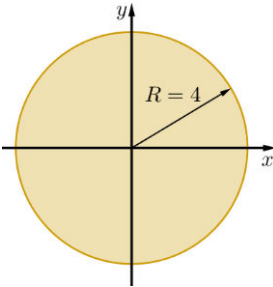
# Double Integrals in Polar Coordinates

## Double Integrals in Polar Coordinates

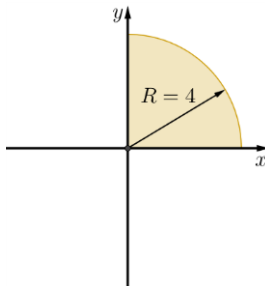
### Questions:

Compute:  $\iint_D \sqrt{x^2 + y^2} dA$  where  $D$  is the domain described in the sketch.

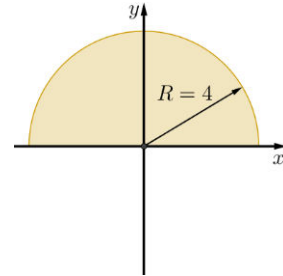
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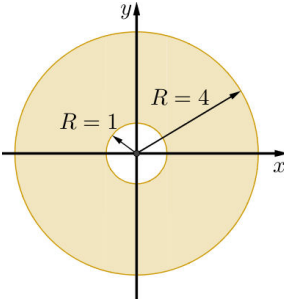
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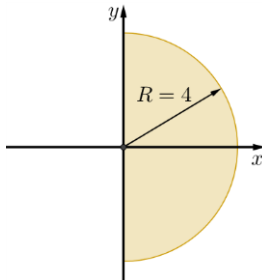
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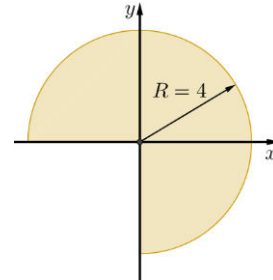
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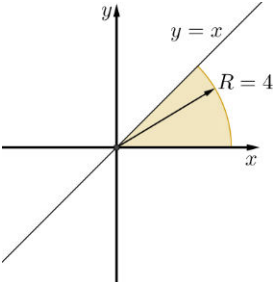
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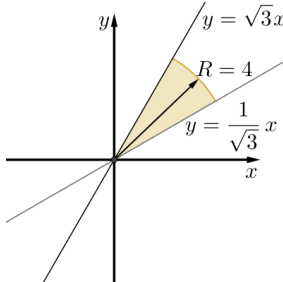
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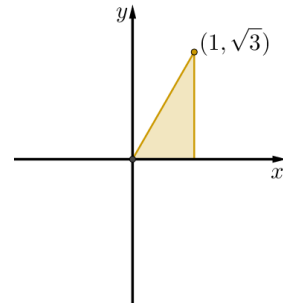
7)



8)



9)



Compute the following integrals by converting to polar coordinates:

$$10) \int_{-1}^1 \int_0^{\sqrt{1-x^2}} dy dx$$

$$12) \int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dx dy$$

$$14) \int_{-a}^a \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy dx$$

$$16) \int_0^6 \int_0^y x dx dy$$

$$18) \int_{-1}^0 \int_{-\sqrt{1-x^2}}^0 \frac{2}{1 + \sqrt{x^2 + y^2}} dy dx$$

$$20) \int_0^{\ln 2} \int_0^{\sqrt{\ln^2 2 - y^2}} e^{\sqrt{x^2 + y^2}} dx dy$$

$$22) \int_0^2 \int_0^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2 + y^2} dy dx$$

$$24) \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2 + y^2 + 1) dx dy$$

$$11) \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy dx$$

$$13) \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (x^2 + y^2) dx dy$$

$$15) \int_0^2 \int_0^{\sqrt{4-y^2}} (x^2 + y^2) dx dy$$

$$17) \int_0^2 \int_0^x y dy dx$$

$$19) \int_{-1}^1 \int_{-\sqrt{1-y^2}}^0 \frac{4\sqrt{x^2 + y^2}}{1 + x^2 + y^2} dx dy$$

$$21) \int_0^1 \int_0^{\sqrt{1-x^2}} e^{-(x^2 + y^2)} dy dx$$

$$23) \int_0^2 \int_{-\sqrt{1-(y-1)^2}}^0 xy^2 dx dy$$

$$25) \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1 + x^2 + y^2)^2} dy dx$$

In each of the following, compute the volume of the described solid:

26) The solid bounded by the sphere  $x^2 + y^2 + z^2 = 9$  and the cylinder  $x^2 + y^2 = 1$ .

27) The solid bounded inside the cylinder  $x^2 + y^2 = 2y$ , below the cone  $z = \sqrt{x^2 + y^2}$ , and above the  $xy$  plane.

28) The solid bounded inside the cylinder  $x^2 + y^2 = x$ , below the paraboloid  $z = 1 - x^2 - y^2$ , and above the  $xy$  plane.

**Answer Key:**

1)  $\frac{128\pi}{3}$

3)  $\frac{64\pi}{3}$

5)  $\frac{64\pi}{3}$

7)  $\frac{16\pi}{3}$

9)  $S = \frac{1}{3} \int_0^{\pi/3} \frac{1}{\cos^3 \theta} d\theta$

11)  $\pi$

13)  $\frac{\pi}{2}$

15)  $2\pi$

17)  $\frac{4}{3}$

19)  $\pi(4 - \pi)$

21)  $\frac{\pi(e-1)}{4e}$

23)  $-\frac{4}{5}$

25)  $\pi$

27)  $\frac{32}{9}$

2)  $\frac{32\pi}{3}$

4)  $42\pi$

6)  $32\pi$

8)  $\frac{32\pi}{9}$

10)  $\frac{\pi}{2}$

12)  $\frac{\pi}{8}$

14)  $\pi a^2$

16)  $36$

18)  $\pi \ln \frac{e}{2}$

20)  $\frac{\pi}{2} \ln \frac{4}{e}$

22)  $\frac{\pi}{2} + 1$

24)  $\pi \ln \frac{4}{e}$

26)  $\frac{(108 - 64\sqrt{2})\pi}{9}$

28)  $\frac{5\pi}{32}$