

Equations

Introduction

Questions:

- 1) For each of the following, determine if the given number is a solution to the given equation.
- a. Is $x = 6$ a solution to $2x + 1 = 3(x - 2) - 5$?
 - b. Is $x = 6$ a solution to $2x + 1 = 3(x - 2) + 1$?
 - c. Is $z = -2$ a solution to $z^2 + z + 1 = 2z + 5$?
 - d. Is $z = -2$ a solution to $z^2 + z + 1 = 2z + 7$?
 - e. Is $y = 1$ a solution to $\frac{y-1}{y^2+1} = 0$?
 - f. Is $w = -4$ a solution to $\frac{w^2 + 5w + 4}{w + 4} = 0$?

Answer Key:

- 1) a. No b. Yes c. No d. Yes e. Yes f. Undefined

Linear Equations with One Unknown

Questions:

1) Solve the following equations:

a. $6x + 2 = 8$

b. $7 - 2x = 7$

c. $2x + x = 24$

d. $2x + 6 = 8 + x$

2) Solve the following equations:

a. $-7x + 5 + 2x = 4x - 13$

b. $6x - 3 + 5 - 7x = x - 5x - 7$

c. $2 - 5x + 7 = -3x + 8$

3) Solve the following equations:

a. $3(x - 1) - 4 = 2$

b. $7x - 4(3 - 4x) = -x$

c. $6(4 - x) - (6 - x) = 3x$

d. $5x - (3x - 7)4 = 21$

4) Solve the following equations:

a. $x(x - 5) = x^2 - 7x + 8$

b. $(7 - x)(1 - x) - (x - 3)^2 = 0$

Answer Key:

1) a. $x = 1$

b. $x = 0$

c. $x = 8$

d. $x = 2$

2) a. $x = 2$

b. $x = -3$

c. $x = \frac{1}{2}$

3) a. $x = 3$

b. $x = \frac{1}{2}$

c. $x = 4\frac{1}{2}$

d. $x = 1$

4) a. $x = 4$

b. $x = -1$

Linear Equations with One Unknown, 0 or Infinite Solutions

Questions:

- 1) Solve the following equation: $6(x-2) = 2x+5+4x$.
- 2) Solve the following equation: $5x-3+x = 4x+2x-3$.

Answer Key:

- 1) No solution.
- 2) All x satisfy the equation.

Linear Equations with One Unknown in Denominator

Questions:

Solve the following equations:

- 1) $\frac{x}{3} - \frac{x}{9} = -4$ 2) $\frac{4x}{15} - \frac{3x}{10} = 1$ 3) $\frac{2}{3}x + \frac{4}{5}x = x - \frac{7}{15}$
- 4) $\frac{5x+1}{6} - \frac{6x-1}{5} = \frac{3x+1}{4} - 1$ 5) $\frac{2}{5}(x-3) - \frac{3}{15}(4-x) = x+2$
- 6) $5\left(\frac{x}{3} - \frac{x}{7}\right) - x = 1$ 7) $\frac{1}{4} - \frac{2}{x} = 0$ 8) $\frac{1}{2} - \frac{x}{x-1} = 0$
- 9) $\frac{3}{x} = \frac{1}{x+2}$ 10) $\frac{5}{2x-1} = \frac{4}{3x+2}$ 11) $\frac{x+5}{3x^2} - \frac{1}{6x} = \frac{1}{x}$
- 12) $\frac{7}{x^2-1} + \frac{2}{x+1} + \frac{3}{2-2x} = 0$ 13) $\frac{3}{(2-x)^2} + \frac{5}{12-3x^2} = 0$

14) Solve each of the following equations and check your answer:

- a. $4x+1=7(2-x)+3x+3$ b. $2(m+4)+6=4(10-2m)$
- c. $\frac{1-2z}{6} + \frac{2}{3} = \frac{3z}{4}$ d. $\frac{2y+5}{y-3} - 4 = \frac{5}{y-3}$
- e. $\frac{5w}{w-1} + \frac{18}{w+2} = 5$ f. $\frac{1}{t-1} + \frac{1}{t+1} = \frac{1}{t^2-1}$

Answer Key:

- 1) $x = -18$ 2) $x = -30$ 3) $x = -1$ 4) $x = 1$
- 5) $x = -10$ 6) $x = -21$ 7) $x = 8$ 8) $x = -1$
- 9) $x = -3$ 10) $x = -2$ 11) $x = 2$ 12) $x = -7$
- 13) $x = -7$

- 14) a. $x = 2$ b. $m = 2.6$ c. $z = \frac{10}{13}$ d. $y = 6$ e. $w = \frac{8}{23}$ f. $t = \frac{1}{2}$

Linear Equations with Two Unknowns

Questions:

Solve the following system of equations:

1)
$$\begin{cases} 3x + y = 11 \\ y = 5 \end{cases}$$

2)
$$\begin{cases} -3x + 2y = -16 \\ x = 5y + 14 \end{cases}$$

3)
$$\begin{cases} 5x - 2y = -2 \\ x + 4y = 4 \end{cases}$$

4)
$$\begin{cases} 2x + 3y = 5 \\ 5x + 7y = 11 \end{cases}$$

5)
$$\begin{cases} y = x - 3 \\ y = 2x + 4 \end{cases}$$

6)
$$\begin{cases} 3y - x + 2 = 4x + 2 - 3y \\ 2x - 3 - y = 5y - 4x + 3 \end{cases}$$

7)
$$\begin{cases} \frac{x-3}{8} - \frac{x+y}{16} = \frac{y-1}{4} \\ 3(2x-y) - 4x - 11 = 0 \end{cases}$$

8)
$$\begin{cases} \frac{3x-1}{4} - \frac{2}{5}(x-y) = \frac{3}{10}(x+3) \\ \frac{x+1}{4} - \frac{y}{2} = 1 \end{cases}$$

9)
$$\begin{cases} x + 3y = 5 \\ x - 3y = 3 \end{cases}$$

10)
$$\begin{cases} 5x + 2y = 14 \\ 5x + 3y = 23 \end{cases}$$

Answer Key:

1) (2,5)

2) (4,-2)

3) (1,0)

4) (-2,3)

5) (-7,-10)

6) (6,5)

7) (7,1)

8) (7,2)

9) $\left(4, \frac{1}{3}\right)$

10) $\left(-\frac{4}{5}, 9\right)$

Linear Equations with Two Unknowns, 0 or Infinite Solutions

Questions:

Solve the following system of equations:

1)
$$\begin{cases} x + 2y = 1 \\ 4x + 8y = 5 \end{cases}$$

2)
$$\begin{cases} 2(x - y) + 4y = 1 + x \\ 2 - 7y + x = 3(x - y) \end{cases}$$

Answer Key:

- 1) No solution.
- 2) Infinite of solutions.

Application of Linear Equations

Questions:

- 1) Jim's plumbing service charges \$40 for a service call plus \$25/hr for repair time.
 - a. Using t , for time in hours, and c , for the total cost, write an equation that describes how much Jim would charge for a service call and repair.
 - b. Solve explicitly for t , in terms of c , using your equation in part a.

- 2) A social group bought a total of 200 orchestra and balcony tickets to a concert for a total of \$2,080. Orchestra tickets cost \$12 each and balcony tickets cost \$8 each. How many of each ticket type did the group bought?

- 3) After cycling at a certain speed for 4 hours, Joe realizes that he could have covered the same distance in 3 hours if he had driven 5 mph faster. What distance did he travel during the 4 hour trip?

- 4) The price of an electrical appliance is \$142.60, after adding a sales tax of 6%. What is the price of the appliance before tax?

- 5) A clothes store is having a 30% off end-of-season sale on all items. The sale price of a shirt is \$19.95. What was the original price of the shirt?

- 6) Two trains start out 280 miles apart and travel towards each other. They meet after $3\frac{1}{2}$ hours. One of the trains travels 10 mph faster than the other. How fast each of them traveled?

- 7) A car and a truck start travelling north simultaneously on the same highway. At the start, the truck is 35 km north of the car. The car travels at 90 km/h and the truck at 70 km/h. After how long the car will catch up with the truck?

- 8) A water pump can fill a reservoir in 8 hours, and a different pump can fill the same reservoir in 13 hours. How long would it take for both pumps, working simultaneously, to fill the reservoir?

- 9) Tom can paint a fence in 14 hours. Tom and Becky can paint the fence together in 9 hours. How long would it take Becky to paint the fence on her own?

- 10) How much Vermouth (17% alcohol) should be mixed with 50 ml. of gin (40% alcohol), to get a 30% alcohol cocktail?
- 11) We have two concentrations of sulfuric acid solutions: 15% and 40% (in water). How much of each should we mix together to get 100 liters of a 25% solution? Assume we have sufficient quantities of the 15% and 40% solutions.
- 12) A rectangular field is surrounded by 90 meters of fence. The field is twice as long as it is wide. What are the dimensions of the field?

Answer Key:

- 1) a. $c = 40 + 25t$ b. $t = \frac{c - 40}{25}$
- 2) 120 orchestra tickets and 80 balcony tickets.
- 3) 60 miles.
- 4) \$134.53
- 5) \$28.50
- 6) B travelled at 35 mph and A travelled at 45 mph.
- 7) After 1.75 hours = 1hr 45min.
- 8) 4 hours 57 min.
- 9) 25 hours 12 min.
- 10) 38.46 ml of Vermouth.
- 11) 60 liters of 15% solution and 40 liters of 40% solution.
- 12) The field is 15m × 30m.

Quadratic Equations

Questions:

Solve the following equations:

1) $x^2 - 36 = 0$

2) $32x^2 - 18 = 0$

3) $-7x^2 - 14x = 0$

4) $5x^2 - x = 0$

5) $x^2 + 3x - 10 = 0$

6) $-x^2 + 10x - 16 = 0$

7) $25x^2 - 20x + 4 = 0$

8) $2x^2 - 6x + 5 = 0$

- 9) The length of a rectangle is 7 inches more than its width.
The diagonal of the rectangle is 17 inches.
Find the dimensions of the rectangle.

Answer Key:

1) $x = \pm 6$

2) $x = \pm \frac{3}{4}$

3) $x = 0, -2$

4) $x = 0, \frac{1}{5}$

5) $x = 2, -5$

6) $x = 2, 8$

7) $x = \frac{2}{5}$

8) No solution

9) 15×8 inches

Disarranged Quadratic Equations

Solve the following equations:

1) $4x^2 - 5x + 7 = 4 - x^2 + 3$

2) $-x(x-5) = (1-3x)(1-x) + 4$

3) $2(x-5)^2 - (2x-3)^2 = 10x + 21$

Answer Key:

1) $x = 1, 0$

2) $x = 1\frac{1}{4}, 1$

3) $x = 1, -10$

Disguised Quadratic Equations

Questions:

Solve the following equations:

1) $\frac{x^2+2}{3x^2+5x} = \frac{3x-1}{9x+15}$

2) $\frac{4x^2-24x+36}{x-3} = 12$

3) $\frac{4x+1}{3} - \frac{x+2}{2} = \frac{2}{x}$

4) $\frac{x^2-9}{x+3} + x = x^2 - 18$

5) $\frac{3}{2x+2} - \frac{2x-5}{2(x-1)^2} - \frac{4}{1-x^2} = 0$

Answer Key:

- 1) $x = -6$ 2) $x = 6$ 3) $x = -1\frac{1}{5}, 2$ 4) $x = 5$
- 5) $x = 0, -5$

Nonlinear Equations

Questions:

Find the solution for the following system of equations:

$$1) \begin{cases} \frac{3}{x} + \frac{1}{y} = 4 \\ \frac{5}{x} - \frac{1}{y} = 4 \end{cases}$$

$$2) \begin{cases} \frac{3}{x} + \frac{3}{y} = 2 \\ \frac{9}{x} - \frac{4}{y} = -7 \end{cases}$$

$$3) \begin{cases} 4x - \frac{7}{y} = -3 \\ 5x + \frac{2}{y} = 7 \end{cases}$$

$$4) \begin{cases} x(y+2) + y = xy - 5 \\ x - y = 2 \end{cases}$$

$$5) \begin{cases} xy = 20 \\ y(3x-4) = 20 \end{cases}$$

$$6) \begin{cases} 5x - 4xy = 22 \\ 6x + xy = -20 \end{cases}$$

$$7) \begin{cases} y = x^2 + 6x - 7 \\ y = 4x + 8 \end{cases}$$

$$8) \begin{cases} y = x^2 + 3x - 2 \\ y - 4x = -2 \end{cases}$$

$$9) \begin{cases} y = 1 - 3x \\ \frac{x^2}{2} + 2y^2 = 2 \end{cases}$$

$$10) \begin{cases} y - 2x = -1 \\ x^2 + 2y^2 = 3 \end{cases}$$

$$11) \begin{cases} xy = 4 \\ 25x^2 + 4y^2 = 100 \end{cases}$$

$$12) \begin{cases} xy = 6 \\ \frac{x^2}{2} + \frac{y^2}{3} = 5 \end{cases}$$

$$13) \begin{cases} y = 1 - 2x^2 \\ 3x^2 - \frac{y^2}{3} = 3 \end{cases}$$

$$14) \begin{cases} y + 4x^2 = 6 \\ 3x^2 - \frac{y^2}{4} = 2 \end{cases}$$

Answer Key:

$$1) (1,1) \quad 2) (-3,1) \quad 3) (1,1) \quad 4) (-1,-3)$$

$$5) (2,10) \quad 6) (-2,4) \quad 7) (3,20), (-5,-12)$$

$$8) (0,-2), (1,2) \quad 9) (0,1), \left(\frac{24}{37}, -\frac{35}{37}\right) \quad 10) (1,1), \left(-\frac{1}{9}, -\frac{11}{9}\right)$$

$$11) \left(\frac{4}{\sqrt{5}}, \sqrt{5}\right), \left(-\frac{4}{\sqrt{5}}, -\sqrt{5}\right), \left(\frac{2}{\sqrt{5}}, 2\sqrt{5}\right), \left(-\frac{2}{\sqrt{5}}, -2\sqrt{5}\right)$$

$$12) (2,3), (-2,-3), (\sqrt{6}, \sqrt{6}), (-\sqrt{6}, -\sqrt{6})$$

$$13) (\sqrt{2}, -3), (-\sqrt{2}, -3), \left(\frac{\sqrt{5}}{2}, -\frac{3}{2}\right), \left(-\frac{\sqrt{5}}{2}, -\frac{3}{2}\right) \quad 14) \emptyset$$

Equations with Roots

Questions:

1) Solve the following equations:

a. $\sqrt{4x-3} = 5$

b. $\sqrt{x+2} = x$

2) Solve the following equations:

a. $\sqrt{3x+1} + x = 13$

b. $2x = 16 - 3\sqrt{x-1}$

3) Solve the following equations:

a. $\sqrt{x-1} \cdot \sqrt{2x-5} = \sqrt{11-x^2}$

b. $\sqrt{2x-1} + 3 = \sqrt{7x+1}$

4) Solve the following equations:

a. $\sqrt{9x-8} - 3\sqrt{x+4} = -2$

b. $\sqrt{2x-3} + \sqrt{3-x} = 2$

5) Solve the following equations:

a. $\sqrt{x+3} + \sqrt{x-2} = \sqrt{4x+1}$

b. $\sqrt{2x-2} + \sqrt{5x-4} = \sqrt{3x-2}$

Answer Key:

1) a. $x = 7$

b. $x = 2$

2) a. $x = 8$

b. $x = 5$

3) a. $x = 3$

b. $x = 5$

4) a. $x = 12$

b. $x = 2\frac{2}{8}, 2$

5) a. $x = 6$

b. $x = 1$

Absolute Value Equations

Questions:

1) Solve the following equations:

a. $|x-4|=6$

b. $|2x+5|=11$

2) Solve the following equations:

a. $x+|4x-3|=2$

b. $2|x-1|=x+3$

3) Solve the following equations:

a. $3|2-x|-|2x+3|=-2$

b. $|x-1|-|3x-1|=0$

4) Solve the following equations:

a. $x+3+|x+3|=2|x-1|$

b. $||x-3|+|2x-1||=4$

5) Solve the following equations:

a. $|5+x|-1=|3x-2|$

b. $|x-4|+|4-x|=4$

Answer Key:

1) a. $x=10,-2$

b. $x=3,-8$

2) a. $x=1,\frac{1}{3}$

b. $x=5,-\frac{1}{3}$

3) a. $x=7,1$

b. $x=0,\frac{1}{2}$

4) a. No solution.

b. $x=0,2$

5) a. $x=-\frac{1}{2},3$

b. $x=6,2$