

Everything You Always Wanted to Know About Functions

The Concept of a Function

Theory sections that includes the following topics:

- General overview about 'what is a function'.
- Notation of functions.
- The domain and definition of a function.
- Graphical Description of a function.
- Positivity and Negativity of a Function.

Common Functions

Theory sections that includes the following topics:

- The Linear Function.
- The Quadratic Function.
- The Exponent Function.
- The Logarithmic Function.
- Basic Functions.

The Domain of Definition of a Function

Theory section which explains about the domain of a function.

The Domain of Basic Functions

Questions:

In exercise 1-8 find the domain of the function for each function described:

1) $y = x^3 - x^2 - 4x + 1$

2) $y = \frac{4x+1}{x+1}$

3) $y = \frac{1}{x^2-4}$

4) $y = \frac{1}{x^3-x}$

5) $y = \frac{x^2}{x^2-x-2}$

6) $y = \sqrt{x-4}$

7) $y = \sqrt{x^2+x-2}$

8) $y = \sqrt[3]{x^2+x-1}$

Final Answers:

1) All x

2) All x

3) $x \neq \pm 2$

4) $x \neq 0, \pm 1$,

5) $x \neq 2, x \neq -1$

6) $x \geq 4$

7) $x \leq -2, x \geq 1$

8) All x

The Domain of Logarithmic and Exponent Functions

Questions:

In exercises 1-10 find the domain of the function for each function described

1) $y = e^{2x^2-4x+1} + x + 8$

2) $y = e^{\frac{1}{x}} + 4x + 3$

3) $y = \ln(x-1) + \ln(x+3)$

4) $y = \sqrt{e^{2x}-4}$

5) $y = \sqrt{e^{2x} + e^x - 2}$

6) $y = \ln(2x^2 + 3x - 5)$

7) $y = \frac{\ln(x+1)}{\ln(x-1)}$

8) $y = \ln\left(\frac{x+3}{x-7}\right)$

9) $y = \sqrt{\ln x + 5}$

10) $y = \frac{1}{\sqrt{\ln^2 x - 5 \ln x + 6}}$

Final Answers:

- 1) All x 2) $x \neq 0$ 3) $x > 1$ 4) $x \geq \frac{1}{2} \ln 4$
5) $x \geq 0$ 6) $x > 1, x < 2.5$ 7) $x > 1$ 8) $x < -3, x > 7$
9) $x \geq e^{-5}$ 10) $0 < x < e^2, x > e^3$

Domain of Trigonometric Functions

Questions:

In exercise 1-5 find the domain of the function for each function described:

- 1) $y = \sin 2x + \cos 3x$ 2) $y = \tan 2x$ 3) $y = \cot 4x$
4) $y = \arcsin(2x)$ 5) $y = \arccos(3x)$

Final Answers:

- 1) All x 2) $x \neq \frac{\pi}{4} + k\frac{\pi}{2}$ 3) $x \neq k\frac{\pi}{4}$ 4) $-\frac{1}{2} \leq x \leq \frac{1}{2}$ 5) $-\frac{1}{3} \leq x \leq \frac{1}{3}$

Domain of Absolute Value Functions

Questions:

In exercise 1-2 find the domain of the function for each function described:

- 1) $y = \frac{1}{\sqrt{|x|}-4}$ 2) $y = \ln(2-|x|)$

Final Answers:

- 1) $x < -4, x > 4$ 2) $x < -2, x > 2$

Domain of a Piecewise Function

Questions:

- 1) find the domain of the function for the function describe $f(x) = \begin{cases} 1/x & x < 0 \\ \sqrt{x} & 0 \leq x < 4 \\ 1/(x-8) & 4 < x < 20 \end{cases}$.

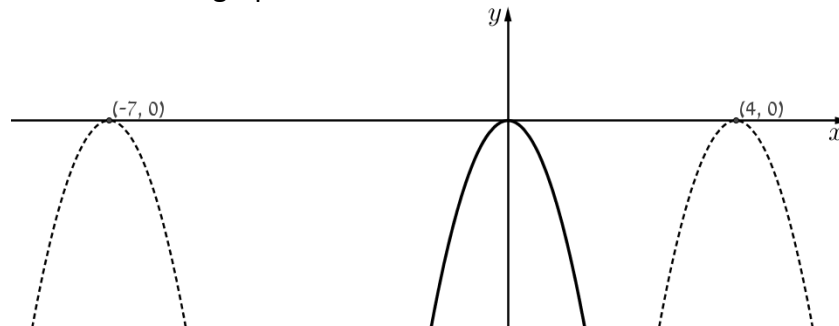
Final Answers:

- 1) $x < 0, 0 \leq x < 4, 4 < x < 8, 8 < x < 20$.

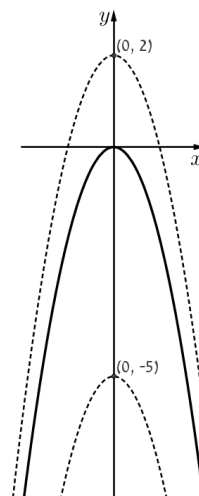
Translation (Shifting) and Reflection of Functions

Questions:

- 1) The accompanying figure shows the graph of $y = -2x^2$ shifted to two new positions. Write equations for the new graphs.



- 2) The accompanying figure shows the graph of $y = -2x^2$ shifted to two new positions. Write equations for the new graphs.



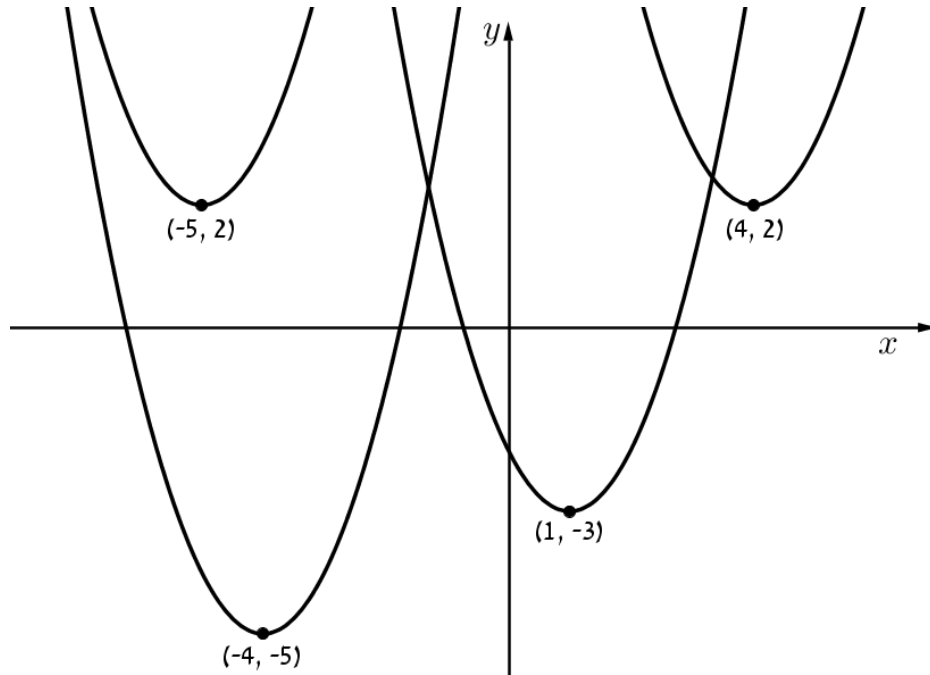
3) Match the equations listed in parts 1-4 to the graphs in the accompanying figure:

1. $y = (x - 4)^2 + 2$

2. $y = (x - 1)^2 - 3$

3. $y = (x + 5)^2 + 2$

4. $y = (x + 4)^2 - 5$



4) In Parts 1-8 tell how many units and in what directions the graphs of the given equations are to be shifted. Give an equation for the shifted graph, then sketch the original and shifted graphs together, labeling each graph with its equation.

1. $y = x^3$ Left 2, down 1

2. $y = x^{\frac{2}{3}}$ Right 1, down 3

3. $y = \sqrt{x}$ Left 1

4. $y = -\sqrt{x}$ Right 4

5. $y = 2x - 7$ Up 4

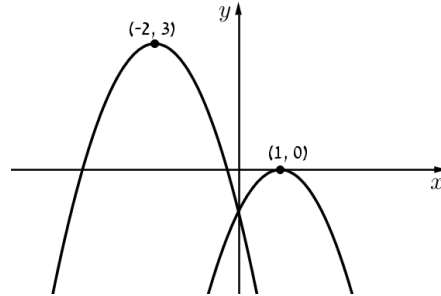
6. $y = \frac{1}{2}(x + 1) + 2$ Down 5, Right 2

7. $y = \frac{1}{x}$ Up 3, right 1

8. $y = \frac{1}{x^2}$ Left 2, down 3

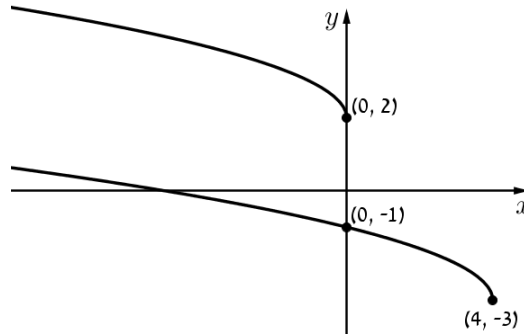
5) Match the equations listed in parts 1-2 to the graphs in the accompanying figure:

1. $y = -(x-1)^2$
2. $y = -(x+2)^2 + 3$



6) Match the equations listed in parts 1-2 to the graphs in the accompanying figure:

1. $y = \sqrt{-x} + 2$
2. $y = \sqrt{4-x} - 3$



7) Graph the functions in parts 1-10:

- | | | | |
|-------------------------|-----------------------|----------------------|--------------------------|
| 1. $y = x^2 - 4x + 6$ | 2. $y = \sqrt{4-x}$ | 3. $y = x-4 $ | 4. $y = 2-x - 3$ |
| 5. $y = 1 + \sqrt{x-3}$ | 6. $y = 3 - \sqrt{x}$ | 7. $y = (x+4)^{2/3}$ | 8. $y = (x-1)^{2/3} - 2$ |
| 9. $y + 1 = x^{2/3}$ | 10. $y = 1 - x^{2/3}$ | | |

8) Graph the functions in parts 1-10:

- | | | |
|---------------------------------|----------------------------|----------------------------|
| 1. $y = \sqrt[3]{x-4} - 2$ | 2. $y = (x+1)^{3/2} + 3$ | 3. $y = \frac{1}{x-6}$ |
| 4. $y = \frac{1}{x} - 6$ | 5. $y = \frac{1}{x} + 5$ | 6. $y = \frac{1}{x+5} - 2$ |
| 7. $y = \frac{1}{(x-2)^2}$ | 8. $y = \frac{1}{x^2} - 2$ | 9. $y = \frac{1}{x^2} + 4$ |
| 10. $y = \frac{1}{(x+4)^2} + 2$ | | |

Final Answers:

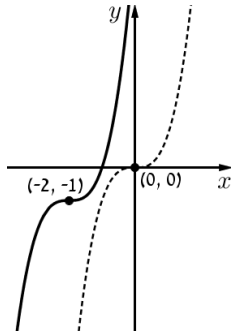
1) $y = -2(x-4)^2$ and $y = -2(x+7)^2$

2) $y = -2x^2 + 2$ and $y = -2x^2 - 5$

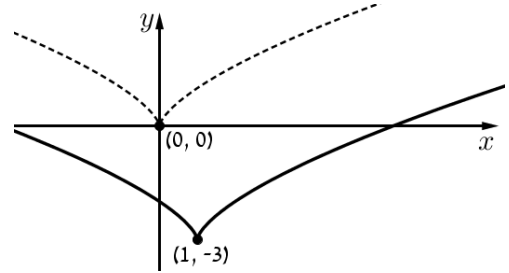
3) D-3 F-4 B-2 A-1

4) Final Drawings are as follows:

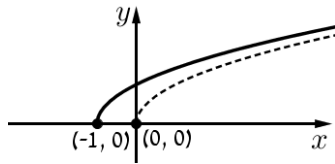
1. $y = (x+2)^3 - 1$



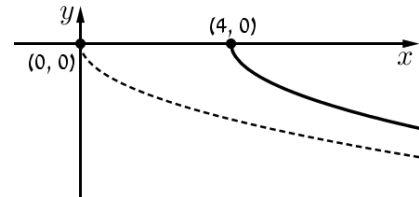
2. $y = (x-1)^{2/3} - 3$



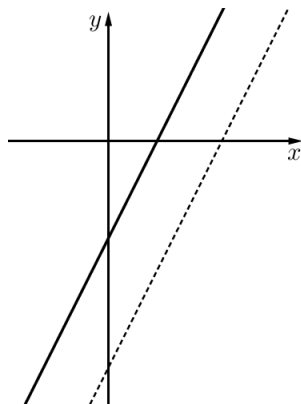
3. $y = \sqrt{x+1}$



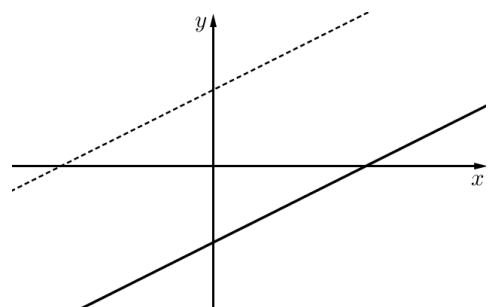
4. $y = -\sqrt{x-4}$



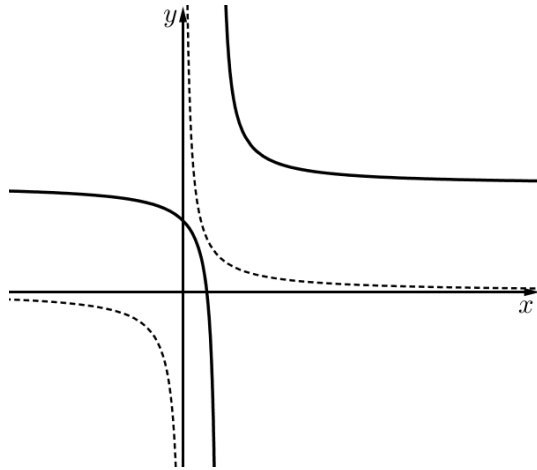
5. $y = 2x - 3$



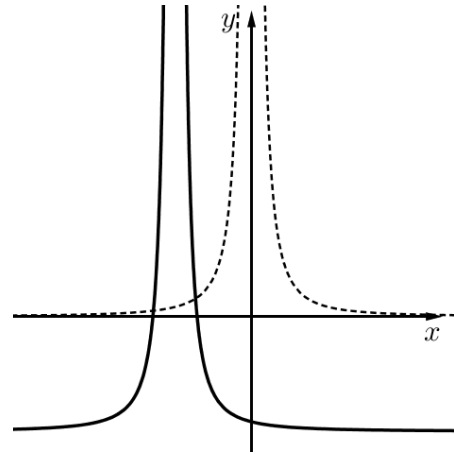
6. $y = \frac{1}{2}(x-1) - 3$



7. $y = \frac{1}{(x-1)} + 3$



8. $y = \frac{1}{(x+2)^2} - 3$

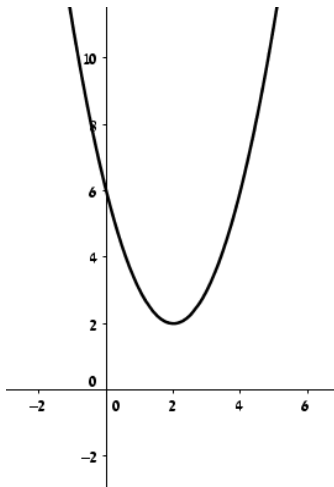


5) B-2 A-1

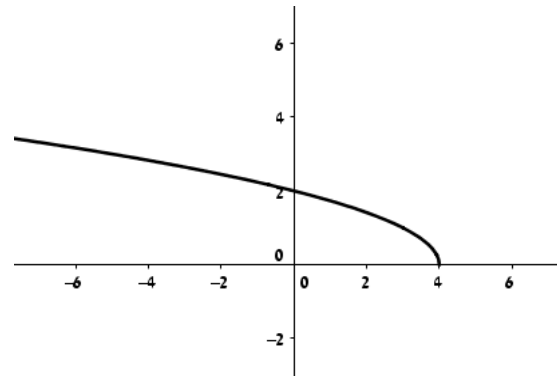
6) A-1 B-2

7) Final Drawings are as follows:

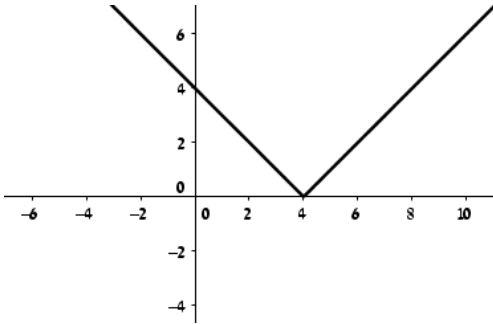
1. $y = x^2 - 4x + 6$



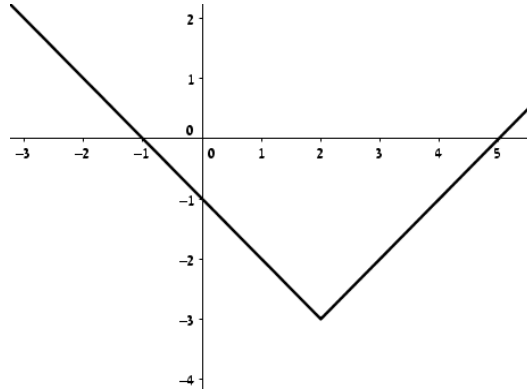
2. $y = \sqrt{4-x}$



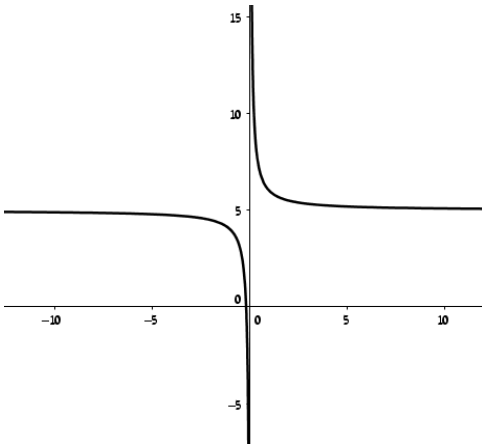
3. $y = |x - 4|$



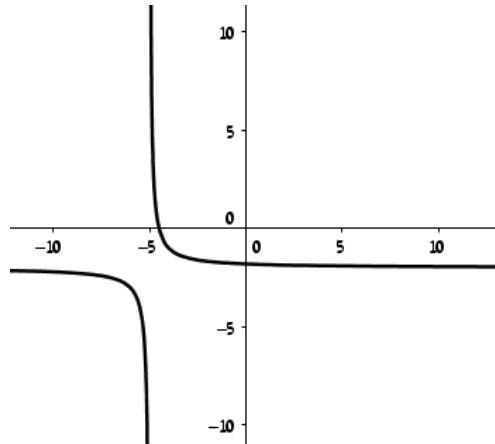
4. $y = |2 - x| - 3$



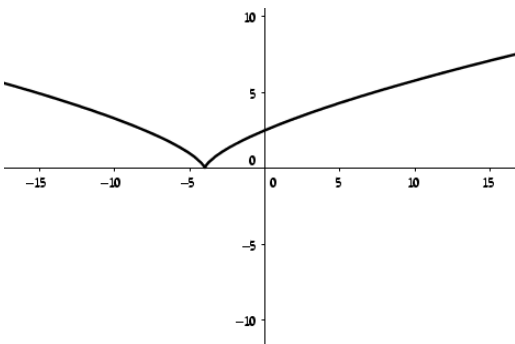
5. $y = \frac{1}{x} + 5$



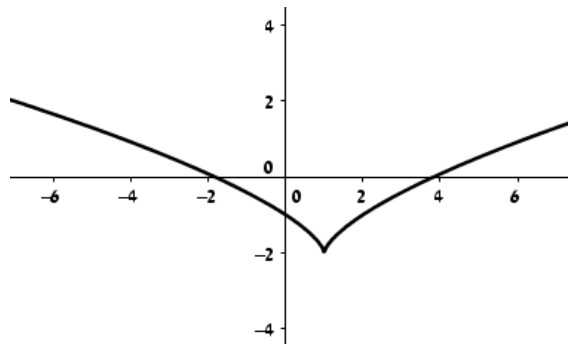
6. $y = \frac{1}{x + 5} - 2$



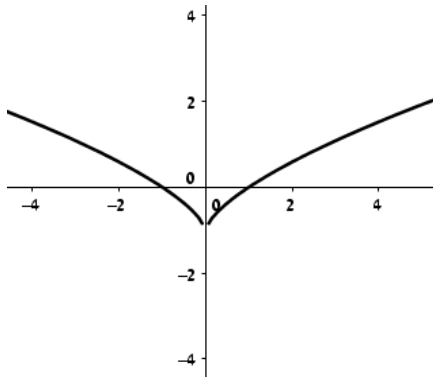
7. $y = (x + 4)^{2/3}$



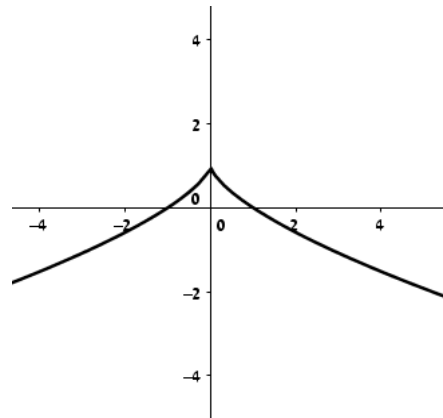
8. $y = (x - 1)^{2/3} - 2$



9. $y + 1 = x^{\frac{2}{3}}$

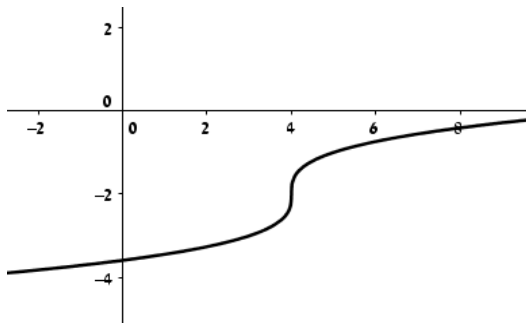


10. $y = 1 - x^{\frac{2}{3}}$

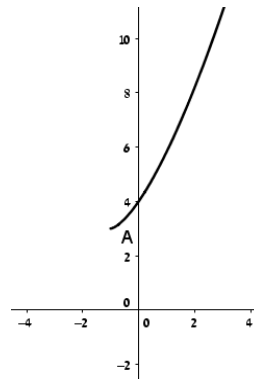


8) As follows:

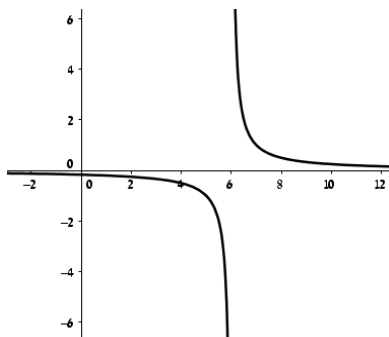
1. $y = \sqrt[3]{x-4} - 2$



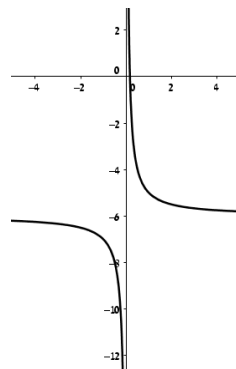
2. $y = (x+1)^{3/2} + 3$



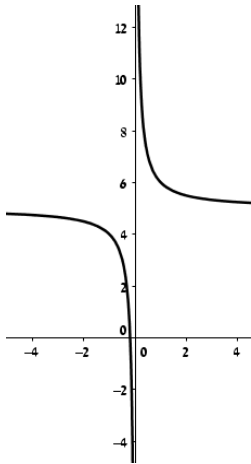
3. $y = \frac{1}{x-6}$



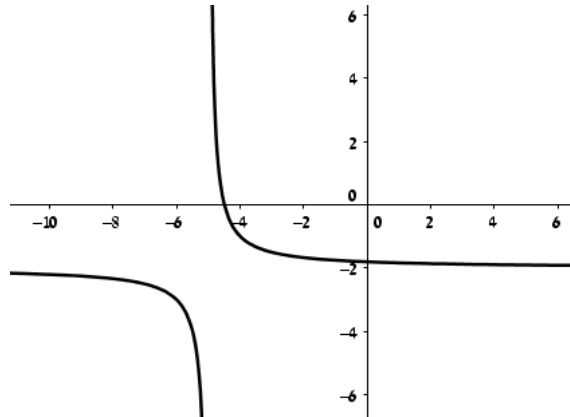
4. $y = \frac{1}{x} - 6$



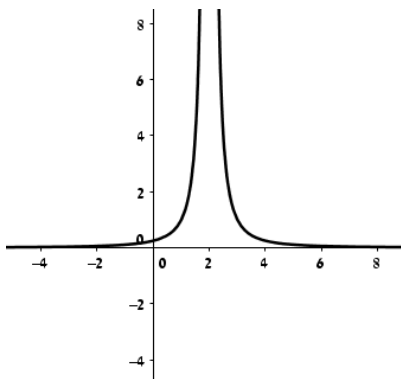
5. $y = \frac{1}{x} + 5$



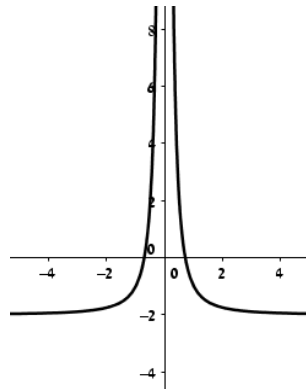
6. $y = \frac{1}{x+5} - 2$



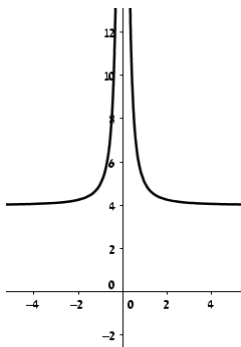
7. $y = \frac{1}{(x-2)^2}$



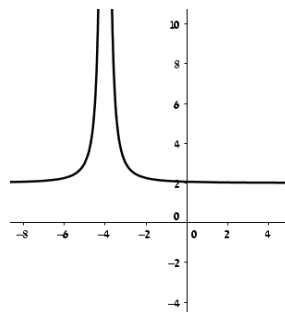
8. $y = \frac{1}{x^2} - 2$



9. $y = \frac{1}{x^2} + 4$



10. $y = \frac{1}{(x+4)^2} + 2$



Composed Functions

Questions:

- 1) If $f(x) = x - 4$ and $g(x) = x^2$ then what is $(f \circ g)(1)$?
- 2) If $f(x) = x - 4$ and $g(x) = x^2$ then what is $(g \circ f)(1)$?
- 3) If $f(x) = x - 4$ and $g(x) = x^2$ then what is $(f \circ g)(x)$?
- 4) If $f(x) = x - 4$ and $g(x) = x^2$ then what is $(g \circ f)(x)$?
- 5) If $f(x) = x - 4$ then what is $(f \circ f)(x)$?
- 6) If $f(x) = x - 4$, $g(x) = x^2$ and $h(x) = 4/x$ then what is $h(g(f(5)))$?
- 7) If $f(x) = x - 4$, $g(x) = x^2$ and $h(x) = 4/x$ then what is $h(g(f(x)))$?
- 8) If $f(x) = x - 4$, $g(x) = x^2$ and $h(x) = 4/x$ then what is $h(g(x))$?
- 9) Given: $f(x) = x^2 + 4$ and $g(x) = \sqrt{x - 4}$
 - a. What is $(f \circ g)(x)$?
 - b. What is the domain of $(f \circ g)(x)$?
 - c. What is the domain of $(g \circ f)(x)$?
- 10) $g(x) = \frac{1 - 2x}{x + k}$, $f(x) = \frac{x + 1}{x + 2}$, k is constant.
Find the value of the constant k for which $f(g(x)) = x$.
- 11) f and g are both defined on the set of real numbers and a is a constant.
 $g(x) = ax - 3$, $f(x) = ax + 5$. If $(f \circ g)(x) = (g \circ f)(x)$ for all values of x , what is the value of a ?
- 12) If $f(x) = \begin{cases} 4x + 3 & x < 5 \\ 2x & x \geq 5 \end{cases}$ and $g(x) = \begin{cases} 3 & x \geq 1 \\ 2 & x < 1 \end{cases}$ then what is $f \circ g$?

Final Answers:

1) -3.

2) 9.

3) $x^2 - 4$.

4) $x^2 - 8x + 16$.

5) $x - 8$.

6) 4.

7) $\frac{4}{(x-4)^2}$.

8) $\frac{4}{x^2}$.

9) a. x b. $x \geq 4$ c. all x .

10) $k = -1$.

11) $a = 1$.

12) $(f \circ g)(x) = \begin{cases} 15 & , x \geq 1 \\ 11 & , x < 1 \end{cases}$.

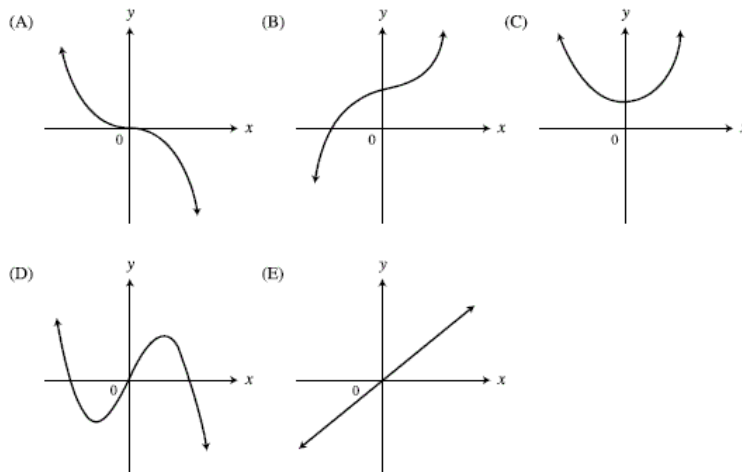
Even and Odd Functions

Questions:

Which of the following functions are odd, are even, are neither?

- | | | |
|------------------------------------|------------------------------------|---------------------------------------|
| 1) $f(x) = 4x^3$ | 2) $f(x) = x^4 + x^{10}$ | 3) $y = 1$ |
| 4) $y = \frac{1}{x} + \sqrt[3]{x}$ | 5) $y = x^2 + x^3$ | 6) $y = \frac{x^3 + 4x}{ x + 1}$ |
| 7) $y = \frac{1}{2}(e^x + e^{-x})$ | 8) $y = \frac{1}{2}(e^x - e^{-x})$ | 9) $y = \ln x + x + 1$ |
| 10) $y = \ln^2 x + x^2$ | 11) $f(x) = \sin x \cdot \cos x$ | 12) $y = \frac{\tan x}{x^2 + \cos x}$ |
| 13) $y = \ln(\cos x)$ | | |

- 14) Using the graph of the function, say whether the function is even, odd, or neither. Give reasons for your answer:



- 15) Prove the following statements:
- The sum of two even functions is even.
 - The sum of two odd functions is odd.
 - The product of two even functions is an even function.
 - The product of two odd functions is an even function.
 - The product of an even function and an odd function is an odd function.

Final Answers:

Odd: (1), (4), (6), (8), (11), (12). Even: (2), (3), (7), (13). Neither: (5), (9), (10).

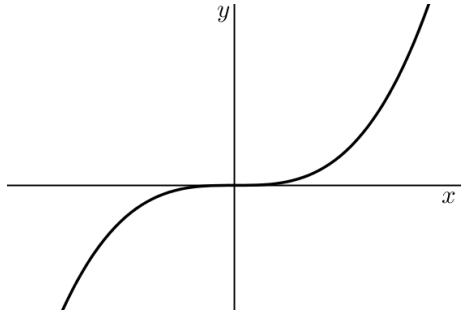
14) a. Odd b. neither c. Even d. Odd e. Odd.

One to One Functions

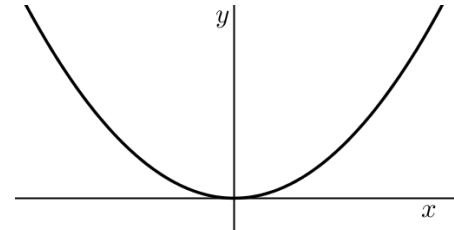
Questions:

1) Which of the functions are one-to-one?

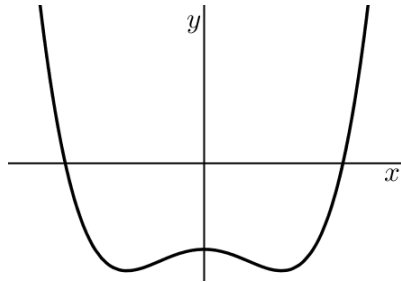
a.



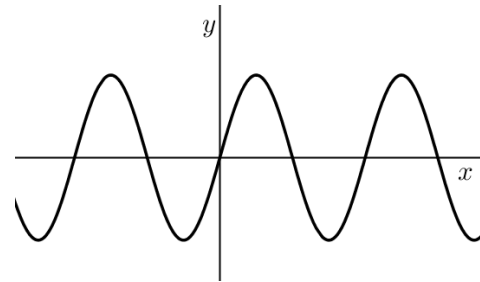
b.



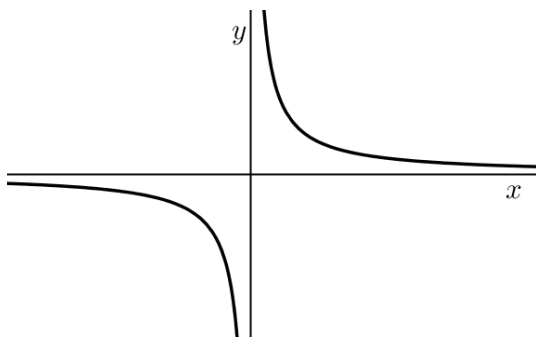
c.



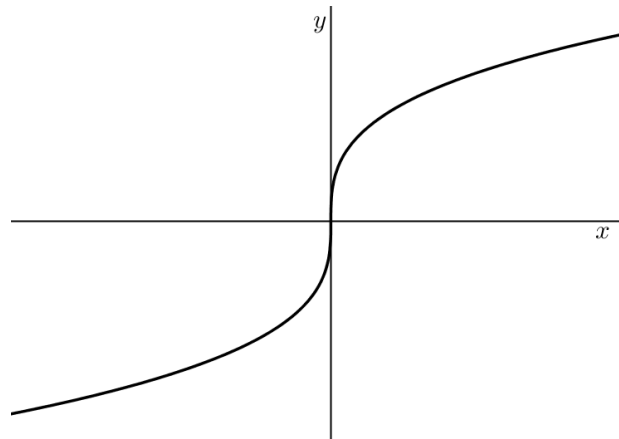
d.



e.



f.



2) Determine whether the given function is one-to-one.

- a. $y = x^2$
- b. $f(x) = \frac{x+1}{2}$
- c. $f(x) = 4x+3$

3) Determine whether the given function is one-to-one.

- a. $f(x) = \frac{2x+1}{4x-2}$
- b. $f(x) = \frac{1}{x^3}$
- c. $f(x) = x^2 \ (x \geq 0)$

4) Determine whether the given function is one-to-one.

- a. $f(x) = 2(x-3)^2 - 4 \ (x \geq 3)$
- b. $f(x) = x^2 - 4x + 5 \ (x \leq 2)$
- c. $f(x) = \frac{x^2}{x^2 + 1} \ (x \geq 0)$

5) Determine whether the given function is one-to-one.

- a. $f(x) = 4 \ln x$
- b. $f(x) = 2 + 3 \ln(x-1)$
- c. $f(x) = 1 + 2e^{2x}$

Final Answers:

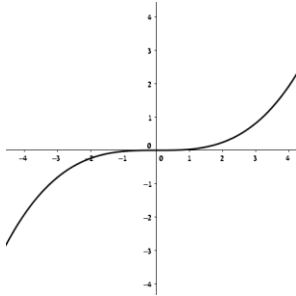
- | | | | | | | |
|----|--------|--------|--------|-------|--------|--------|
| 1) | a. yes | b. no | c. no | d. no | e. yes | f. yes |
| 2) | a. no | b. yes | c. yes | | | |
| 3) | a. yes | b. yes | c. yes | | | |
| 4) | a. yes | b. yes | c. yes | | | |
| 5) | a. yes | b. yes | c. yes | | | |

The Inverse of a Function

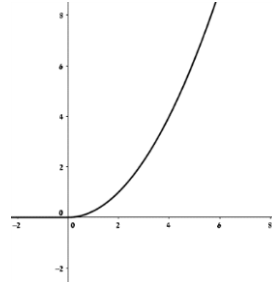
Questions:

- 1) Each of sections a-f shows the graph of a function $y = f(x)$. Copy the graph and draw in the line $y = x$, then use symmetry with respect to the line $y = x$ to add the graph of f^{-1} to your sketch.

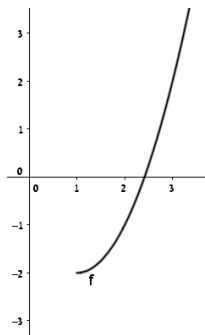
a.



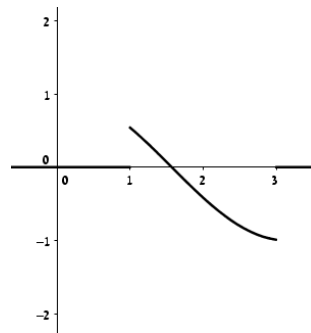
b.



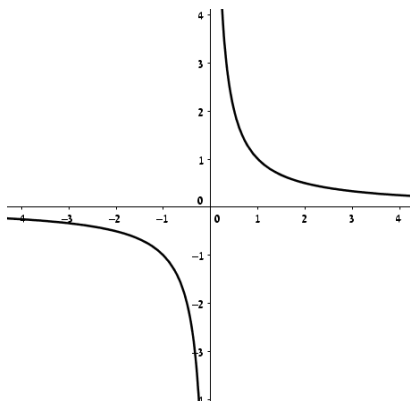
c.



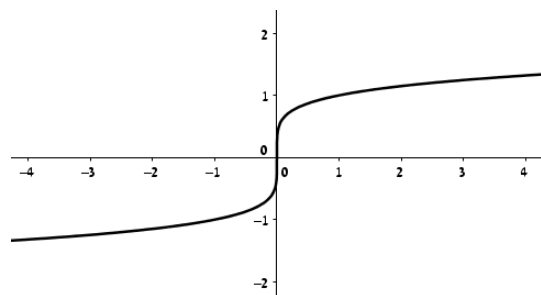
d.



e.



f.



- 2) Each of sections a-c gives a formula for a function $y = f(x)$.

In each case, find $f^{-1}(x)$ and identify the Image of the function f .

As a check, show that $f(f^{-1}(x)) = x$.

- $y = x^2$
- $f(x) = (x+1)/2$
- $f(x) = 4x+3$

- 3) Each of sections a-c gives a formula for a function $y = f(x)$.

In each case, find $f^{-1}(x)$ and identify the Image of the function f .

As a check, show that $f(f^{-1}(x)) = x$.

- $f(x) = \frac{2x+1}{4x-2}$
- $f(x) = \frac{1}{x^3}$
- $f(x) = x^2 \quad (x \geq 0)$

- 4) Each of sections a-c gives a formula for a function $y = f(x)$.

In each case, find $f^{-1}(x)$ and identify the Image of the function f .

As a check, show that $f(f^{-1}(x)) = x$.

- $y = 2(x-3)^2 - 4 \quad (x \geq 3)$
- $y = x^2 - 4x + 5 \quad (x \leq 2)$
- $y = \frac{x^2}{x^2 + 1} \quad (x \geq 0)$

- 5) Each of sections a-c gives a formula for a function $y = f(x)$.

In each case, find $f^{-1}(x)$ and identify the Image of the function f .

As a check, show that $f(f^{-1}(x)) = x$.

- $f(x) = 4 \ln x$
- $f(x) = 2 + 3 \ln(x-1)$
- $f(x) = 1 + 2e^{2x}$

- 6) Find the inverse function of: $f(x) = \log_2\left(x + \frac{1}{x}\right), x > 1$.

Final Answers:

1) Graphs at the video.

2) a. no inverse , $y \geq 0$

b. $y = 2x - 1$, all y

c. $y = \frac{x-3}{4}$, all y .

3) a. $y = \frac{2x+1}{4x-2}$, $y \neq \frac{1}{2}$

b. $\sqrt[3]{\frac{1}{x}}$, $y \neq 0$

c. $y = \sqrt{x}$, $y \geq 0$.

4) a. $y = 3 + \sqrt{\frac{x+4}{2}}$, $y \geq -4$

b. $y = 2 - \sqrt{x-1}$, $y \geq 1$

c. $y = \sqrt{\frac{-x}{x-1}}$, $0 \leq y \leq 1$.

5) a. $y = e^{\frac{x}{4}}$, all y

b. $y = e^{\frac{x-2}{3}} + 1$, all y

c. $\frac{1}{2} \log\left(\frac{x-1}{2}\right)$, $y > 1$.

6) $f^{-1}(x) = \frac{2^x + \sqrt{2^{2x} - 4}}{2}$, $x \geq 1$.

Piecewise Defined Functions

Questions:

In exercise 1-3 graph the function described:

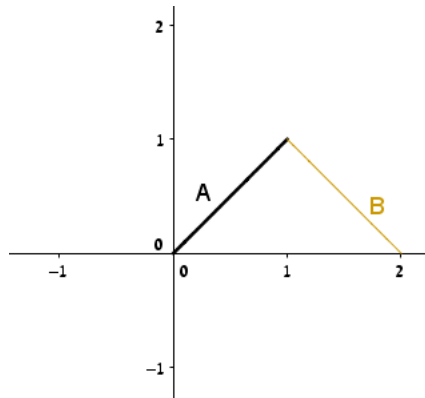
$$1) f(x) = \begin{cases} x & , 0 \leq x \leq 2 \\ 4-x & , 2 \leq x \leq 3 \end{cases}$$

$$2) g(x) = \begin{cases} 2-x & , 0 \leq x \leq 1 \\ 3-x & , 1 \leq x \leq 2 \end{cases}$$

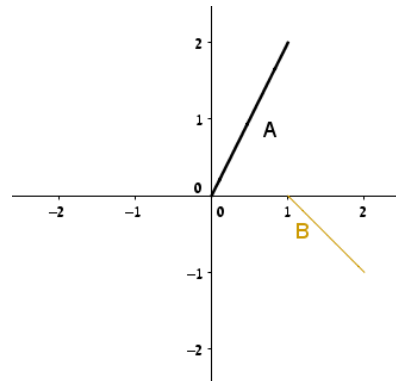
$$3) f(x) = \begin{cases} x^2 & , 0 \leq x \leq 1 \\ \frac{1}{x} & , 1 \leq x \leq 2 \end{cases}$$

4) Find a formula for each function graphed:

a.

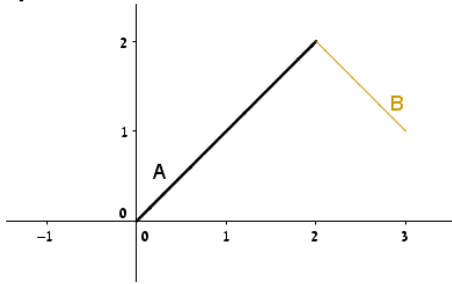


b.

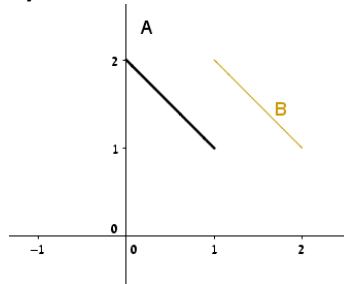


Final Answers:

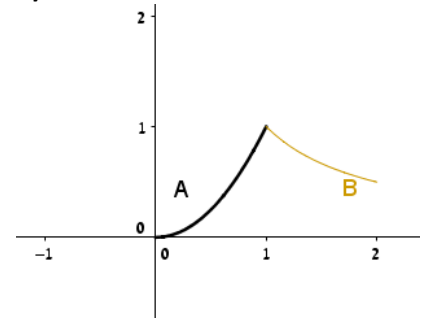
1)



2)



3)



4)
$$y = \begin{cases} x & , 0 \leq x \leq 1 \\ -x+2 & , 1 < x < 2 \end{cases}$$

5)
$$y = \begin{cases} 2x & , 0 \leq x < 1 \\ -x+1 & , 1 \leq x < 2 \end{cases}$$

The Absolute Value Function

Questions:

In exercise 1-3, graph the function described:

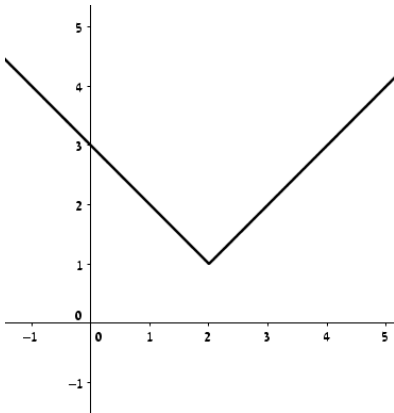
1) $f(x) = 1 + |x - 2|$

2) $f(x) = x^2 + 2|x + 1| + 1$

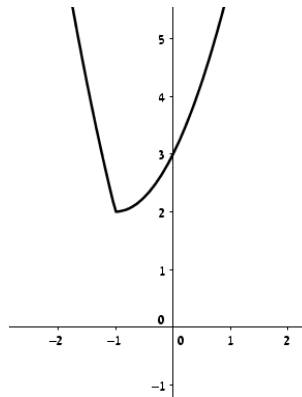
3) $f(x) = |x^2 + x - 2|$

Final Answers:

1)



2)



3)

