

Exponents, Exponential Equations

Exponents

Questions:

1) Simplify each of the following and write the answers with only positive exponents:

a. $a^3b^2a^4b^5$

b. $\frac{x^7y^5}{y^2x^3}$

c. $(x^2)^3(y^2)^4x^2y^3$

d. $(2xy^2)^3(2x)^4$

e. $\left(\frac{a^2}{b^3}\right)^3 \cdot \left(\frac{b}{a^2}\right)^2$

f. $\left(\frac{4x^2y^3}{x^4y^5}\right)^2$

2) Simplify each of the following and write the answers with only positive exponents:

a. $a^{-3}b^{-2}a^4b^5$

b. $\frac{x^7y^{-5}}{y^2x^{-3}}$

c. $(x^{-2})^3(y^{-1})^4x^{-2}y^3$

d. $(2c^{-1}d^{-2})^3(2c)^4$

e. $\left(\frac{m^{-2}}{n^3}\right)^3 \cdot \left(\frac{n^{-3}}{m^2}\right)^{-2}$

f. $\left(\frac{4x^2y^{-3}}{x^{-4}y^5}\right)^{-2}$

3) Simplify each of the following and write the answers with only positive exponents:

a. $\left(\frac{a^{-2}}{4v^{-\frac{1}{2}}}\right)^{\frac{1}{2}}$

b. $\left(\frac{x^{-\frac{1}{2}}y^{-5}}{y^{-25}x^{-3}}\right)^{\frac{1}{5}}$

c. $\left(\frac{a^{-2}b^{\frac{1}{2}}}{c^3d^{-2}}\right)^{\frac{1}{6}}$

d. $\frac{(2c^{-1}d^{-2})^{\frac{1}{3}}}{(2c^{-4})^{\frac{1}{4}}}$

4) Evaluate each of the following:

a. $\sqrt[4]{16}$

b. $\sqrt[3]{125}$

c. $\sqrt[3]{64^2}$

d. $\sqrt[5]{32^4}$

e. $\sqrt[3]{-8}$

f. $\sqrt[5]{-32}$

g. $\sqrt[3]{-27}$

h. $\sqrt[5]{-243}$

5) Simplify each of the following and write the answers with only positive exponents:

a. $9^{\frac{1}{2}}$

b. $16^{\frac{1}{4}}$

c. $81^{\frac{1}{3}}$

d. $-8^{\frac{1}{3}}$

e. $32^{\frac{1}{5}}$

f. $-125^{\frac{1}{3}}$

g. $16^{\frac{3}{4}}$

h. $27^{\frac{2}{3}}$

i. $\left(\frac{243}{32}\right)^{\frac{4}{5}}$

6) Write each of the following radicals in exponent form:

a. $\sqrt[4]{16}$

b. $\sqrt[10]{5x}$

c. $\sqrt{x^2 + y^2}$

d. $\sqrt[3]{(a+b)^5}$

7) Simplify each of the following. Assume that x , y , and z are positive:

a. $\sqrt{x^6}$

b. $\sqrt[9]{y^6}$

c. $\sqrt{18x^3y^7}$

d. $\sqrt{x^3}$

e. $\sqrt[3]{x^2y^3z^4}$

f. $\sqrt[4]{x^{15}y^5z^{20}}$

g. $\sqrt[3]{x^3y^6}$

h. $\sqrt[5]{x^{15}y^5}\sqrt[5]{z^{20}}$

8) Multiply each of the following. Assume that x is positive:

a. $(\sqrt{x} + 2)(\sqrt{x} - 2)$

b. $(\sqrt{x} + 2)(\sqrt{x} - 5)$

c. $(4\sqrt{x} + \sqrt{y})(2\sqrt{x} - 5\sqrt{y})$

9) Rationalize the denominator for each of the following. Assume that x is positive:

a. $\frac{5}{\sqrt{x}}$

b. $\frac{2}{\sqrt[3]{x}}$

c. $\sqrt[4]{\frac{2}{x^3}}$

d. $\frac{5}{\sqrt{x-1}}$

e. $\frac{2}{3\sqrt{x-4}}$

f. $\frac{5}{2\sqrt{x} + \sqrt{5}}$

Answer Key:

- 1) a. a^7b^7 b. x^4y^3 c. x^8y^{11} d. $128x^7y^6$ e. $\frac{a^2}{b^7}$
 f. $\frac{16}{x^4y^4}$
- 2) a. ab^3 b. $\frac{x^{10}}{y^7}$ c. $\frac{1}{x^8y}$ d. $\frac{128c}{d^6}$ e. $\frac{1}{m^2n^3}$
 f. $\frac{y^{16}}{16x^{12}}$
- 3) a. $\frac{\sqrt[4]{v}}{2a}$ b. $\sqrt{xy^4}$ c. $\frac{\sqrt[12]{b^3}\sqrt[3]{d}}{\sqrt[3]{a}\sqrt{c}}$ d. $\frac{\sqrt[12]{2^3}\sqrt[3]{c^2}}{\sqrt[3]{d^2}}$
- 4) a. 2 b. 5 c. 16 d. 16 e. -2 f. -2
 g. -3 h. -3
- 5) a. 3 b. 2 c. $3^3\sqrt{3}$ d. -2 e. 2 f. -5
 g. 8 h. 9 i. $5\frac{1}{16}$
- 6) a. 2 b. $(5x)^{\frac{1}{10}}$ c. $(x^2 + y^2)^{\frac{1}{2}}$ d. $(a+b)^{\frac{5}{3}}$
- 7) a. x^3 b. $\sqrt[3]{y^2}$ c. $3\sqrt{2}x\sqrt{xy^3}\sqrt{y}$ d. $x^{\frac{3}{5}}$ e. $\sqrt[3]{x^2}yz\sqrt[3]{z}$
 f. x^3yz^4 g. xy^2 h. x^3yz^4
- 8) a. $x-4$ b. $x-3\sqrt{x}-10$ c. $8x-18\sqrt{x}\sqrt{y}-5y$
- 9) a. $\frac{5\sqrt{x}}{x}$ b. $\frac{2\sqrt[3]{x^2}}{x}$ c. $\frac{\sqrt[4]{2}\sqrt[4]{x}}{x}$ d. $\frac{5(\sqrt{x}+1)}{x-1}$
 e. $\frac{2(3\sqrt{x}+4)}{9x-16}$ f. $\frac{5(2\sqrt{x}-\sqrt{5})}{4x-5}$

Exponential Equations - Same Base

Questions:

1) Solve the following equations:

a. $2^{3x} = 64$

b. $5^{3x-6} = 125$

c. $9^{x-1} = 27^{x+2}$

d. $49^{x-2} = 7^{3x-4}$

2) Solve the following equations:

a. $8^{4x-2} = \left(\frac{1}{2}\right)^{3x-3}$

b. $10^{6x-2} = 0.001^{1.5x-2}$

3) Solve the following equations:

a. $\left(\frac{25}{9}\right)^{3-x} = \left(\frac{27}{125}\right)^{x-2}$

b. $\left(\frac{81}{16}\right)^{-x} = \left(\frac{2}{3}\right)^{5-x^2}$

4) Solve the following equations:

a. $\frac{8}{2^x} \cdot 0.25^{2x^2+1} = \frac{1}{16}$

b. $\left(\frac{5}{3}\right)^{1-x} \cdot \left(\frac{3}{5}\right)^{-\frac{1}{2}} = \left(\frac{9}{25}\right)^{2x}$

5) Solve the following equations:

a. $\left(\frac{1}{\sqrt{2}}\right)^{3-x} = 16^{-x}$

b. $(\sqrt{3})^{3-x} = 81^{0.5x+1}$

6) Solve the following equations:

a. $\left(\frac{1}{25\sqrt{5}}\right)^{-x} = 0.2^{\frac{1}{2}x-2}$

b. $\sqrt[3]{4^x} = \sqrt[6]{0.125}$

7) Solve the following equations:

a. $\sqrt{\frac{1}{32}} \cdot \left(\frac{1}{2}\right)^{6x-x^2} = 16\sqrt{2}$

b. $\frac{\sqrt{x+1} \sqrt[3]{36^{0.5x-1}}}{36} = \sqrt{6}$

8) Solve the following equations:

a. $27^{-1} \cdot (3^x)^x = \frac{1}{9^{-x}}$

b. $0.5^{2x-4} = \left(\frac{1}{4^{x-3}}\right)^{\frac{8}{x}}$

9) Solve the following equations:

a. $\sqrt[x]{4^{x^2+x}} = \frac{16^x}{64}$

b. $\sqrt[x]{\sqrt{x}81} = 243\left(\frac{1}{3^x}\right)^x$

10) Solve the following equations:

a. $\left((4^x)^x\right)^x \cdot 32^{x^2} = 8^x$

b. $3^x \cdot \frac{1}{27^{\sqrt{x}}} = 9^5$

11) Solve the following equations:

a. $192 \cdot 3^{2x} - 27 \cdot 4^{2x+1} = 0$

b. $16 \cdot 3^{x^2+x} - 9 \cdot 4^{x^2+x} = 0$

12) Solve the following equations:

a. $2 \cdot 4^x + 5 \cdot 4^x = 112$

b. $3 \cdot 5^x - 5^x = \frac{2}{25}$

13) Solve the following equations:

a. $2^{x+1} + 2^x = 48$

b. $\frac{\sqrt[x+1]{36^{0.5x-1}}}{36} = \sqrt{6}$

14) Solve the following equations:

a. $2^{x+3} - 2^x = 28$

b. $\frac{1}{2}4^{x+2} + 4^{x+1} = \frac{3}{16}$

15) Solve the following equations:

a. $4 \cdot 5^{x-1} + 5^x = 0.36$

b. $3 \cdot 2^x - 5 \cdot 2^{x-2} - 2^{x-1} = 40$

16) Solve the following equations:

a. $5 \cdot 6^{x-1} - 3^x \cdot 2^{x-1} = 72$

b. $2^x \cdot 5^{x+2} - 2^{x+2} \cdot 5^{x+1} = 50$

Answer Key:

- 1) a. $x=2$ b. $x=3$ c. $x=-8$ d. $x=0$
- 2) a. $x=\frac{3}{5}$ b. $x=\frac{16}{21}$
- 3) a. $x=0$ b. $x=1,-5$
- 4) a. $x=1,-1\frac{1}{4}$ b. $x=-\frac{1}{2}$
- 5) a. $x=\frac{1}{3}$ b. $x=-1$
- 6) a. $x=\frac{2}{3}$ b. $x=-\frac{3}{4}$
- 7) a. $x=7,-1$ b. $x=-3$
- 8) a. $x=3,-1$ b. $x=6,4$
- 9) a. $x=4$ b. $x=2,1$
- 10) a. $x=0,\frac{1}{2},-3$ b. $x=25$
- 11) a. $x=1$ b. $x=1,-2$
- 12) a. $x=2$ b. $x=-2$
- 13) a. $x=4$ b. No solution.
- 14) a. $x=2$ b. $x=-3$
- 15) a. $x=-1$ b. $x=5$
- 16) a. $x=2$ b. $x=1$

Exponential Equations - Substitution

Questions:

1) Solve the following equations:

a. $2 \cdot 2^{2x} - 9 \cdot 2^x + 4 = 0$

b. $25^x - 6 \cdot 5^x + 5 = 0$

2) Solve the following equations:

a. $3 \cdot 81^{0.5x} + 5 \cdot 3^x - 2 = 0$

b. $36^{x+0.5} - 6^{x+1} = 36^{0.5x} - 6^0$

3) Solve the following equations:

a. $100^{-x+0.5} + \frac{9}{10^x} = 1$

b. $\left(6\frac{1}{4}\right)^x - 2\left(\frac{5}{2}\right)^{x+1} + \frac{25}{4} = 0$

4) Solve the following equations:

a. $2^{x+2} + 2^{-x+2} = 10$

b. $4^{x+1.5} - 2 \cdot 4^{2-x} = 60$

5) Solve the following equations:

a. $27^x - 3^{2x+1} = 9 \cdot 9^x - 3 \cdot 3^{x+2}$

b. $5 \cdot 6^x - 3 \cdot 4^x - 2 \cdot 9^x = 0$

6) Solve the following equations:

a. $\frac{1}{3^x+1} - \frac{2}{9^x-1} = 0$

b. $\frac{124}{4^{x+1}-4} + \frac{8}{4^{x-1}} = 2$

7) Solve the following equations:

a. $\frac{9}{2-5^x} - \frac{2}{5^{x+1}} = 3$

b. $4 - \frac{2}{9^{x-1} - \frac{1}{9}} = -\frac{5}{9^{x-\frac{1}{2}}}$

8) Solve the following equations:

a. $1 - \frac{48}{3^{x+2} - 36} = \frac{3^{-x+1}}{4 - 3^x}$

b. $\frac{5 \cdot 2^x - 10}{4^x - 2^x - 2} = 1$

9) Solve the following equations:

a. $49^{2\sqrt{x}} - 8 \cdot 7^{2\sqrt{x}} + 7 = 0$

b. $\sqrt{2^{2x+2}} + \frac{1}{4^{0.5x-1}} = 9$

10) Solve the following equations:

a. $e^{2x} - 2e^x + 1 = 0$

b. $e^{2x} - e^{x+1} - e^x + e = 0$

Answer Key:

1) a. $x = 2, -1$

b. $x = 1, 0$

2) a. $x = -1$

b. $x = 0, -1$

3) a. $x = 1$

b. $x = 1$

4) a. $x = \pm 1$

b. $x = 1.5$

5) a. $x = 1, 2$

b. $x = 0, 1$

6) a. $x = 1$

b. $x = 2\frac{1}{2}, -\frac{1}{2}$

7) a. $x = -1$

b. $x = \frac{1}{2}$

8) a. $x = 2, -1$

b. $x = 2, 1$

9) a. $x = \frac{1}{4}, 0$

b. $x = 2, -1$

10) a. $x = 0$

b. $x = 1, 0$

Scientific Notation

Questions:

- 1) Convert the following into scientific notation and then perform the computations.

Express your answer in scientific notation: $\frac{(24,000,000)(0.009)}{0.00015}$.

- 2) The speed of light is approximately 1.86×10^5 miles per second.
If the planet Mars is 1.416×10^8 miles from the sun, how long does it take light from the sun to reach Mars? Express your answer using scientific notation.
- 3) The mass of Pluto is approximately 1.3×10^{22} kilograms.
If one ton is 888.9 kilograms, what is the weight of the Pluto in tons?
Express your answer using scientific notation.

Answer Key:

- 1) $1.44 \cdot 10^9$
- 2) $7.613 \cdot 10^2$ secs.
- 3) $1.46 \cdot 10^{19}$ tons.