proprep™

Workbook

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Extrema in 2 Variables

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Questions:

In each of the following functions find the critical points and classify them as maximum, minimum or saddle.

1) $f(x,y)=8x^3+12xy+3y^2-18x$

2) $f(x,y)=x^3+y^3-3x-12y+20$

3) $f(x,y)=x^3+y^3-3xy+4$

4) $f(x,y)=3x-x^3-2y^2+y^4$

5) $f(x,y)=e^{4y-x^2-y^2}$

6) $f(x,y)=y\sqrt{x}-y^2-x+6y$

7) $f(x,y)=\dfrac{x^2y^2-8x+y}{xy}$

8) $f(x,y)=e^x\cos y$

9) Given the surface $z=x^3+y^3-3xy+4$, find the equations of its tangent planes which are horizontal.

10) From all the open boxes whose volume is 32cm^3, compute the dimensions of the one with least surface area.

11) Find the shortest distance from the point $(1,2,3)$ to the plane $-2x-2y+z=0$, and the point on the plane closest to the above point.

12) A manufacturer sells calculators in China and in the USA.
The production cost of a calculator is $\$6$ in China and $\$8$ in the USA.
The marketing manager estimates the demands Q_1 and Q_2 for calculators in China and the USA, respectively, as: $Q_1=119-30P_1+20P_2$, $Q_2=144+16P_1-24P_2$
P_1 and P_2 are the sale prices of a calculator in China and in the USA, respectively.
What should P_1 and P_2 be, in order to maximize the profit? What is the profit?

Answer Key:

1) $(-0.5,1)$ Saddle; $(1.5,-3)$ Minimum

2) $(1,2)$ Minimum; $(-1,-2)$ Maximum ; $(-1,2)$, $(1,-2)$ Saddle

3) $(0,0)$ Saddle ; $(1,1)$ Minimum

4) $(-1,1)$, $(-1,-1)$ Minimum ; $(1,0)$ Maximum ; $(-1,0)$, $(1,1)$, $(1,-1)$ Saddle

5) $(0,2)$ Maximum

6) $(4,4)$ Maximum

7) $(-0.5,4)$ Maximum

8) No critical points

9) $z=3$, $z=4$

10) Width - 4cm ; Length - 4cm ; Height - 2cm

11) Minimum distance 1 unit, closest point $\left(\frac{1}{3},\frac{4}{3},\frac{10}{3}\right)$

12) $P1=\$10,\ P2=\$12,$ maximum profit \$288.