

Workbook



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Functions of Several Variables

Function of Several Variables

Questions:

1) For each of the following functions, find its domain of definition, sketch it, and sketch its level curves/surfaces as appropriate.

a. $f(x, y) = \frac{y}{x}$

b. $f(x, y) = \ln x + \ln y$

c. $f(x, y) = x^2 + y^2$

d. $f(x, y) = \sqrt{1 - x^2 - y^2}$

e. $f(x, y) = \ln(x^2 - y)$

f. $f(x, y) = x\sqrt{y}$

g. $f(x, y, z) = x^2 + y^2 + z^2$

h. $f(x, y, z) = z^2 - x^2 - y^2$

2) Compute the following limits:

i. $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^3 y)}{x^3 y}$

j. $\lim_{(x,y) \rightarrow (3,2)} \frac{\sin(xy - 6)}{x^2 y^2 - 36}$

k. $\lim_{(x,y) \rightarrow (1,2)} \frac{\arctan(x + y - 3)}{\ln(x + y - 2)}$

l. $\lim_{(x,y) \rightarrow (0,0^+)} (x^2 + y) \ln(x^2 + y)$

m. $\lim_{(x,y) \rightarrow (1^+, 1^+)} \frac{\sin(\sqrt{x + 2y - 3})}{x + 2y - 3}$

n. $\lim_{(x,y) \rightarrow (1,2)} \frac{\sqrt{2x + y - 3} - 1}{2x + y - 4}$

o. $\lim_{(x,y) \rightarrow (1,1)} \frac{xy - y^2}{\sqrt{x} - \sqrt{y}}$

p. $\lim_{(x,y,z) \rightarrow (0,1,2)} \frac{\sin(x(y^2 + z^2))}{xy^2}$

3) Compute the following limits:

q. $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{(x^2 + y^2)^2}{x^4 + y^2}$

s. $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x^3 + y^2}{x^2 + y^2}$

u. $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x^2 y}{x^4 + y^2}$

w. $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{\sin(xy)}{x^2 + y^2}$

r. $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} |y|^x$

t. $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x}{y}$

v. $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x^3 y}{2x^6 + y^2}$

x. $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0 \\ z \rightarrow 0}} \frac{xyz}{x^2 + y^4 + z^4}$

4) Compute the following limits:

y. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y}{x^2 + y^2}$

aa. $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(xy)}{\sqrt{x^2 + y^2}}$

cc. $\lim_{(x,y) \rightarrow (0,0)} \frac{3x^2 - x^2 y^2 + 3y^2}{x^2 + y^2}$

ee. $\lim_{(x,y) \rightarrow (0,0)} y \ln(x^2 + y^2)$

z. $\lim_{(x,y) \rightarrow (\infty, \infty)} \frac{x - y}{x^2 + yx + y^4}$

bb. $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 + y^4}{x^2 + y^2}$

dd. $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(\sqrt{x^2 + y^2})}{\sqrt[3]{x^2 + y^2}}$

ff. $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^3 + y^3 + z^3}{x^2 + y^2 + z^2}$

5) For each of the following functions, check if it's continuous at $(0,0)$.

If it's not continuous there, is the discontinuity removable?

a. $f(x, y) = \begin{cases} \frac{\sin(x^2 + y^2)}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 2 & (x, y) = (0, 0) \end{cases}$

b. $f(x, y) = \begin{cases} \frac{x^3 + y^3}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$

Answer Key:

- 1) a. $x \neq 0$, The plane without the y -axis b. The 1st quadrant without the axes
c. The whole plane d. $x^2 + y^2 \leq 1$, The unit circle
e. $y < x^2$ f. $y \geq 0$, The upper half-plane
g. The whole 3D-space h. The whole 3D-space
- 2) a. 1 b. $\frac{1}{12}$ c. 1 d. 0 e. 3 f. 0
g. 0 h. 0
- 3) In each section the limit doesn't exist
- 4) a-d. 0 e. 3 f-h. 0
- 5) a. Discontinuous; removable if we redefine $f(0,0) = 1$ b. Continuous