proprep™

Workbook

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Green's Theorem

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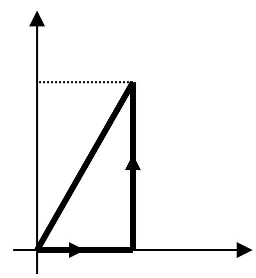
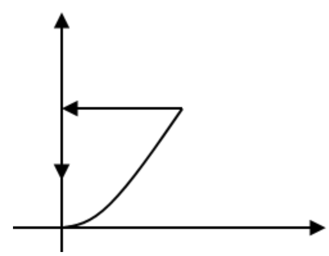
Questions:

In each of the exercises 1-3 verify Green's Theorem: $\oint_C f dx + g dy = \iint_R \left(g_x - f_y\right) dA$.

1) $\oint_C x^2 y dx + x dy$; the path C is as in the illustration:



2) $\oint_C (x - y^2) dx + dy$; the path C is as in the illustration:



3) $\oint_C \left(x^2 - xy^3\right) dx + \left(y^2 - 2xy\right) dy$; C traces out, anticlockwise, the square with vertices: $(0,0),(2,0),(2,2),(0,2)$.

4) Compute the work done by a force field $\mathbf{F}(x, y) = \left(e^x - y^3\right)\mathbf{i} + \left(\cos y + x^3\right)\mathbf{j}$, on a particle which moves anticlockwise on the unit circle $x^2 + y^2 = 1$ and completes one revolution.

5) Compute the integral $\int_C \left(e^y - \tan\frac{x}{2}\right) dx + \left(xe^y + y\cos y^2\right) dy$, where C is the clockwise union of the parts of the curves $y = 8 - x^2$, $y = x^2$ between the y-axis and their intersection in the first quadrant.

6) Compute the integral $\int_C -2e^{2x-y}\cos y\,dx + \left(e^{2x-y}(\sin y + \cos y) + 2xy\right)dy$ where C is the semi-ellipse $\{x^2 + 4y^2 = 4,\ y \ge 0\}$ from the point $(2,0)$ to the point $(-2,0)$.

7) Answer the following:

a. Prove that the area bounded by a simple closed curve C is given by $\frac{1}{2}\oint_C x\,dy - y\,dx$.

b. Compute the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, using the formula $A = \frac{1}{2}\oint_C x\,dy - y\,dx$.

Answer Key:

1) The common value is 0.5

2) The common value is 0.8

3) The common value is 8

4) 1.5π

5) $0.5\sin 64$

6) $\frac{8}{3} + 2\left(e^4 - \frac{1}{e^4}\right)$

7) a. Refer to the video b. πab