

Workbook

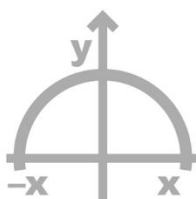


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Infinite Series

Infinite Geometric Series

Questions:

- 1) For the following series determine if the series converges or diverges. If the series converges give its value.

a. $\sum_{n=1}^{\infty} (0.44)^n$

b. $\sum_{n=0}^{\infty} \frac{4^n}{7^{n+1}}$

c. $\sum_{n=1}^{\infty} (-1)^n \frac{5^n}{4^{n+2}}$

d. $\sum_{n=0}^{\infty} (-4) \left(\frac{3}{4}\right)^{2n}$

e. $\sum_{n=1}^{\infty} \frac{4^n + (-5)^n}{7^n}$

f. $\sum_{n=4}^{\infty} 2^{3n+4} 3^{1-2n}$

g. $\sum_{n=3}^{\infty} \frac{(-5)^{3-n}}{8^{2-n}}$

h. $\sum_{n=2}^{\infty} 2^{3n+4} 5^{1-n}$

i. $2^{-2} + 2^{-4} + 2^{-6} + \dots$

- 2) For the following series determine if the series converges or diverges. If the series converges give its value.

a. $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$

b. $\frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \dots$

c. $\sum_{n=1}^{\infty} \frac{1}{16n^2 + 8n - 3}$

d. $\ln\left(1 + \frac{1}{1}\right) + \ln\left(1 + \frac{1}{2}\right) + \ln\left(1 + \frac{1}{3}\right) + \ln\left(1 + \frac{1}{4}\right) + \dots$

Answer Key:

1) a. Converges to $\frac{11}{14} = 0.7857$ b. Converge to $\frac{1}{3} = 0.333$ c. Diverges

d. Converge to $-\frac{64}{7} = -9.14$ e. Converge to $\frac{11}{12}$ f. Converge to $269\frac{169}{243}$

g. Diverge h. Diverges i. Converge to

$\frac{1}{3} = 0.333$

2) a. Converge to 0.5 b. Converge to $\frac{3}{4}$ c. Converge to $\frac{1}{12}$

d. Diverge

The Harmonic Series and The P-Series

Questions:

For each of the following series determine if the series converges or diverges:

a. $\sum_{n=1}^{\infty} \frac{3}{5n}$

b. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

c. $\sum_{n=1}^{\infty} \frac{1}{n^4}$

d. $\sum_{n=1}^{\infty} \frac{1}{n^e}$

e. $\sum_{n=1}^{\infty} \frac{10}{\sqrt[3]{n^4}}$

f. $\sum_{n=10}^{\infty} n^{-2/3}$

Answer Key:

- a. Diverges
- b. Diverges
- c. Converges
- d. Converges
- e. Converges
- f. Diverges

Algebraic Properties of Series

Questions:

For each of the following series determine if the series converges or diverges:

a. $\sum_{n=1}^{\infty} \frac{10+n^2}{n^3}$

b. $\sum_{n=1}^{\infty} \frac{4n+10}{n^2}$

c. $\sum_{n=0}^{\infty} \left(\frac{4^n}{7^{n+1}} + n^{-1.5} \right)$

Answer Key:

- a. Diverges
- b. Converges
- c. Converges

The Divergence Test

Questions:

For each of the following series determine if the series converges or diverges:

1) $\sum_{n=1}^{\infty} \frac{4n+5}{7n+8}$

2) $\sum_{n=1}^{\infty} 1$

3) $\sum_{n=1}^{\infty} \cos(\ln n)$

4) $\sum_{n=1}^{\infty} n$

5) $\sum_{n=1}^{\infty} \frac{e^n}{n^3}$

6) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

Answer Key:

1-6) Diverges

The Integral Test

Questions:

For the following series determine if the series converges or diverges:

a. $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$

b. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+5}}$

c. $\sum_{n=1}^{\infty} ne^{-n}$

d. $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

e. $\sum_{n=2}^{\infty} \frac{1}{n \ln^3 n}$

f. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

Answer Key:

- a. Diverges
- b. Diverges
- c. Converges
- d. Converges
- e. Converges
- f. Diverges

The Comparison Test \ The Limit Comparison Test

Questions:

For the following series determine if the series converges or diverges:

a.
$$\sum_{n=1}^{\infty} \frac{5n^2 + 4n + 8}{14n^5 + 10n^3 + 4n^2 + 10n + 1}$$

b.
$$\sum_{n=1}^{\infty} \frac{1 \cdot 2 \cdot 3 \cdot \dots \cdot n}{n \cdot n \cdot n \cdot \dots \cdot n}$$

c.
$$\sum_{n=1}^{\infty} \frac{2n^3 + n^2 + 4n + 1}{\sqrt{n^{10} + 4n^4 + n^2 + n + 1}}$$

d.
$$\sum_{n=1}^{\infty} \frac{4n + 5}{\sqrt{n^4 + 2n^3 + n^2 + 4n + 1}}$$

e.
$$\sum_{n=1}^{\infty} \frac{2^n - 2}{3^n + 2n}$$

f.
$$\sum_{n=1}^{\infty} \frac{5 \sin^2 n}{n!}$$

g.
$$\sum_{n=1}^{\infty} \left(\sqrt{n^2 + 1} - n \right)$$

h.
$$\sum_{n=1}^{\infty} \left(1 - \cos \frac{1}{n} \right)$$

i.
$$\sum_{n=1}^{\infty} \frac{\sqrt{n} \ln n}{n^2 + 1}$$

j.
$$\sum_{n=1}^{\infty} \frac{5n^2 + 4n + 8}{14n^5 + 10n^3 + 4n^2 + 10n + 1}$$

k.
$$\sum_{n=1}^{\infty} \frac{1 \cdot 2 \cdot 3 \cdot \dots \cdot n}{n \cdot n \cdot n \cdot \dots \cdot n}$$

l.
$$\sum_{n=1}^{\infty} \frac{2n^3 + n^2 + 4n + 1}{\sqrt{n^{10} + 4n^4 + n^2 + n + 1}}$$

m.
$$\sum_{n=1}^{\infty} \frac{4n + 5}{\sqrt{n^4 + 2n^3 + n^2 + 4n + 1}}$$

n.
$$\sum_{n=1}^{\infty} \frac{2^n - 2}{3^n + 2n}$$

o.
$$\sum_{n=1}^{\infty} \frac{5 \sin^2 n}{n!}$$

p.
$$\sum_{n=1}^{\infty} \left(\sqrt{n^2 + 1} - n \right)$$

q.
$$\sum_{n=1}^{\infty} \left(1 - \cos \frac{1}{n} \right)$$

r.
$$\sum_{n=1}^{\infty} \frac{\sqrt{n} \ln n}{n^2 + 1}$$

Answer Key:

- | | | | |
|--------------|--------------|--------------|--------------|
| a. Converges | b. Converges | c. Converges | d. Diverges |
| e. Converges | f. Converges | g. Diverges | h. Converges |
| i. Converges | j. Converges | k. Converges | l. Converges |
| m. Diverges | n. Converges | o. Converges | p. Diverges |
| q. Converges | r. Converges | | |

The Ratio Test \ The Root Test

Questions:

For the following series determine if the series converges or diverges:

- | | | |
|--|---|--|
| a. $\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$ | b. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ | c. $\sum_{n=1}^{\infty} \frac{(n+3)!}{n! \cdot 3^n}$ |
| d. $\sum_{n=1}^{\infty} \frac{(2n)!}{n!(2n)^n}$ | e. $\sum_{n=1}^{\infty} \frac{3^n(1+n^2)}{n!}$ | f. $\sum_{n=1}^{\infty} n^{1000} e^{-n}$ |
| g. $\sum_{n=2}^{\infty} \frac{4^n(n^2+4n+5)}{3^n \ln n}$ | h. $\sum_{n=1}^{\infty} \frac{(4n^2+5n+1)^n}{4^n n^{2n}}$ | i. $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ |

Answer Key:

- | | | |
|--------------|-----------------|--------------|
| a. Converges | b. Converges | c. Converges |
| d. Converges | e. Converges | f. Converges |
| g. Diverges | h. Inconclusive | i. Converges |

The Alternating Series Test

Questions:

For the following series determine if the series converges or diverges:

a. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{4n+1}}$

b. $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$

c. $\sum_{n=1}^{\infty} (-1)^n \frac{n+2}{n^2+n}$

Answer Key:

- a. Converges
- b. Converges
- c. Converges

Absolute and Conditional Convergence of Series

Questions:

- 1) Determine if the following series is absolutely convergent, conditionally convergent or divergent:

a. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n+1}}$

b. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2 + 1}$

c. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos n}{n^2}$

d. $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n}$

e. $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$

f. $\sum_{n=1}^{\infty} \left(\frac{-1}{\ln n} \right)^n$

g. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n(n+1)}}$

- 2) Prove or disprove:

- If $\sum a_n$ converges and $\sum b_n$ diverges, then $\sum (a_n + b_n)$ diverges.
- If $\sum a_n$ diverges and $\sum b_n$ diverges, then $\sum (a_n + b_n)$ diverges.
- If $\sum (a_n)^2$ converges, then $\sum a_n$ converges.
 - If $\sum (a_n)^2$ converges and $\sum a_n$ converges, then $\sum a_n$ converges absolutely.
- If $\sum a_n$ converges and positive, then $\sum \frac{1}{a_n}$ diverges.
- If $\sum a_n$ converges, then $\sum (a_n)^2$ converges.

Answer Key:

- 1) a. conditionally converges b. converges absolutely c. converges absolutely
 d. conditionally converges e. conditionally converges f. converges absolutely
 g. conditionally converges
- 2) a-e. Refer to the videos.