

Workbook



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L'Hopital's Rule

L'Hopital's Rule – Zero over Zero, Inf over Inf $\frac{\infty}{\infty}$

Questions:

In exercises 1-30, find the limit:

1) $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^2 - 9}$

2) $\lim_{x \rightarrow -5} \frac{2x^2 - 50}{2x^2 + 3x - 35}$

3) $\lim_{x \rightarrow 4} \frac{\sqrt{2x+1} - \sqrt{x+5}}{x-4}$

4) $\lim_{x \rightarrow 3} \frac{\sqrt{x^2+7} - 4}{\sqrt{x-2} - 1}$

5) $\lim_{x \rightarrow 1} \frac{\sqrt[3]{2x^2-1} - \sqrt{x}}{x-1}$

6) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$

7) $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x} \quad (a, b > 0)$

8) $\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2}$

9) $\lim_{x \rightarrow 0} \frac{2e^x - x^2 - 2x - 2}{2x^3}$

10) $\lim_{x \rightarrow 1} \frac{\ln x - x + 1}{x^2 - 2x + 1}$

11) $\lim_{x \rightarrow \infty} \frac{\ln\left(\frac{x^2+1}{x^2-1}\right)}{1/x^2}$

12) $\lim_{x \rightarrow 0} \frac{\ln^2(x+1) + x}{x}$

13) $\lim_{x \rightarrow 0} \frac{\tan x}{x}$

14) $\lim_{x \rightarrow 0} \frac{\sin(ax^2)}{bx^2}$

15) $\lim_{x \rightarrow 0} \frac{\sin(ax)}{\sin(bx)}$

16) $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}$

17) $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$

18) $\lim_{x \rightarrow 0} \frac{\sqrt{1+\sin x} - \sqrt{\cos x}}{x}$

19) $\lim_{x \rightarrow 0} \frac{1 - \cos(1 - \cos x)}{x^4}$

20) $\lim_{x \rightarrow 0} \frac{e^x \sin x - x(1+x)}{x^3}$

21) $\lim_{x \rightarrow 0} \frac{\ln(\cos x^2)}{x^4}$

22) $\lim_{x \rightarrow 0} \frac{\arctan(x^2 + 3x)}{\arcsin(x^2 - 4x)}$

23) $\lim_{x \rightarrow 0} \frac{\sin x}{\sinh x}$

24) $\lim_{x \rightarrow 0} \frac{2 \cosh x - 2}{1 - \cos 2x}$

25) $\lim_{x \rightarrow \infty} \frac{x^2 + 1}{2x^2 + x + 3}$

26) $\lim_{x \rightarrow 0} \frac{e^x}{x}$

27) $\lim_{x \rightarrow \infty} \frac{\ln x + x + 1}{e^x}$

28) $\lim_{x \rightarrow \infty} \frac{(\ln x)^2 + 2 \ln x - 3}{x}$

29) $\lim_{x \rightarrow \infty} \tanh x$

30) $\lim_{x \rightarrow 0^+} \frac{\ln(\sin x)}{\ln(\tan x)}$

Answer Key:

- | | | | | |
|--------------------|----------------------------------|-------------------|-------------------|--------------------|
| 1) $\frac{5}{6}$ | 2) $\frac{20}{17}$ | 3) $\frac{1}{6}$ | 4) $\frac{3}{2}$ | 5) $\frac{5}{6}$ |
| 6) 1 | 7) $\ln\left(\frac{a}{b}\right)$ | 8) $\frac{1}{2}$ | 9) $\frac{1}{6}$ | 10) $-\frac{1}{2}$ |
| 11) 2 | 12) 1 | 13) 1 | 14) $\frac{a}{b}$ | 15) $\frac{a}{b}$ |
| 16) $\frac{1}{6}$ | 17) $\frac{1}{2}$ | 18) $\frac{1}{2}$ | 19) $\frac{1}{8}$ | 20) $\frac{1}{3}$ |
| 21) $-\frac{1}{2}$ | 22) $-\frac{3}{4}$ | 23) 1 | 24) $\frac{1}{2}$ | 25) $\frac{1}{2}$ |
| 26) ∞ | 27) 0 | 28) 0 | 29) 1 | 30) 1 |

L'Hopital's Rule – Zero Times Inf $0 \cdot \infty$

Questions:

In exercises 1- 5 find the limit:

- | | | |
|--------------------------------------------------------------------------|------------------------------------------------------------------------------------|----------------------------------------------------------|
| 1) $\lim_{x \rightarrow 0} (1 - \cos x) \cot x$ | 2) $\lim_{x \rightarrow 0^+} x \cdot \ln x$ | 3) $\lim_{x \rightarrow 3^+} (x^2 - 9) \cdot \ln(x - 3)$ |
| 4) $\lim_{x \rightarrow \infty} x \cdot \ln\left(\frac{x+3}{x-3}\right)$ | 5) $\lim_{x \rightarrow \infty} x \cdot \left[\sqrt{1 + \frac{5}{x}} - 1 \right]$ | |

Answer Key:

- | | | | | |
|------|------|------|------|--------|
| 1) 0 | 2) 0 | 3) 0 | 4) 6 | 5) 2.5 |
|------|------|------|------|--------|

L'Hopital's Rule – $0^0, 1^\infty, \infty^0$

Questions:

In exercises 1 - 13 find the limit:

- | | | |
|------------------------------------------------------------------------------|---------------------------------------------------------|---------------------------------------------------------|
| 1) $\lim_{x \rightarrow 1} x^{\frac{1}{x-1}}$ | 2) $\lim_{x \rightarrow 0^+} (ax)^x (a > 0)$ | 3) $\lim_{x \rightarrow 2^+} (2x-4)^{x-2}$ |
| 4) $\lim_{x \rightarrow \infty} \left(\frac{x^2+1}{x^2-1} \right)^{x^2}$ | 5) $\lim_{x \rightarrow 0^+} x^{\sin x}$ | 6) $\lim_{x \rightarrow 0} (1 + \tan 3x)^{\frac{1}{x}}$ |
| 7) $\lim_{x \rightarrow 0^+} (\sin x)^{\tan x}$ | 8) $\lim_{x \rightarrow 0^+} (x)^{\tan x}$ | 9) $\lim_{x \rightarrow 0} (1+x^2)^{\cot^2 x}$ |
| 10) $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{\frac{1}{x^2}}$ | 11) $\lim_{x \rightarrow 0} (\cos x)^{2-\frac{1}{x^4}}$ | 12) $\lim_{x \rightarrow 0} (\cos x^n)^{\frac{1}{x^4}}$ |
| 13) $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^{\frac{1}{x^2}}$ | | |

Answer Key:

- | | | | | | |
|------------------------|------|------|-----------------------|------------------------|-----------------------|
| 1) e | 2) 1 | 3) 1 | 4) e^2 | 5) 1 | 6) e^3 |
| 7) 1 | 8) 1 | 9) 1 | 10) $e^{\frac{1}{3}}$ | 11) $e^{-\frac{1}{2}}$ | 12) $e^{\frac{1}{2}}$ |
| 13) $e^{-\frac{1}{6}}$ | | | | | |

L'Hopital's Rule – $\infty - \infty$

Questions:

Find the limit:

1) $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$

2) $\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$

3) $\lim_{x \rightarrow 0^+} [\ln(3x) - \ln(\sin 5x)]$

4) $\lim_{x \rightarrow \infty} \sqrt{x^2 + x + 1} - x$

5) $\lim_{x \rightarrow -\infty} \sqrt{x^2 + x + 1} + x$

Answer Key:

1) 0

2) 0.5

3) $\ln \frac{3}{5}$

4) 0.5

5) -0.5