proprep™

Workbook

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Line Integrals

General Questions with Line Integrals

Questions:

1) Compute the following:

a. $\int_C \left(1-x^2\right)ds$ Where $C:\ x=\cos t,\ \ y=\sin t,\ \ 0\le t\le 2\pi$

b. $\int_C xds$ Where $C:\ x=t-\sin t,\ \ y=1-\cos t,\ \ 0\le t\le \pi$

c. $\int_C (x+y)ds$ Where C is the line segment joining $\mathrm{O}(0,0)$ with $\mathrm{A}(1,2)$.

d. $\int_C \left(x+y^2\right)ds$ Where C is the perimeter of $\Delta\mathrm{OAB}:\mathrm{O}(0,0)$, $\mathrm{A}(0,1)$, $\mathrm{B}(1,0)$

2) Compute the following:

a. $\int_C \left(x^2+y^2+z^2\right)ds$ where $C: x=\cos t,\ y=\sin t,\ z=t\ \ \ 0\le t\le \pi$.

b. $\int_C \left(x^3+3z\right)ds$ where $C:\ x=t,\ \ y=\frac{1}{\sqrt{2}}t^2,\ \ z=\frac{1}{3}t^3\ \ \ 0\le t\le 3$.

3) Compute the length of the curve: $x^{\frac{2}{3}}+y^{\frac{2}{3}}=1$.

4) A coil made of thin wire is expressed by $x=\cos t,\ y=\sin t,\ z=2t\ \ (0\le t\le \pi)$.
Compute the **mass** of the coil if the density function is $\delta(x,y,z)=kz\ \ (k>0)$.

5) Compute the following:

a. $\int_C 2xydx+\left(x^2+y^2\right)dy$; $C:\ x=\cos t,\ \ y=\sin t\ \ 0\le t\le \frac{\pi}{2}$

b. $\int_C (2x+y)dx+\left(x^2-y\right)dy$; $C:\ x=t,\ \ y=t^2\ \ 0\le t\le 1$

6) Compute $\int_C ydx + x^2dy$ where C is the path from point $(0,0)$ to point $(2,4)$ given by the equation:

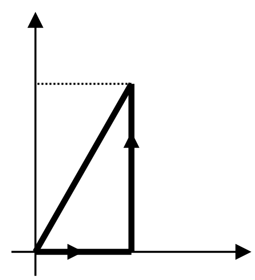
a. $y = 2x$

b. $y = x^2$

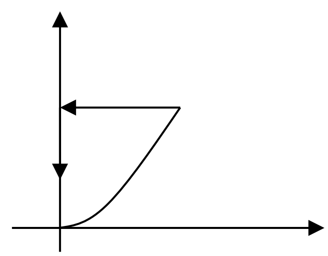
7) Compute $\int_{(1,1)}^{(4,2)} (x+y)dx + (y-x)dy$ along each of the following curves:

a. The parabola $y^2 = x$
b. A line segment
c. The line segments from $(1,1)$ to $(1,2)$ and then to $(4,2)$
d. The curve: $x = 2t^2 + t + 1, \quad y = t^2 + 1 \quad 0 \le t \le 1$

8) Compute $\int_C x^2ydx + xdy$ where the path C is as in the figure:



9) Compute $\int_C (x - y^2)dx + dy$ where C is as in the figure:



10) If $\mathbf{F}(x,y,z) = (3x^2 - 6yz)\mathbf{i} + (2y + 3xz)\mathbf{j} + (1 - 4xyz^2)\mathbf{k}$,

compute the line integral $\int_C F \cdot dr$ from $(0,0,0)$ to $(1,1,1)$ along the paths:

a. $x = t,\ y = t^2,\ z = t^3$.
b. The lines from $(0,0,0)$ to $(0,0,1)$, then to $(0,1,1)$ and then to $(1,1,1)$.
c. The line from $(0,0,0)$ to $(1,1,1)$.

11) Compute the line integral $\int_C F \cdot dr$, where:

a. $F(x,y) = \langle x^2y^3, -y\sqrt{x} \rangle, \quad C: r(t) = \langle t^2, -t^3 \rangle, \quad 0 \le t \le 1$

b. $F(x,y,z) = \langle \sin x, \cos y, xz \rangle$, $C: r(t) = \langle t^3, -t^2, t \rangle, \quad 0 \le t \le 1$

12) Answer the following:

a. Compute the work done by the force field $\mathbf{F}(x,y) = x^3y\mathbf{i} + (x-y)\mathbf{j}$ on a particle which moves along the parabola $y = x^2$ from $(-2,4)$ to $(1,1)$.

b. How would your answer change if the particle moved from $(1,1)$ to $(-2,4)$?

13) Compute the work done by the force field $\mathbf{F}(x,y,z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$ on a particle which moves along the path $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ $(0 \le t \le 1)$.

Remark on Notation:

A line integral of type II has different notation in the technical literature:

$$\int_C \mathbf{F} \cdot dr = \int_C (f,g,h) \cdot (dx,dy,dz) = \int_C fdx + gdy + hdz$$

$$\int_C \mathbf{A} \cdot dr = \int_C (A_1,A_2,A_3) \cdot (dx,dy,dz) = \int_C A_1dx + A_2dy + A_3dz$$

Answer Key:

1) a . π b . $\frac{16}{3}$ c . $\frac{3\sqrt{5}}{2}$ d . $\frac{5}{6}\left(\sqrt{2}+1\right)$

2) a . $2\sqrt{2}\pi\left(1+\frac{4\pi^2}{3}\right)$ b . $\frac{567}{2}$

3) 6

4) $\sqrt{5}k\pi^2$

5) a . $\frac{1}{3}$ b . $\frac{4}{3}$

6) a . $\frac{28}{3}$ b . $\frac{32}{3}$

7) a . $\frac{34}{3}$ b. 11   c.14   d. $\frac{32}{3}$

8) $\frac{1}{2}$

9) $\frac{4}{5}$

10) a. 2   b. -3   c. $\frac{6}{5}$

11) a. $-\frac{59}{105}$ b. $\frac{6}{5}-\sin 1-\cos 1$

12) a. 3   b. -3

13) 1