

Workbook



Tale of Contents

Matrices	3
Matrix and Basic Operations on Matrices	3
Matrix Inverse and its Applications	6
Properties of the Matrix Inverse.....	8



Matrices

Matrix and Basic Operations on Matrices

Questions

1) For each of the following matrix “calculations”, say whether it’s undefined or give the order [size] of the result:

- | | |
|---|---|
| a. $A_{4 \times 6} + B_{4 \times 6}$ | b. $A_{4 \times 6} \cdot B_{4 \times 6}$ |
| c. $A_{4 \times 6} \cdot C_{6 \times 2} - D_{4 \times 2}$ | d. $A_{4 \times 6} \cdot E_{6 \times 4} - B_{4 \times 6}$ |
| e. $B_{4 \times 6} + A_{4 \times 6} \cdot B_{4 \times 6}$ | f. $E_{6 \times 4} (B_{4 \times 6} + A_{4 \times 6})$ |
| g. $(E_{6 \times 4} + A_{4 \times 6}^T) D_{4 \times 2}$ | h. $E_{6 \times 4}^T \cdot B_{4 \times 6}$ |
| i. $E_{6 \times 4} \cdot A_{4 \times 6} \cdot C_{6 \times 2}$ | j. $E_{6 \times 4} (B_{4 \times 6} - A_{4 \times 6})$ |

2) Solve the matrix equation $\begin{bmatrix} x+2y & 3x-2y \\ 2x-5y & 2x+8y \end{bmatrix} = \begin{bmatrix} 2-2z & 5+z \\ -4-3z & 3-12z \end{bmatrix}$, for x, y, z .

3) Given the matrices $C = \begin{bmatrix} 1 & 4 & 2 \\ 4 & 1 & 5 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 4 & 2 \\ 1 & 0 & -1 \\ 4 & 2 & 10 \end{bmatrix}$, $E = \begin{bmatrix} 4 & 1 & 1 \\ -1 & 0 & 1 \\ 4 & 1 & -1 \end{bmatrix}$.

Evaluate:

- | | | |
|------------------------|--------------------|---------|
| a. $E + D$ | b. $E - D + I_3$ | c. $5C$ |
| d. $2D + 4E \cdot I_3$ | e. $2tr(D^2 - 2E)$ | |

Given the matrices $A = \begin{bmatrix} 4 & 0 \\ 1 & 2 \\ -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 4 & 2 \\ 4 & 1 & 5 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 4 & 2 \\ 1 & 0 & -1 \\ 4 & 2 & 10 \end{bmatrix}$.

- | | | |
|---------------|------------------------------------|--------------------------|
| f. $4C^T + A$ | g. $\frac{1}{2}A^T + \frac{1}{4}C$ | h. $I_2 \cdot B \cdot C$ |
| i. $trC^T C$ | j. $D \cdot A \cdot B \cdot C$ | |

Answer Key

- 1) a. $C_{4 \times 6}$ b. Undefined. c. $B_{4 \times 2}$ d-e. Undefined.
 f. $C_{6 \times 6}$ g. $B_{6 \times 6}$ h. Undefined. i. $B_{6 \times 2}$ j. $C_{6 \times 6}$

2) $x=2, y=1, z=-1$

3) a. $\begin{bmatrix} 5 & 5 & 3 \\ 0 & 0 & 0 \\ 8 & 3 & 9 \end{bmatrix}_{3 \times 3}$ b. $\begin{bmatrix} 4 & -3 & -1 \\ -2 & 1 & 2 \\ 0 & -1 & -10 \end{bmatrix}_{3 \times 3}$ c. $\begin{bmatrix} 5 & 20 & 10 \\ 20 & 5 & 25 \end{bmatrix}$

d. $\begin{bmatrix} 18 & 12 & 8 \\ -2 & 0 & 2 \\ 24 & 8 & 16 \end{bmatrix}$ e. 230 f. $\begin{bmatrix} 8 & 16 \\ 17 & 6 \\ 7 & 21 \end{bmatrix}_{3 \times 2}$ g. $\begin{bmatrix} 2\frac{1}{4} & 1\frac{1}{2} & 0 \\ 1 & 1\frac{1}{4} & 1\frac{3}{4} \end{bmatrix}$

h. $\begin{bmatrix} 8 & 17 & 13 \\ -8 & -2 & -10 \end{bmatrix}_{2 \times 3}$ i. 63 j. $\begin{bmatrix} -32 & 82 & -22 \\ 48 & 87 & 75 \\ -48 & 108 & -36 \end{bmatrix}$

4) a. $\underbrace{\begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & -4 \\ 6 & 4 & 1 \end{bmatrix}}_A \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_x = \underbrace{\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}}_b$ b. $\underbrace{\begin{bmatrix} 2 & -3 & 1 & 1 \\ 4 & 1 & 2 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & -2 & -4 & 0 \end{bmatrix}}_A \underbrace{\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}}_x = \underbrace{\begin{bmatrix} 1 \\ 4 \\ 1 \\ 10 \end{bmatrix}}_b$

5) a. $\begin{cases} 4x - 2y + 4z = 1 \\ x - y + z = 2 \\ x - 6y + 3z = 3 \end{cases}$ b. $\begin{cases} -2y + 4z = 1 \\ x - 5y + z = 2 \\ x - 6y - z = 3 \end{cases}$ c. $\begin{cases} (4+k)x - 2y + 4z = 1 \\ x + (-1+k)y + z = 2 \\ x - 6y + (3+k)z = 3 \end{cases}$

d. $\begin{cases} 3x - 2y + 4z = 0 \\ x - 2y + z = 0 \\ x - 6y + 2z = 0 \end{cases}$ e. $\begin{cases} 2x + y + z = 3 \\ -2x - 3y - 6z = 6 \\ 4x + y + z = 9 \end{cases}$

- 6) a. All three are true. b. (ii) Is true. c. All three are true.
 d-e. Solution in the recordings.

Matrix Inverse and its Applications

Questions

Find the inverse of the matrix:

1) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

2) $A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$

3) $A = \begin{bmatrix} 4 & 1.5 \\ 2 & 1 \end{bmatrix}$

4) $A = \begin{bmatrix} 1 & 0 & 2 \\ 4 & -1 & 8 \\ 2 & 1 & 3 \end{bmatrix}$

5) $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & -1 \\ 5 & 2 & 3 \end{bmatrix}$

6) $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & -2 & 2 \\ 5 & -3 & 4 \end{bmatrix}$

7) $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 2 & 3 & 4 \end{bmatrix}$

Find the values of k for which the following matrixes are invertible:

8) $\begin{bmatrix} 1 & -1 & 1 \\ 5 & -7 & k^2 + 3 \\ 3 & -1 & k + 3 \end{bmatrix}$

9) $\begin{bmatrix} 1 & 1 & 1 & 1 & k \\ 1 & 1 & 1 & k & 1 \\ 1 & 1 & k & 1 & 1 \\ 1 & k & 1 & 1 & 1 \\ k & 1 & 1 & 1 & 1 \end{bmatrix}$

10) Solve the following SLE by using the Inverse Matrix method: $\begin{cases} 2x - y + z = 3 \\ 3x - 2y + 2z = 5 \\ 5x - 3y + 4z = 11 \end{cases}$

11) Solve the following SLE by using the Inverse Matrix method: $\begin{cases} x + 4y + 2z + 4t = 1 \\ x + 2y - z = 0 \\ y + z + t = 1 \\ x + 3y - z - 2t = 0 \end{cases}$



Answer Key

1) $\begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$

2) $\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$

3) $\begin{bmatrix} 1 & -1.5 \\ -2 & 4 \end{bmatrix}$

4) $\begin{bmatrix} -11 & 2 & 2 \\ 4 & -1 & 0 \\ 6 & -1 & -1 \end{bmatrix}$

5) $\begin{bmatrix} 8 & -1 & -3 \\ -5 & 1 & 2 \\ -10 & 1 & 4 \end{bmatrix}$

6) $\begin{bmatrix} 2 & -1 & 0 \\ 2 & -3 & 1 \\ -1 & -1 & 1 \end{bmatrix}$

7) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{4} & \frac{1}{4} \end{bmatrix}$

8) $k \neq -2, k \neq 1$

9) $k = 1, k = -4$

10) $x = 1, y = 2, z = 3$

11) $x = -13, y = 42, z = -5, t = 2$

Properties of the Matrix Inverse

Questions

- 1) Assume that all the matrices are invertible and of order n , and extract X :
- a. $AXC = D$ b. $A^{-1}XC = A^{-1}DC$ c. $P^{-1}X^T P = A$
- d. $C^{-1}(A+X)D^{-2} = I$ e. $(A-AX)^{-1} = X^{-1}C$ f. $ABC^T X^{-1} BA^T C = AB^T$

- 2) Given that $B = \begin{bmatrix} 1 & 2 \\ 4 & 9 \end{bmatrix}$ and that $B^2 X (2B)^{-1} = B + I$. Find X .

All matrices are 2×2 .

- 3) Given that $B^{-1} = \begin{bmatrix} 1 & 0 & 2 \\ 4 & -1 & 8 \\ 2 & 1 & 3 \end{bmatrix}$ and that $BYB^T = B^{-1} + B$. Find Y .

All matrices are 3×3 .

- 4) Given that $A^{-1} = \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix}$ and that $5A^T B (I + 2A)^{-2} = (7A)^{-2}$. Find B .

All matrices are 2×2 .

- 5) Prove the following:

- a. Given: A is a square matrix satisfying $A^2 - 5A - 2I = 0$.
Prove that A is invertible and express A^{-1} in terms of A and I .
- b. Given: A is a square matrix satisfying $(A - 3I)(A + 2I) = 0$.
Prove that A is invertible and express A^{-1} in terms of A and I .

- 6) Prove the following:

- a. Given: A is a square matrix satisfying $A^2 - 5A - 2I = 0$.
Prove that A is invertible and express A^{-1} in terms of A and I .
- b. Given: A is a square matrix satisfying $(A - 3I)(A + 2I) = 0$.
Prove that A is invertible and express A^{-1} in terms of A and I .



- 7) Given: $A = \begin{bmatrix} -1 & 3 & 0 \\ 3 & -1 & 0 \\ -2 & -2 & 6 \end{bmatrix}$, $p(x) = x^3 - 4x^2 - 20x + 48$.
- Compute $p(A)$.
 - Using part a. above, prove that A is invertible and express A^{-1} in terms of A and I .
- 8) Given: A is a square matrix satisfying $A^4 = 0$.
- Prove that A is noninvertible.
 - Prove that the matrix $I - A$ is invertible and find its inverse.
- 9) Given: $\begin{cases} P^{-1}AP = B \\ Q^{-1}BQ = C \end{cases}$
- Prove that there exists an invertible matrix D such that $D^{-1}AD = C$.

Answer Key

- 1) a. $X = A^{-1}DC^{-1}$ b. $X = D$ c. $X = (P^{-1})^T A^T P^T$
- d. $X = CD^2 - A$ e. $X = (A + C^{-1})^{-1} A$ f. $X = BA^T C \cdot (B^T)^{-1} BC^T$
- 2) $X = \begin{bmatrix} 20 & -4 \\ -8 & 4 \end{bmatrix}$
- 3) $Y = \begin{bmatrix} 22 & 86 & 38 \\ 64 & 246 & 114 \\ 56 & 222 & 92 \end{bmatrix}$
- 4) $B = \frac{1}{245} \begin{bmatrix} 264 & 450 \\ 448 & 768 \end{bmatrix}$
- 5) Solution in the recording.
- 6) Solution in the recording.
- 7) a. $P(A) = 0$ b. Solution in the recording.
- 8) Solution in the recording.
- 9) Solution in the recording.

