proprep™

Workbook

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Systems of Linear Equations

Systems of Linear Equations

Questions

1) For the following augmented matrix perform the indicated elementary row operations: $\left[\begin{array}{cc|c} 3 & -2 & 1 \\ 2 & 0 & -1 \end{array}\right]$

a. $2R_1 \to R_1$
b. $R_1 \leftrightarrow R_2$
c. $R_2 + R_1 \to R_2$
d. $R_2 + 2R_1 \to R_2$

2) For the following augmented matrix perform the indicated elementary row operations: $\left[\begin{array}{ccc|c} 1 & -2 & 0 & 3 \\ 4 & 0 & -1 & 2 \\ 3 & 1 & 2 & -4 \end{array}\right]$

a. $2R_3 \to R_3$
b. $R_1 \leftrightarrow R_3$
c. $R_3 - R_1 \to R_3$
d. $R_2 - 2R_1 \to R_2$

3) For the following augmented matrix perform the indicated elementary row operations: $\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ -2 & 0 & -1 & 2 \\ -1 & 2 & 2 & -1 \end{array}\right]$.

a. $\frac{1}{2}R_3 \to R_3$
b. $R_2 \leftrightarrow R_3$
c. $R_2 - 4R_1 \to R_2$
d. $R_3 + 2R_1 \to R_3$

For the following systems of equations, convert the system into an augmented matrix and use the augmented matrix techniques to determine the solution to the system, or to determine if the system is inconsistent:

4) $\begin{cases} 2x+7y=13 \\ 2x+5y=11 \end{cases}$

5) $\begin{cases} 2x+3y=7 \\ 4x-5y=3 \end{cases}$

6) $\begin{cases} 2x+3y=8 \\ 5x-4y=-3 \end{cases}$

7) $\begin{cases} 4x+8y=20 \\ 3x+6y=15 \end{cases}$

8) $\begin{cases} -6x+3y=15 \\ 10x-5y=-25 \end{cases}$

9) $\begin{cases} 8x-4y=10 \\ -6x+3y=1 \end{cases}$

10) $\begin{cases} x+2y+3z=-11 \\ 2x+3y-z=-5 \\ 3x+y-z=2 \end{cases}$

11) $\begin{cases} 2x-y-3z=5 \\ 3x-2y+2z=5 \\ 10x-6y-2z=32 \end{cases}$

12) $\begin{cases} x+2y+3z=3 \\ 4x+6y+16z=8 \\ 3x+2y+17z=1 \end{cases}$

13) $\begin{cases} x+3y=2 \\ 2x+y=-1 \\ x-y=-2 \end{cases}$

14) $\begin{cases} 4x-7y=0 \\ 8x-14y=2 \\ -16x+28y=0 \end{cases}$

15) $\begin{cases} 3x-2y=1 \\ -9x+6y=-3 \\ 6x-4y=2 \end{cases}$

16) $\begin{cases} x+2y+2z=2 \\ 3x-2y-z=5 \\ 2x-5y+3z=-4 \\ 2x+8y+12z=0 \end{cases}$

17) Solve the following SLE:

a. $\begin{cases} z_1+iz_2+(1-i)z_3=1+4i \\ iz+z_2+(1+i)z_3=2+i \\ (-1+3i)z_1+(3-i)z_2+(2+4i)z_3=5-i \end{cases}$

Over the field $\mathbb{C}$ (Complex numbers).

b. $\begin{cases} z_1+iz_2+(1-i)z_3=1+4i \\ iz+z_2+(1+i)z_3=2+i \\ (-1+3i)z_1+(3-i)z_2+(2+4i)z_3=5-i \end{cases}$

Over the field $\mathbb{R}$ (Real numbers).

Answer Key

1) a. $\left[\begin{array}{cc|c} 3 & -2 & 1 \\ 2 & 0 & -1 \end{array}\right] \underset{2R_1 \to R_1}{\longrightarrow} \left[\begin{array}{cc|c} 6 & -4 & 2 \\ 2 & 0 & -1 \end{array}\right]$

b. $\left[\begin{array}{cc|c} 3 & -2 & 1 \\ 2 & 0 & -1 \end{array}\right] \underset{R_1 \leftrightarrow R_2}{\longrightarrow} \left[\begin{array}{cc|c} 2 & 0 & -1 \\ 3 & -2 & 1 \end{array}\right]$

c. $\left[\begin{array}{cc|c} 3 & -2 & 1 \\ 2 & 0 & -1 \end{array}\right] \underset{R_2+R_1 \to R_2}{\longrightarrow} \left[\begin{array}{cc|c} 3 & -2 & 1 \\ 5 & -2 & 0 \end{array}\right]$

d. $\left[\begin{array}{cc|c} 3 & -2 & 1 \\ 2 & 0 & -1 \end{array}\right] \underset{R_2+2R_1 \to R_2}{\longrightarrow} \left[\begin{array}{cc|c} 3 & -2 & 1 \\ 8 & -4 & 1 \end{array}\right].$

2) a. $\left[\begin{array}{ccc|c} 1 & -2 & 0 & 3 \\ 4 & 0 & -1 & 2 \\ 3 & 1 & 2 & -4 \end{array}\right] \underset{2R_3 \to R_3}{\longrightarrow} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 3 \\ 4 & 0 & -1 & 2 \\ 6 & 2 & 4 & -8 \end{array}\right]$

b. $\left[\begin{array}{ccc|c} 1 & -2 & 0 & 3 \\ 4 & 0 & -1 & 2 \\ 3 & 1 & 2 & -4 \end{array}\right] \underset{R_1 \leftrightarrow R_3}{\longrightarrow} \left[\begin{array}{ccc|c} 3 & 1 & 2 & -4 \\ 4 & 0 & -1 & 2 \\ 1 & -2 & 0 & 3 \end{array}\right]$

c. $\left[\begin{array}{ccc|c} 1 & -2 & 0 & 3 \\ 4 & 0 & -1 & 2 \\ 3 & 1 & 2 & -4 \end{array}\right] \underset{R_3-R_1 \to R_3}{\longrightarrow} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 3 \\ 4 & 0 & -1 & 2 \\ 2 & 3 & 2 & -7 \end{array}\right]$

d. $\left[\begin{array}{ccc|c} 1 & -2 & 0 & 3 \\ 4 & 0 & -1 & 2 \\ 3 & 1 & 2 & -4 \end{array}\right] \underset{R_2-2R_1 \to R_2}{\longrightarrow} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 3 \\ 2 & 4 & -1 & -4 \\ 3 & 1 & 2 & -4 \end{array}\right]$

3) a. $\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ -2 & 0 & -1 & 2 \\ -1 & 2 & 2 & -1 \end{array}\right] \underset{\frac{1}{2}R_3 \to R_3}{\longrightarrow} \left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ -2 & 0 & -1 & 2 \\ -0.5 & 1 & 1 & -0.5 \end{array}\right]$

b. $\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ -2 & 0 & -1 & 2 \\ -1 & 2 & 2 & -1 \end{array}\right] \underset{R_2 \leftrightarrow R_3}{\longrightarrow} \left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ -1 & 2 & 2 & -1 \\ -2 & 0 & -1 & 2 \end{array}\right]$

c. $\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ -2 & 0 & -1 & 2 \\ -1 & 2 & 2 & -1 \end{array}\right] \underset{R_2-4R_1 \to R_2}{\longrightarrow} \left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ -6 & -8 & -17 & 2 \\ -1 & 2 & 2 & -1 \end{array}\right]$

d. $\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ -2 & 0 & -1 & 2 \\ -1 & 2 & 2 & -1 \end{array}\right] \underset{R_3+2R_1 \to R_3}{\longrightarrow} \left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ -2 & 0 & -1 & 2 \\ 1 & 6 & 10 & -1 \end{array}\right]$

4) $(3,1)$

5) $(2,1)$

6) $(1,2)$

7) $(5-2t,t)$, ∞ solutions.

8) $(-2.5+0.5t,t)$, ∞ solutions.

9) No solution.

10) $(1,-3,-2)$

11) No solution.

12) ∞ solutions.

13) $(-1,1)$

14) No solution.

15) ∞ solutions.

16) $(2,1,-1)$

17) a. $z_1 = 1+i-(1-i)t,\ z_2 = 3,\ z_3 = t$ b. $z_1 = 2,\ z_2 = 3,\ z_3 = -1$

SLE with Parameter

Questions

Determine the values of k for which the system below has no solutions, exactly one solution, or infinitely many solutions:

1) $\begin{cases} x-y+z=1 \\ 5x-7y+\left(k^2+3\right)z=k^2+1 \\ 3x-y+\left(k+3\right)z=3 \end{cases}$

2) $\begin{cases} x+ky+z=1 \\ x+y+kz=1 \\ kx+y+z=1 \end{cases}$

3) $\begin{cases} x+2ky+z=0 \\ 3x+y+kz=2 \\ x+9ky+5z=-2 \end{cases}$

4) $\begin{cases} 2x-y+z=0 \\ x+2y-z=0 \\ 5x+\left(1-k\right)y+k^2z=1 \end{cases}$

5) $\begin{cases} kx-y=1 \\ \left(k-2\right)x+ky=-2 \\ \left(k^2-1\right)z=9 \end{cases}$

6) $\begin{cases} x+ky+3z=2 \\ kx-y+z=4 \\ 3x+y+\left(2+k\right)z=0 \end{cases}$

7) $\begin{cases} 2x+ky=3 \\ \left(k+3\right)x+2y=k^2+5 \\ 6x+3ky=7k^2+2 \end{cases}$

8) $\begin{cases} 2x-3y+z=1 \\ 4x+\left(k^2-5k\right)y+2z=k \end{cases}$

9) $\begin{cases} 3x+4y-z=2 \\ kx-2y+z=-1 \\ x+8y-3z=k \\ 2x+6y-2z=0.5k+1 \end{cases}$

Determine the values of a and b, for which the systems below has no solutions, exactly one solution, or infinitely many solutions:

10) $\begin{cases} 2x+4y+az=-1 \\ x+2y+4z=-4 \\ x+2y-4z=0 \\ x+2y+6z=-2b \end{cases}$

11) $\begin{cases} x+y-z+t=1 \\ ax+y+z+t=b \\ 3x+2y+at=1+a \end{cases}$

12) Determine the following:

a. Relationship between the values of a, b, c and d for which the system below has exactly one solution.

b. The values of b, c and d for which the system below has infinitely many solutions for all values of a.

$$\begin{cases} x+az=1 \\ y+2z=2 \\ bx+cy+dz=3 \end{cases}$$

13) Given the SLE $\begin{cases} x+y-z=1 \\ 3x-7y+\left(k^2+1\right)z=k^2-1 \\ 4x-6y+\left(k+2\right)z=4 \end{cases}$.

a. Write the matrix corresponding to the system.

b. Bring the matrix to row-echelon form.

c. Find the values of k for which the SLE has [no, one, infinitely many solution(s)].

Continuing with the SLE (in echelon-form): $\begin{cases} x+\ y-z=1 \\ -10y+\left(k^2+4\right)z=k^2-4 \\ \left(-k^2+k+2\right)z=4-k^2 \end{cases}$

d. Write the general solution for the case when there are infinitely many solutions.

e. For which value of k does the SLE have a solution with $z=0$?

f. For which value of k does the SLE have a unique solution with $z=0$?

Given the SLE $\begin{cases} x+y-z=1 \\ 3x-7y+\left(k^2+1\right)z=k^2-1 \\ 4x-6y+\left(k+2\right)z=4 \end{cases}$

g. For which value of k will $\left(x,y,z\right)=\left(1,2,3\right)$ be a solution of the 3^{rd} equation?

h. For which value of k is $\left(x,y,z\right)=\left(1,2,3\right)$ a solution of the SLE?

Answer Key

1) One solution: $k \neq -2,\ k \neq 1$; Infinitely many solutions: $k=-2$; No solutions: $k=1$.

2) One solution: $k \neq 1,\ k \neq -2$; Infinitely many solutions: $k=1$; No solutions: $k=-2$.

3) One solution: $k \neq -1,\ k \neq \frac{4}{7}$; Infinitely many solutions: $k=-1$; No solutions: $k=\frac{4}{7}$.

4) One solution: $k \neq 1,\ k \neq -0.4$; No solutions: $k=1$ or $k=-0.4$.

5) No solutions: $k=1$, $k=-1$,$k=-2$; Otherwise: Single solutions.

6) One solution: $k=-1$, $k=2$, $k=-3$; Otherwise: No solutons.

7) One solution: $k \neq -1$; Infinitely many solutions: $k=1$; No solutions: $k \neq \pm 1$.

8) Infinitely many solutions: $k \neq 3$; No solutions: $k=3$.

9) One solution: $k \neq 1$; No solutions: $k=1$.

10) No solutions: $5-2b \neq 0$ or $3+0.5a \neq 0$, $b \neq 2.5$ or $a \neq -6$;

Infinitely many solutions: $b=2.5$ and $a=-6$.

11) No solutions: $a=2,\ b \neq 2$; Infinitely many solutions: $a \neq 2$ or $a=b=2$.

12) a. One solution: $d-ab-2c \neq 0$;

b. Infinitely many solutions for all a : $b=0,\ c=1.5,\ d=3$.

13) a. $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 3 & -7 & k^2+1 & k^2-1 \\ 4 & -6 & k+2 & 4 \end{array}\right]$ b. $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & -10 & k^2+4 & k^2-4 \\ 0 & 0 & -k^2+k+2 & 4-k^2 \end{array}\right]$

c. One solution: ,$k \neq 2,\ k \neq -1$; Infinitely many solutions: $k=2$; No solutions: $k=-1$.

d. $x=1+0.2t,\ y=0.8t,\ z=t$ e. $x=1,\ y=0,\ z=0$ f. $k=-2$

g. $k=2$ h. No value of k.

SLE over Zp

Questions

1) Solve the following:

a. System of linear equations $\begin{cases} 2x - y = 3 \\ x + 2y = 4 \end{cases}$

over the field $\mathbb{R}$.

b. Same SLE over $\mathbb{Z}_5 = \{0,1,2,3,4\}$.

2) Solve the SLE $\begin{cases} x + 2y + 3z = 1 \\ 2x + 4y + 4z = 2 \\ 3x + z = 0 \end{cases}$

over the field $\mathbb{Z}_5 = \{0,1,2,3,4\}$.

3) Solve the SLE $\begin{cases} 3x + y + 4z = 3 \\ 4x + 3y + 3z = 4 \\ 2x + 4z = 0 \end{cases}$

over the field $\mathbb{Z}_5 = \{0,1,2,3,4\}$.

4) Solve the SLE $\begin{cases} x + 4y + 2z + 4t = 1 \\ x + 2y - z = 0 \\ y + z + t = 1 \\ x + 3y - z - 2t = 0 \end{cases}$

over the field $\mathbb{Z}_7 = \{0,1,2,3,4,5,6\}$.

5) Given the system $\begin{cases} x + ky + z = 1 \\ x + y + kz = 1 \\ kx + y + z = 1 \end{cases}$

over the field $\mathbb{Z}_3 = \{0,1,2\}$.

Determine the values of k for which the system has:
exactly one solution; no solutions; infinitely many solutions; other.

6) Given the system $\begin{cases} x-y+z=1 \\ 3y+\left(k^2+3\right)z=k^2+1 \\ 3x-y+\left(k+3\right)z=3 \end{cases}$

over $\mathbb{Z}_5=\{0,1,2,3,4\}$.

Determine the values of k, for which the system has:

a. No solutions.

b. Exactly one solution.

c. Infinitely many solutions.

d. Other.

Answer Key

1) a. $(2,1)$

b. $x=4+3t,\ y=t;$ Just 5 solutions (not ∞) for $t=0,1,2,3,4$:

$(4,0)(2,1)(0,2)(3,3)(1,4)$.

2) $(0,3,0)$

3) $(0,3,0)$

4) $x=1,\ y=4,\ z=2,\ t=\ 2$

5) One solution: $k=0,2$, There are no Infiniate solutions and No solutions.

6) a. No solutions: $k=1$.

b. One solution: $k\neq 3,\ k\neq 1$.

c. There are no infinite solutions.

d. Five Solutions: $k=3$.