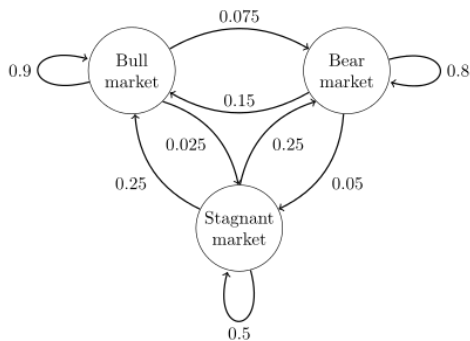


# Markov Chains

## Markov Chains

### Questions:

- 1) Suppose today is sunny. Then what's the probability it will be sunny/rainy:
  - a. Tomorrow?
  - b. In 2 days?
  - c.  $n$  days from now?
  - d. After  $\infty$  days?
  
- 2) A hypothetical stock market, during a given week, can exhibit one of three trends: a bull market, a bear market, or a stagnant market. The 3 trends are our 3 "states". The probability of a trend the following week depends only on the trend in this week, as in the following transition diagram:



- a. Labelling the state space  $\{1 = \text{bull}, 2 = \text{bear}, 3 = \text{stagnant}\}$ , what is the transition matrix?
- b. Find the steady state of this system.

### Answer Key

1) a.  $u_1 = u_0 A = [1 \ 0] \begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix} = [0.8 \ 0.2]$  In words: sunny 80%, rainy 20%.

b.  $u_2 = u_0 A^2 = [1 \ 0] \begin{bmatrix} 0.76 & 0.24 \\ 0.72 & 0.28 \end{bmatrix} = [0.76 \ 0.24]$

c.  $u_n = \frac{1}{4} [3 + 0.2^n \quad 1 - 0.2^n]$

d.  $[0.75 \ 0.25]$

2) a. Let's call it  $A$ , then:  $A = \begin{bmatrix} 0.9 & 0.075 & 0.025 \\ 0.15 & 0.8 & 0.05 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$ .

b. Bull market 62.5%, bear market 31.25%, stagnant market 6.25%.