proprep™

Workbook

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Partial Derivative

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Questions:

1) Compute the first order partial derivatives of the following functions:

a. $f(x,y)=4x^3-3x^2y^2+2x+3y$

b. $f(x,y)=x^5\ln y$

c. $(\text{only } f_x)\ f(x,y)=\dfrac{x^2y^4\left(\sqrt{y}+5\ln y\right)}{y^2+5y+y^y}$

d. $f(x,y)=\left(x^2+y^3\right)\cdot\left(2x+3y\right)$

e. $f(x,y)=\dfrac{x^2-3y}{x+y^2}$

f. $f(x,y)=\sin(xy)$

g. $f(x,y)=\arctan(2x+3y)$

h. $f(r,\theta)=r\cos\theta$

i. $f(x,y,z)=xy^2z^3$

j. $f(u,v,t)=e^{uv}\sin ut$

2) Compute the second order partial derivatives of the following functions:

a. $f(x,y)=4x^2-x^2y^2+4x+10y$

b. $f(x,y)=x^4\ln y$

c. $f(x,y)=\sin(10x+4y)$

d. $f(x,y,z)=xyz$

3) Answer the following questions:

a. Find the partial derivatives of the function below at the point $(0,0)$.

b. Is the function continuous at $(0,0)$?

c. If a function is partially differentiable at a point, must it also be continuous at that point?

$$f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$$

4) Is the function from Exercise 3 differentiable at $(0,0)$?

5) Check the differentiability of the following functions at $(0,0)$:

a. $$f(x,y)=\begin{cases}\dfrac{x^3+y^3}{2x^2+y^2} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$$

b. $$f(x,y)=\begin{cases}\left(x^2+y^2\right)\sin\left(\dfrac{1}{\sqrt{x^2+y^2}}\right) & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$$

6) Check the differentiability of the following function in its domain of definition:

$$f(x,y)=\begin{cases}e^{-\frac{1}{x^2+y^2}} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$$

Notation:

$$f=f(x,y)\quad\Rightarrow\quad\begin{array}{ll} f_x=\dfrac{\partial f}{\partial x}=f_1 & f_y=\dfrac{\partial f}{\partial y}=f_2\\ f_{xx}=\dfrac{\partial^2 f}{\partial x^2}=f_{11} & f_{yy}=\dfrac{\partial^2 f}{\partial y^2}=f_{22}\\ f_{xy}=\dfrac{\partial^2 f}{\partial y\partial x}=f_{12} & f_{yx}=\dfrac{\partial^2 f}{\partial x\partial y}=f_{21}\end{array}$$

Answer Key:

1) a. $f_x = 12x^2 - 6xy^2 + 2$, $f_y = -6x^2 y + 3$ b. $f_x = 5x^4 \ln y$, $f_y = \frac{x^5}{y}$

c. $f_x = 2x\frac{y^4\left(\sqrt{y} + 5\ln y\right)}{y^2 + 5y + y^y}$ d. $f_x = 6x^2 + 6xy + 2y^3$, $f_y = 6xy^2 + 12y^3 + 3x^2$

e. $f_x = \frac{x^2 + 2xy^2 + 3y}{\left(x + y^2\right)^2}$, $f_y = \frac{-3x + 3y^2 - 2x^2 y}{\left(x + y^2\right)^2}$

f. $f_x = \cos(xy) \cdot y$, $f_y = \cos(xy) \cdot x$ g. $f_x = \frac{2}{1 + (2x + 3y)^2}$, $f_y = \frac{3}{1 + (2x + 3y)^2}$

h. $f_r = \cos\theta$, $f_\theta = -r\sin\theta$ i. $f_x = y^2 z^3$, $f_y = 2xyz^3$, $f_z = 3xy^2 z^2$

j. $f_u = e^{uv}\left[v\sin ut + t\cos ut\right]$, $f_v = u \cdot e^{uv} \cdot \sin ut$, $f_t = u \cdot e^{uv} \cdot \cos ut$

2) a. $f_x = 8x - 2xy^2 + 4$, $f_{xx} = 8 - 2y^2$, $f_y = -2x^2 y + 10$, $f_{yy} = -2x^2$, $f_{xy} = -4xy$, $f_{yx} = -4xy$

b. $f_x = 4x^3 \ln y$, $f_{xx} = 12x^2 \ln y$, $f_y = \frac{x^4}{y}$, $f_{yy} = -\frac{x^4}{y^2}$, $f_{xy} = \frac{4x^3}{y}$, $f_{yx} = \frac{4x^3}{y}$

c. $f_x = 10\cos(10x + 4y)$, $f_{xx} = -100\sin(10x + 4y)$, $f_y = 4\cos(10x + 4y)$,

$f_{yy} = -16\sin(10x + 4y)$, $f_{xy} = -40\sin(10x + 4y)$, $f_{yx} = -40\sin(10x + 4y)$

d. $f_x = yz$, $f_{xx} = 0$, $f_{xy} = z$, $f_{xz} = y$, $f_y = xz$, $f_{yx} = z$, $f_{yy} = 0$, $f_{yz} = x$, $f_z = xy$,

$f_{zx} = y$, $f_{zy} = x$, $f_{zz} = 0$

3) a. The partial derivatives at the point $(0,0)$ equal zero.

b. The function is not continuous at $(0,0)$.

c. A partially differentiable function at a point is not necessarily continuous at that point.

4) Not differentiable.

5) a. Not differentiable b. Differentiable.

6) Differentiable in all its domain.