proprep™

# Workbook

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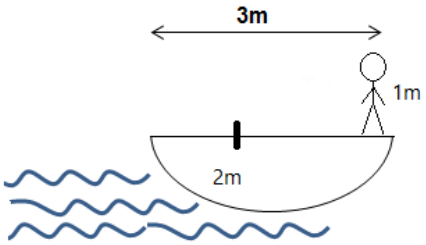
# Center of Mass

## Center of a Mass

### Questions

**1) Man on a Boat.**

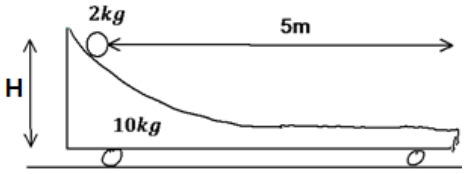
A man stands at one end of a boat of length $3\text{m}$.
The man weighs $70\text{kg}$ and the boat weighs $100\text{kg}$.
The man moves $2\text{m}$ up the boat.
How much does the boat move?
Ignore friction between boat and water.
Given: $m_1 = 70\text{kg}$, $m_2 = 100\text{kg}$.




**2) Ball on Ramp.**

A ball is resting on a sloped ramp which is also at rest.
The height of the ball is $1\text{m}$, and it is $5\text{m}$ away from the edge of the ramp.
The mass of the ramp, $m_1 = 10\text{kg}$ and the mass of the ball, $m_2 = 2\text{kg}$.

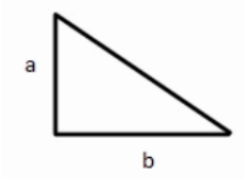
a. Find the displacement of the ramp when the ball reaches its edge.
b. Find the velocity of the bodies if it is given that the velocity of the ball at the edge of the ramp is only is the $x$ direction.



# Calculating the Center of Mass Using Integrals

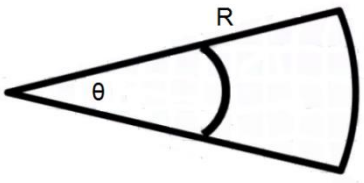
## Questions:

**3) Triangles Center of Mass.**
Find the triangle's center of mass.



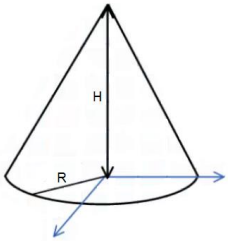

**4) Gate Center of Mass.**
An electric gate of mass $m$ and length $l$ is resting on an axis, which is located a distance $d$ from its edge. Explain why a heavy mass is attached to one end of the gate and find it's mass, if its length is $L$.

**5) Sector Center of Mass.**
Find the center of mass of the sector with uniform density and $\measuredangle\theta°$.




**6) Area of Segment.**
Find the area of a segment.

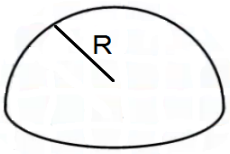
**7) Center of Mass of a Solid Cone.**
Find the center of mass of a solid cone with uniform density.




**8) Half Hoop with Two Masses.**
Find the center of mass of the half hoop of mass $M$ and radius $R$, when at each end there is also ball of mass $m$ attached.

**9) Center of Mass Half Sphere Full**

Find the center of mass of a filled half sphere.




**10) Center of Mass of a Rod**

Find the center of mass of a non-homogeneous rod of length $L$.

Density from left end: $\lambda(x) = \lambda_0 \frac{x}{L}$.


## Answer Key

**1)** $x = \frac{14}{17}\text{m}$

**2)** a. $\Delta x_1 = -\frac{10}{12}\text{m}$ b. $u_1 = \frac{2}{\sqrt{3}}\text{m/s}, \quad u_2 = \frac{-10}{\sqrt{3}}\text{m/s}$

**3)** $x_{\text{cm}} = \frac{1}{3}b$

**4)** $M = \frac{\frac{L}{2}m - dm}{\frac{L}{2} + d}$

**5)** $x_{\text{cm}} = \frac{4R\sin\frac{\theta_0}{2}}{3\theta_0}$

**6)** $S = \frac{\theta R^2}{2}$

**7)** $Z_{\text{cm}} = \frac{H}{4}$

**8)** $y_{cm_{Hoop}} = \frac{\sqrt{2}MR}{\pi(M+2m)}$