proprep™

# Workbook

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# Momentum and Impulse

## Impulse

$$J_{\sum F} = \int \sum \vec{F} \, , \mathrm{dt} = \triangle \vec{P}$$

### Questions

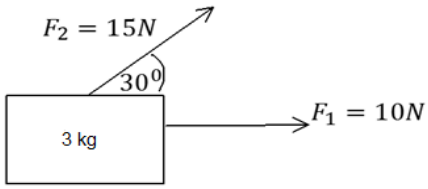
**1) Calculating Impulse.**

A football player kicks a ball with a mass of $2\mathrm{kg}$ with a constant force of 50 Newton's.
The player is touching the ball for 0.2 seconds.
What will be the velocity with which the ball travels?

**2) Two Forces on an Object.**

You are given an object with a mass of $3\mathrm{kg}$.
Two forces are acting upon the object as described in the diagram for a period of 0.5 seconds.

a. Find the impulse of each force.
b. Find the total impulse applies to the object.
c. Find the velocity of the object after the forces have acted upon it, assuming that the object starts at rest.




**3) Impulse of an Average Force.**

A ball with mass of $1\mathrm{kg}$ is thrown at a wall with a velocity of $2\,\mathrm{m/s}$.
The ball hits the wall and comes back with identical velocity.

a. Find the Impulse applied to the ball.
b. Who applied the impulse described above?
c. Calculate the average normal force that the wall exerted if the period of exertion was 0.2 seconds

## Conserving Momentum and External Forces

**4) Conservation of Momentum.**

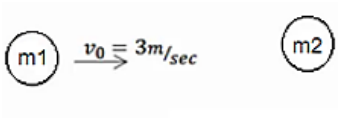
A ball with a mass $m_1$ and velocity $v_0$ collides with a second ball with a mass of $m_2$.

After the collision the first ball at a $\sphericalangle 45°$ downwards with respect to the $x$-axis and the second ball files at a $\sphericalangle 30°$ with respect to the $x$-axis.

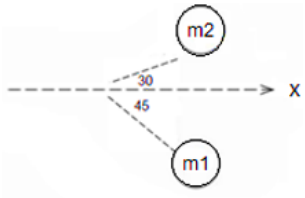
Given: $m_1 = 3\text{kg}$, $m_2 = 2\text{kg}$, $v_0 = 3\text{m/s}$.

a. Find the velocity of each object after the collision.
b. Find the Impulse applied to each object.

Before collision:




After collision :




## Advanced Exercises

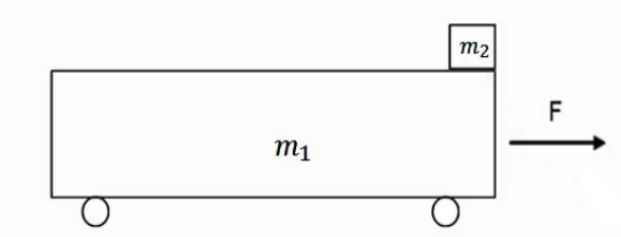
**5) An Object on a Trolley and Loss of Energy.**

Force $F$ is a constant force pulling a trolley with a mass of $m_1$ without friction.

On top of the Trolley is the mass $m_2$. There is friction between the two objects.

Given: $\mu_s$, $\mu_k$, $m_1$, $m_2$.

a. What is the maximum force that $F$ can reach without $m_2$ slipping relative to $m_1$?
   Assume that Force $F$ is larger than the maximum from part a.
   Also assume the $F$ is acting for a given time $T$ and that $m_2$ doesn't fall off on $m_1$.
b. What is the acceleration, velocity, and position of $m_1$ and $m_2$ as a function of time until the time $T$?
c. How much energy is lost during this time?
d. Find the terminal velocity of the objects $(t > T)$ before the $m_2$ fails.

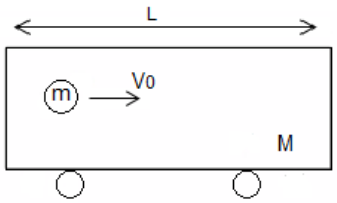


## From the Trolley and Lab Perspective

$M = am$

### Questions

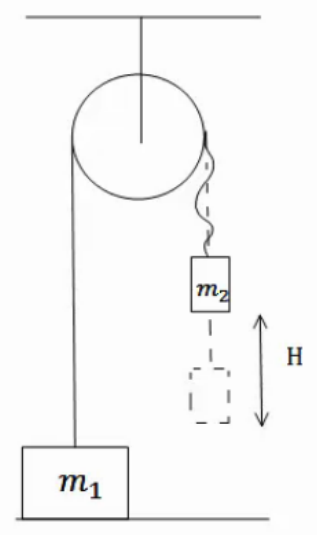
**6) Elastic collision.**




a. What is the velocity of the objects after collision? Check for $a = 0, 1, \infty$.

b. How much time will pass between the first and second collisions?

**7) Two Masses on a Pulley with a Loose Rope.**




Tow masses, $m_1$ and $m_2$, are hanging from an ideal, frictionless pulley. $m_1$ is resting on the ground, while $m_2$ is hanging in the air. $m_2$ is raised an additional distance $H$ from the ground such that the rope holding the two masses hangs loose.
The system is at rest as described until $m_2$ is dropped.

a. Find the velocity of $m_2$ before the rope is fully taut.

b. Assume that the rope has reached full tension and that it is not elastic.
Find the total change in momentum of both objects from the moment just before the rope reaches full tension to the moment just after the rope reaches full tension and both masses move.

c. Find the impulse applied to the pulley be the ceiling during part a.

d. How high will $m_1$ rise, assuming that $m_1 > m_2$ and that $m_2$ doesn't reach the ground?

e. How much impulse does the ceiling apply to the pulley from the time $t = 0$, until $m_1$ reaches its maximum height?

## Answer Key

**1)** $v_f = 0.5\text{m/sec}$

**2)** a. $\vec{J}_1 = 5N\cdot\sec\hat{x}, \quad J_{2x} = 7.3\frac{\sqrt{3}}{2}\hat{x}, \quad J_{2y} = 7.5\cdot\frac{1}{2}\hat{y}$ b. $|J| = 12.1N\sec$

c. $v_{xf} = \frac{11.5}{3}\frac{\text{m}}{\text{sec}}, \quad v_{yf} = \frac{3.75}{3}\frac{\text{m}}{\text{sec}}$

**3)** a. $1_{\text{kg}}\cdot\left(-4\frac{\text{m}}{\text{sec}}\right) = -4N\cdot\sec = \vec{J}$ b. The Normal Force. c. $\vec{\vec{N}} = -20N\hat{x}$

**4)** a. $v_1 = 0.98\text{m/s}, \quad \text{v}_2 = 2.078\text{m/s}$

b. $J_{1x} = -6.921N\cdot\sec, \quad J_{1y} = -2.079N\cdot\sec, \quad -J_{2x} = J_{1x}, \quad -J_{2y} = J_{1y}$

**5)** a. $F = \mu sg\left(m_1 + m_2\right)$

b. Acceleration: $a_1 = \frac{F}{m_1} - \frac{m_2}{m_1}\mu_{kg}, \quad a_2 = \mu_{kg}$, velocity: $v_1 = a_1 t, \quad v_2 = a_2 t$,

Position as a function of time: $x_1 = \frac{1}{2}\vec{a}_1 t^2, \quad x_2 = \frac{1}{2}\vec{a}_2 t^2$

c. $E_F = \frac{1}{2}m_2 v_2^2(T) + \frac{1}{2}m_1 v_1^2(T)$ d. $U_F = \frac{FT}{m_1 + m_2}$

**6)** a. $\alpha = 0 \Rightarrow u_1 = v_0, \quad u_2 = 2v_0; \quad \alpha = 1 \Rightarrow u_1 = 0, \quad u_2 = v_0; \quad \alpha = \infty \Rightarrow u_1 = -v_0, \quad u_2 = 0$

b. $t = \frac{L}{u_2 - u_1}$

**7)** a. $v_2 = \sqrt{2gH}$ b. $\Delta P_T = J_{collision} = 2\int T dt = \frac{2m_1 m_2}{m_1 + m_2}\sqrt{2gH}$

c. $J_{ceiling} = \int T_2 dt = \int 2T dt$ d. $h = \frac{m_2}{m_1 - m_2}\sqrt{\frac{H}{2g}}$

e. Solution in the recording.