

# Workbook



## Table of Contents

Rigid Body .....	2
Angular Momentum of a Rigid Body .....	2
Rotational Energy of a Rigid Body.....	3
Analysis Through Forces And Moments, Rolling Without Slipping .....	3
Rolling with Slipping.....	4
End of Chapter Question.....	5

# Rigid Body

## Angular Momentum of a Rigid Body

### Questions

**1) Ball and Disk Collision.**

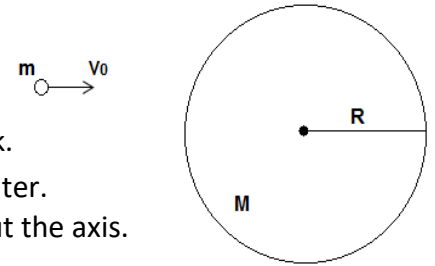
A disk of mass  $M$  and radius  $R$  is at rest and attached to a frictionless axis at its center.

A small ball of mass  $m$  moves at a velocity  $v_0$  towards the disk.

The ball hits the disk from the left, a distance  $d$  above its center.

The ball sticks on to the disk, which then begins rotating about the axis.

What is the initial angular velocity of the system?



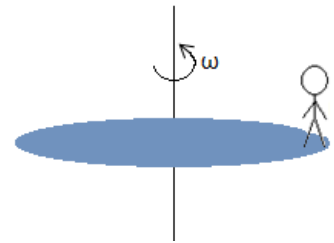
**2) Man Jumps off a Disk.**

A disk of radius  $R$  and mass  $M$  is rotating about an axis at its center a constant angular velocity  $\omega_0$ .

A man of mass  $m$  stands at the edge of the disk.

The man jumps off the disk. His velocity at this moment is  $v_0$ , in the radial direction, relative to the ground.

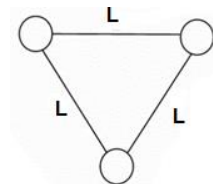
What is the disk's angular velocity after the jump?



**3) Three Balls.**

Three identical balls of mass  $m$  are placed at the corners of an equilateral triangle. The balls are conjoined by three massless rods of length  $L$  (the sides of the triangle).

- a. Find the systems centre of mass.



It is now given that the system moves with angular velocity  $\omega$  about the centre of mass. At some moment, when the system is in the position described by the diagram, the lower ball disconnects from the system.

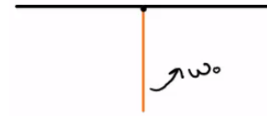
- b. Find the velocity of the disconnected ball after the break off.
- c. Find the velocity of the remaining system's centre of mass.
- d. Find the angular velocity of the remaining system about its centre of mass.

## Rotational Energy of a Rigid Body

### Questions

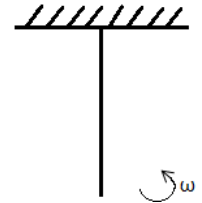
**4) Rotating Rod.**

A rod, of length  $L$  and mass  $M$ , is attached to the ceiling.  
The rod rotates at an initial angular velocity of  $\omega_0$ .  
What is the maximum angle which the rod will reach?



**5) Ball Hits Rod.**

A ball of mass  $m$  hits a rod, which is attached to the ceiling, a distance  $x$  from the rod's axis of rotation.  
The rod is of length  $L$  and mass  $M$ .  
a. What is the angular velocity of the system right after collision?  
b. What is the maximum angle the rod will reach?  
c. Find which length of  $x$  will cause the force being applied by the ceiling to the rod to be equal to 0.

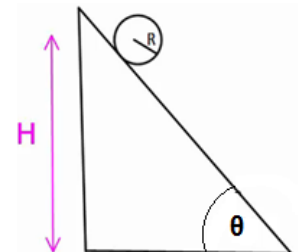


## Analysis Through Forces And Moments, Rolling Without Slipping

### Questions

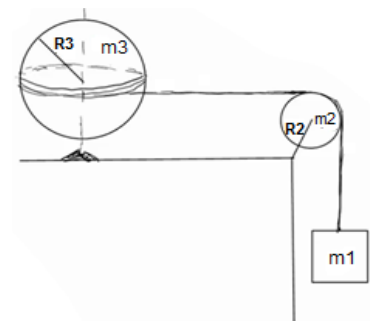
**6) Ball on a Slope.**

A ball of radius  $R$  is placed at a height  $H$  on a slope of angle  $\theta^\circ$ .  
The ball begins to roll down the slope without slipping.  
a. What is the velocity of the ball at the bottom of the slope?  
b. What is the ball's acceleration?



**7) Ball and Pulley.**

A ball is nailed to a table.  
It rotates around the axis perpendicular to the table.  
A rope is wound around the center of the ball, it rests on a non-ideal pulley system. A mass  $m_1$ , is attached to its end.  $m_2$  and  $R_2$  are the mass and radius of the pulley, and  $m_3$  and  $R_3$  are the mass and radius of the ball.  
The system begins at rest.  
Find each body's acceleration, as well as the tension in the rope.

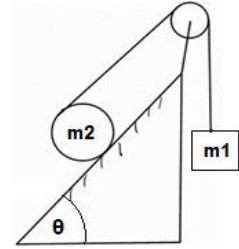


8) **Yoyo and a Mass.**

A yo-yo (a ball with a string wound around it) of mass  $m_2$  and radius  $R$  is placed on a slope of angle  $\theta^\circ$ .

The yo-yo's string is attached, via an ideal pulley, to mass  $m_1$ .

We are told that the yo-yo rolls, without slipping, down the slope and that there is friction between the yo-yo and the slope.



- In which direction is the static friction? Find the movement of the system.
- Find the accelerations of the bodies and the size of the friction.

9) **Falling Horizontal Rod.**

A rod of mass  $M$  (with uniform density) and length  $L$  is hung from one end to a wall such that it is free to rotate about the point of attachment.

The rod is released from a horizontal position.

- Find the angular acceleration and the acceleration of the rod's center of mass at the moment of release.
- Find the force that the axis (connecting the rod to the wall) exerts at the moment of release.

$L, M$

The rod falls until it is perpendicular to the ground.

- Find the angular acceleration of the rod at this moment (when the rod is perpendicular to the ground).
- Repeat sections a. and b., but this time the rod is perpendicular to the ground.

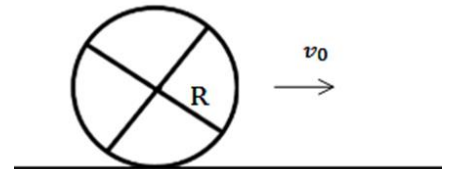


**Rolling with Slipping**

10) **Ball Slipping without Rotation.**

A homogenous ball of mass  $M$  and initial velocity without rotating (no angular velocity).

Find its final velocity if it is known that the coefficient of friction is kinetic.

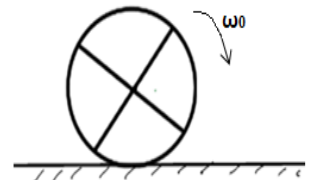


11) **Spinning Ball.**

A homogenous ball of mass  $M$  is held in the air, where it rotates about its center of mass with an angular velocity  $\omega_0$ .

The ball is lowered to the ground whilst rotating.

What is the ball's final velocity, if the coefficient of friction is  $\mu_k$ ?



## End of Chapter Question

---

### 12) Falling Pencil.

A pencil stands perpendicular to the table.

The pencil begins to fall to the right.

When the angle between the pencil and the axis perpendicular to the table reaches  $\theta_1$  the pencil begins to slip.

- a. For all angles  $\theta$  such that  $\theta < \theta_1$ :
  - i. What is the angular velocity of the pencil?
  - ii. What is the angular acceleration of the pencil?
  - iii. Find the acceleration vector of the pencil's center of mass.
  - iv. Find the size and direction of the frictional force.
  - v. Find the normal force.
- b. Find the static coefficient of friction,  $\mu_s$ .



Answer Key

$$1) \quad \omega = \frac{mv_0 d}{R^2 \left( \frac{1}{2}M + m \right)}$$

$$2) \quad \omega = \frac{\left( \frac{1}{2}M + m \right) \omega_0}{\frac{1}{2}M}$$

$$3) \quad a. \quad y_{cm} = \frac{\sqrt{3}L}{6} = \frac{L}{2\sqrt{3}}$$

$$b. \quad \vec{v}_3 = -\frac{\omega L}{\sqrt{3}} \hat{x}$$

$$c. \quad \vec{v}_{cm} = \frac{\omega L}{2\sqrt{3}} \hat{x}$$

$$d. \quad \omega' = \frac{2\omega}{L} \left( \frac{2L}{3} - \frac{1}{6} \right)$$

$$4) \quad \cos \theta = 1 - \frac{L\omega_0^2}{3g}$$

$$5) \quad a. \quad \omega = \frac{mu_0 x}{mx^2 + \frac{1}{3}ML^2}$$

$$b. \quad \cos \theta = -\frac{\frac{1}{2}I_r \omega^2}{g \left( \frac{ML}{2} + mx \right)} + 1$$

$$c. \quad x = \frac{\frac{mu_0}{2} - \frac{ML}{2}}{m}$$

$$6) \quad a. \quad v_{cm} = \sqrt{\frac{10}{7}gh}$$

$$b. \quad a_x = \frac{5}{3}g \sin \theta$$

7) Solution in the recording.

8) Solution in the recording.

$$9) \quad a. \quad \alpha = \frac{3g}{2L}, \quad a_{cm} = \frac{3g}{4} \hat{y}, \quad a_x = 0 \quad b. \quad F_x = 0, \quad F_y = \frac{1}{4}mg \quad c. \quad \omega = \sqrt{\frac{3g}{L}}$$

$$d. \quad \alpha = 0, \quad a_{cm} = -\frac{3g}{2}; \quad F_x = 0, \quad F_y = -\frac{1}{2}Mg$$

$$10) \quad v_f = \frac{5}{7}v_0$$

$$11) \quad v \left( \frac{2\omega_0 R}{7\mu_k g} \right) = \frac{2}{7}\omega_0 R$$

$$12) \quad a. (i) \quad \omega = \sqrt{3\frac{g}{L}(1 - \cos \theta)} \quad (ii) \quad \alpha = \frac{3g}{2L} \sin \theta \quad (iii) \quad \vec{a} = -3\frac{g}{2}(1 - \cos \theta) \hat{r} + \frac{3g}{4} \sin \theta \hat{\theta}$$

$$(iv) \quad f_s = \frac{3}{2}g \left( \frac{1}{2} \sin \theta \cos \theta + \cos \theta - 1 \right)$$

$$(v) \quad N = mg \left( 1 - \frac{3}{2}(1 - \cos \theta) \cos \theta - \frac{3}{4} \sin^2 \theta \right)$$

$$b. \quad \mu_R = \frac{3 \left( \frac{1}{2} \sin \theta_1 \cos \theta_1 + \cos \theta_1 - 1 \right)}{2m \left( 1 - \frac{3}{2} \cos \theta_1 + \frac{3}{2} \cos^2 \theta_1 - \frac{3}{4} \sin^2 \theta_1 \right)}$$