proprep™

Workbook

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Special Relativity

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Questions

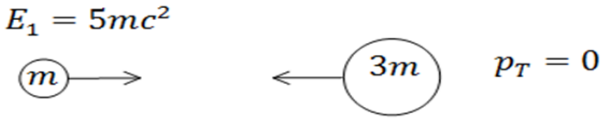
1) Neutron Arriving.

Find the energy needed for a neutron to reach Earth from a distance of five light-years away, given that the lifespan of a neutron is 881 seconds, and its mass is $Mn = 940\frac{MeV}{C^2}$.

2) Collision.

A particle of mass m collides with another particle of mass $3m$. The first particle had an initial energy of $5mc^2$ before the collision. The total momentum of the two particles, Relative to the lab, is 0. As a result of the collision, the two particles become a new, third **particle**, at rest.

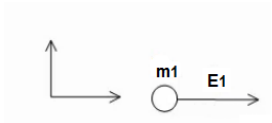
a. What is the kinetic energy of particle number 1?
b. Calculate the Lorentz Factor of the particles before the collision, and the kinetic energy of the second particle.
c. What is the mass of the third particle, formed after the collision?



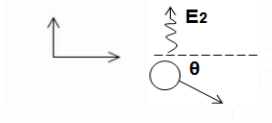
3) Particle Splits to Photon and another Particle.

A particle of energy E_1 and resting mass m_1 is moving in the lab in the positive direction of the x-axis.
At some moment the particle breaks down and transforms in to a photon and particle 2. The photon moves in the positive y direction and has an energy of E_γ.

Before:



After:



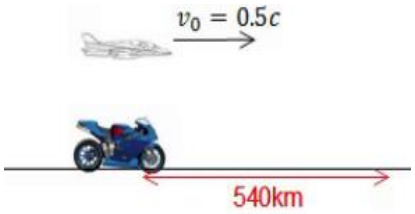
a. What is the momentum of the particle before the decay?
b. At what angle is the momentum of particle 2 relative to the x-axis?
c. Find a new frame of reference, $2'$, where the photon will be ejected in the opposite direction to that of particle 2.
What is the velocity of S' compared to the lab's frame of reference?

4) Photon Collides with Proton to form Pion.

A proton, at rest in the lab, is hit by a photon.
What is the minimum energy required from the photon,
in order for a pion (π) and a proton to be formed?

The masses of the proton and pion, M_π and M_p, are given.

5) Velocity and Position of a Motorbike.



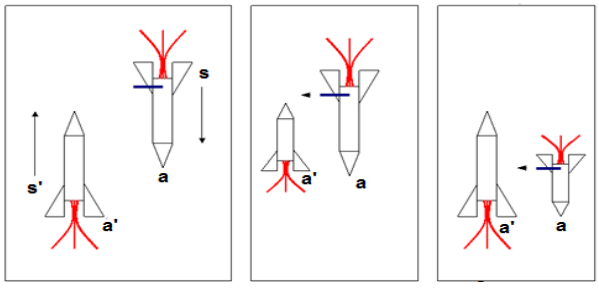
A motorbike travels at a constant velocity in a straight line.
An observer on the ground records that the motorbike travelleda distance of 540km.
Another observer is in an airplane travelling at $v = 0.5\text{c}$ in the same direction as the motorbike.
This observer records that the motorbike travels a total of 0.01 seconds.

a. What is the velocity of the motorbike in the Earth's reference frame?
b. What is the distance travelled by the motorbike according to the observer in the airplane?

6) Cannon Fired.

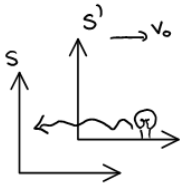
Two rockets with identical rest lengths pass fly by each other. The S rocket has a cannon at its base positioned perpendicularly to its direction of travel; see Diag. 1.
The S rocket fires its cannon at the exact moment that the point a (on its head) reaches point a' (the base of rocket S'). Due to rocket S' having a shorter length than the rest length of S, it is assumed that the shot fired by the cannon on S - will miss S'; see Diag. 2.
However, in the S' reference frame, rocket S has a shorter length than the rest length of rocket S', and so the cannon shot by S will hit S'; see Diag. 3.
Resolve the paradox.

1: **2:** **3:**



7) The Relativistic Doppler Effect.

A light source moving with a velocity v_0 emits a pulse of light.
The duration of the light emission in the rest frame is τ.
Find the relation between τ and T (duration of light emission in lab).



8) Rod Emits Light.

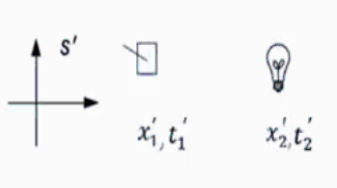
A rod of rest length l_0 is moving with a velocity v relative to Earth.

At $t=0$, the left edge of the rod is at $x=x'=0$. At this moment the rod emits light from its right edge. At $t=\tau$ the rod emits light from its left edge.

Calculate the time difference between the two events as seen from Earth.

Note: consider the time taken for the light to reach the observer.

9) Causality.



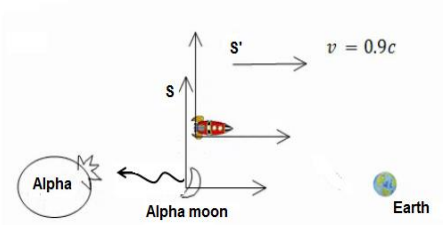
Two events took place in the S' system, which moves with a velocity v. The first event was the lifting of a switch at time t'_1 and position x'_1.

The second event was an emission of light at time t'_2 and position x'_2.

Event 1 took place before event 2 in the S' System $t'_1 < t'_2$.

Find the times at which the events took place in the S' system.

10) Explosion on Alpha.



A rocket leaves the star Alpha, and travels back to Earth. On the way, the rocket passes next to Alpha's moon and observes a strong electromagnetic pulse heading towards the star.

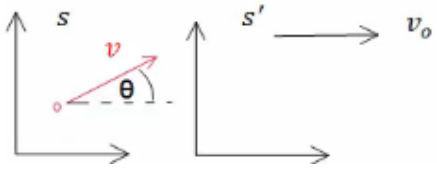
It is known that aggressive aliens live on the moon.

1.3 seconds later, the rocket sees a huge explosion go off on Alpha.

The distance between Alpha and its moon is 500 million meters, as recorded in the rocket's system. The velocity of the rocket relative to Alpha and its moon is $0.9c$.

a. What is the time span between observing the electromagnetic wave and the actual explosion, relative to Alpha and its moon?
b. What is the significance of your answer to a.?
c. Did the pulse cause the explosion, or was it the other way around?

11) Aberration.



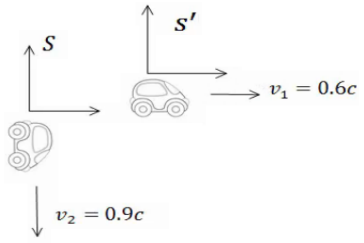
A particle moves with a velocity v at and angle $\theta°$, relative to the x-axis in the S reference frame.

$\vec{v} = (v\cos\theta, v\sin\theta, 0)$

Find the direction of its speed, relative to S'.

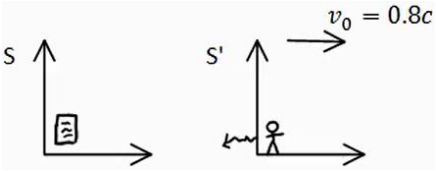
12) Relative Velocity Between Cars.

Two cars travel perpendicularly to one another. The velocity of the first car is $0.6c$ and the velocity of the second car is $0.9c$. Find the relative velocity between the two.



13) Examiner Goes on a Walk.

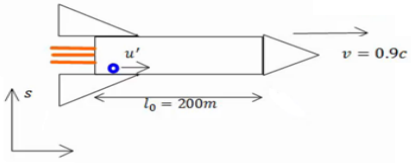
The physics exam began at $9{:}00$, at which time the examiner went on a walk at a velocity of $0.8c$. After an hour, relative to her watch, she sent a radio signal to the students saying that time is up and to put down their pencils. How long was the exam, according to the students?



14) Ball Rolls in a Rocket.

A rocket with a rest length of 200m travels at $0.9c$ relative to an inertial reference frame, S. A small ball rolls along the rocker's length at a velocity of $u' = 0.4c$ in the x direction, as recorded by an observer in the rocket.

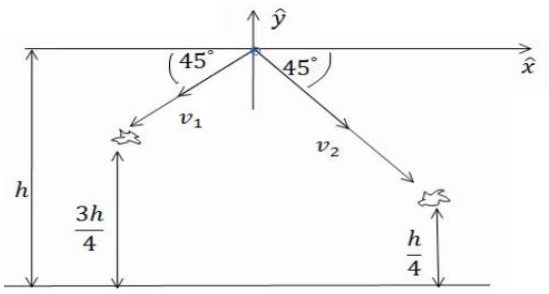
a. What is the velocity of the ball relative to an observer in S?
b. How long will it take to the ball to travel from one end of the rocket to the other, relative to S?
c. What distance has the ball traveled, relative to S?



15) Particles are Created and Decay.

Two particles are created at height h above the ground. Particle A is emitted at an angle of $225°$ to the x-axis, and particle B is emitted at $45°$ to the x-axis.

Particle A decays at time T and height $\frac{3h}{4}$ and particle B decays at $2T$ at height $\frac{h}{4}$. Ignore gravity.



a. Express the velocities of the two particles as a function of h and T.
b. Find the life span of each partied in their rest frame.
c. Find a reference frame S', which moves in the positive x direction, such that both particles decay at the same time.
d. What is the distance between the two decays in S'?

Answer Key

1) $E_n = 1.69 \cdot 10^8 MeV$

2) a. $E_{k_1} = 4mc^2$ b. $E_{k_2} = 3mc^2\left(\sqrt{\frac{11}{3}} - 1\right)$ c. $m_3 = 6.91m$

3) a. $\vec{p}_1 = \sqrt{\left(\frac{E_1}{c}\right) - m_1 c^2}\,\hat{x}$ b. $\tan\theta \frac{\frac{E_\gamma}{c}}{\sqrt{\left(\frac{E_1}{c}\right)^2 - m_1^2 c^2}}$ c. $v_0 = \sqrt{1 - \left(\frac{mc^2}{E_1}\right)^2} \cdot c$

4) $E_\gamma = \frac{1}{2mp}\left(m_\pi^2 + 2m_\pi m_p\right)c^2$

5) a. $v_1 = 5.65 \cdot 10^7 \text{ m/s}$ b. $x_2^1 = -10.32 \cdot 10^5 \text{ m}$

6) Solution in the recording.

7) Solution in the recording.

8) $\Delta t = \gamma_0\left(1+\beta\right)\left(\tau - \frac{L_0}{c}\right)$

9) Solution in the recording.

10) a. $t_3 = -3.525 \text{sec}$

b. The explosion was before the pulse reaches Alpha and before pulse was shot.

c. Nither; both events were independent.

11) $\tan\theta' = \frac{v\sin\theta}{\left(v\cos\theta - v_0\right)\gamma_0}$

12) $v_{2x}' = -0.6c, \quad v_{2y}' = -0.72c$

13) $\Delta t = 3.415 \cdot 10^4 \text{ sec}$

14) a. $v_x = 0.907c$ b. $t_1 = 39.62\mu s$ c. $x_1 = 10.78 \text{km}$

15) a. $v_A = \frac{h}{2\sqrt{2}T}, \quad v_B = \frac{3h}{4\sqrt{2}T}$ b. $\tau_A = T\sqrt{1 - \frac{h^2}{\gamma T^2 c^2}}, \quad \tau_B = 2T\sqrt{1 - \frac{9h^2}{64T^2c^2}}$

c. $v_0 = \frac{c^2 T}{h}$ d. $d'^2 = \frac{\frac{5h^4}{4} - 3c^2T^2h^2 + c^4T^4}{h^2 - c^2T^2}$