

Workbook



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Polar Coordinates

Introduction

Questions:

- 1) Given the point with polar coordinates $(r, \theta) = \left(3, \frac{\pi}{5}\right)$, find three other sets of coordinates for the same point with different angles θ in the range $-2\pi \leq \theta \leq 2\pi$.

Conversion between Polar and Cartesian Coordinates

Questions:

- 2) Convert the following sets of Cartesian coordinates to polar coordinates:
- a. $(1, \sqrt{3})$ b. $(-2, -2)$ c. $(0, -5)$ d. $(-3, 4)$
- 3) Convert the following sets of polar coordinates to Cartesian coordinates:
- a. $\left(\sqrt{8}, \frac{3\pi}{4}\right)$ b. $\left(-4, \frac{2\pi}{3}\right)$ c. $\left(0, \frac{\pi}{2}\right)$ d. $\left(6, -\frac{\pi}{3}\right)$

Conversion between Polar and Cartesian Coordinates

Questions:

- 4) Convert the following equations in x, y to polar form in r, θ :
- a. $2x - 5x^3 = 1 + xy$ b. $x^2 + y^2 = 6y$ c. $x = 3$ d. $y = -4$.

- 5) Convert the following polar equations in r, θ to Cartesian form in x, y :
- a. $r = -8\cos\theta$
 - b. $6r^3 \sin\theta = 4 - \cos\theta$
 - c. $r = 2$
 - d. $\theta = \frac{\pi}{4}$
- 6) Sketch the graph of $r = 2 + 4\sin\theta$.

Tangent Lines in Polar Coordinates

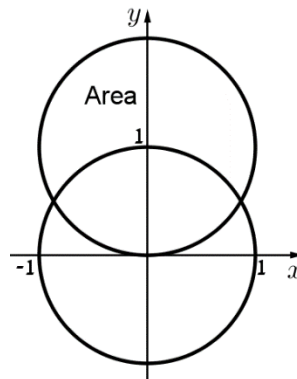
Questions:

- 7) Find the equation(s) of the tangent line(s) to $r = \sin(4\theta)\cos(\theta)$ at $\theta = \frac{\pi}{6}$.
- 8) Find the equation(s) of the tangent line(s) to $r = 1 - 2\sin\theta$ at the origin.

Area in Polar Coordinates

Questions:

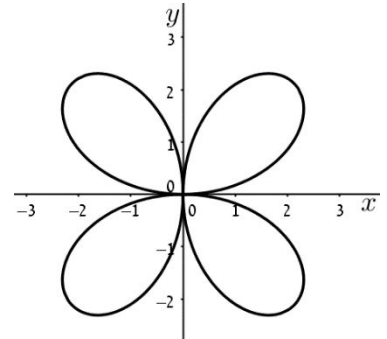
- 9) Find the area inside the graph of $r = 6 + 4\cos\theta$ and to the left of the y -axis.
- 10) Find the area between the circles $r = 1$ and $r = 2\sin\theta$ as in the figure:



Arc Length in Polar Coordinates

Questions:

- 11) Find the length of the circumference of the cardioid $r = 1 + \cos \theta$.
- 12) Find the arc length of one petal of the rose $r = 6 + 4 \cos \theta$.
 [No need to evaluate the integral]



Surface Area in Polar Coordinates

Questions:

- 13) Find the surface area obtained by revolving the curve $r = \cos \theta$, $0 \leq \theta \leq \frac{\pi}{2}$ about the x -axis
- 14) Find the surface area obtained by revolving the curve $r = 4 + 4 \sin \theta$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ about the y -axis.

Answer Key:

- 1) $\left(3, -\frac{9\pi}{5}\right)$ $\left(-3, \frac{6\pi}{5}\right)$ $\left(-3, -\frac{4\pi}{5}\right)$
- 2) a. $\left(2, \frac{\pi}{3}\right)$ b. $\left(2\sqrt{2}, \frac{5\pi}{4}\right)$ c. $\left(5, \frac{3\pi}{2}\right)$ d. (5, 2.214)
- 3) a. (-2, 2) b. $(2, -2\sqrt{3})$ c. (0, 0) d. $(3, -3\sqrt{3})$
- 4) a. $1 + r^2 \cos \theta \sin \theta$ b. $r = 6 \sin \theta$ c. $r \cos \theta = 3$ d. $r \sin \theta = -4$
- 5) a. $(x+4)^2 + y^2 = 16$ b. $4\sqrt{x^2 + y^2} - x$ c. $x^2 + y^2 = 4$ d. $y = x$
- 6) see figure:
- 7) $y = \frac{1}{3\sqrt{3}}x + \frac{1}{4}$
- 8) $y\left(\theta = \frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}x$, $y\left(\theta = \frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{3}x$
- 9) $A = 22\pi - 48$
- 10) $A = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$
- 11) $L = 8$
- 12) $L = 3 \int_0^{\frac{\pi}{2}} \sqrt{\sin^2 2\theta + 4 \cos^2 2\theta} d\theta$
- 13) $S = \pi$
- 14) $\frac{512\pi}{5}$

