

# Polynomial Functions

## Polynomial Division

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### Questions:

1) Use long division to perform the following division problems:

a. Divide  $4x^3 + 5x^2 - x - 11$  by  $4x^2 + x - 6$ .

b. Divide  $3x^4 + 10x^3 + 17x^2 + 10x + 24$  by  $3x^2 + 14x + 8$ .

2) Use long division to evaluate the following problems:

a. 
$$\frac{x^5 - 8x^4 + 15x^3 + 20x^2 - 76x + 48}{x^2 - x - 6}$$

b. 
$$\frac{x^5 - 8x^4 + 15x^3 + 20x^2 - 76x + 48}{x^3 - 2x^2 - 5x + 6}$$

3) Use long division to solve the following division problems:

a. Divide  $3x^3 + 3x^2 + 3x + 1$  by  $x + 6$ .

b. Divide  $x^3 + 12x^2 + 10x + 8$  by  $4x^2 + 3x + 1$ .

4) Use long division to solve the following division problems:

a. Divide  $x^4 + 4x^3 + 3x^2 - 4x - 3$  by  $x^2 + 10x - 9$ .

b. Divide  $3x^4 - 12x^3 + 8x^2 - 10x + 2$  by  $x^3 + x^2 + 7x + 1$ .

5) Use long division to solve the following division problems:

a. Divide  $x^3 + x^2 - 10x + 8$  by  $x^2 + 3x - 4$

b. Divide  $2x^4 - x^3 - 14x^2 + 19x - 6$  by  $2x^2 - 3x + 1$

6) Use long division to evaluate the following:

a. 
$$\frac{2x^6 - 13x^5 - 31x^3 + 31x^4 + 8x - 4 + 7x^2}{(2x+1)(x-2)}$$

b. 
$$\frac{2x^7 + 25x^4 + 120x^3 + 270x + 270x^2 + 81}{(x-1)^2}$$

7) Use long division to evaluate the following:

a. 
$$\frac{x^4 - ax^3 + x - a}{x - a}$$

b. 
$$\frac{x^3 + (1-b)x^2 - (2+b)x + 2b}{x - 2 + x^2}$$

**Answer Key:**

1) a.  $x+1+\frac{4x-5}{4x^2+x-6}$       b.  $x^2 - \frac{4}{3}x + 9\frac{2}{9} - \frac{108\frac{4}{9}x + 49\frac{7}{9}}{3x^2 + 14x + 8}$

2) a.  $x^3 - 7x^2 + 14x - 8$       b.  $x^2 - 6x + 8$

3) a.  $3x^2 - 15x + 93 - \frac{557}{x+6}$       b.  $4x^2 + 3x + 1 + \frac{21x+83}{16(4x^2+3x+1)}$

4) a.  $x^2 - 6x + 72 + \frac{-778x+645}{x^2+10x-9}$       b.  $3x - 15 + \frac{2x^2+92x+17}{x^3+x^2-7x+1}$

5) a.  $x - 2$       b.  $x^2 + x - 6$

6) a.  $x^4 - 5x^3 + 9x^2 - 7x + 2$       b.  $2x^5 + 4x^4 + 6x^3 + 33x^2 + 80x - 43$

7) a.  $x^3 + 1$       b.  $x - b$

## Polynomial Roots

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### Questions:

1) Solve the following equations:

a.  $x^3 - 2x^2 + x - 2 = 0$

b.  $x^3 + 2x^2 - 5x - 6 = 0$

c.  $x^3 - 2x^2 - 5x + 6 = 0$

d.  $x^4 - x^3 - 7x^2 + x + 6 = 0$

e.  $x^4 - x^3 - 11x^2 + 9x + 18 = 0$

f.  $x^3 - 7x^2 + 14x - 8 = 0$

g.  $x^3 + x^2 - 17x + 15 = 0$

2) Solve the following equations:

a.  $2x^3 + 3x^2 - 8x + 3 = 0$

b.  $2x^3 + x^2 - 2x - 1 = 0$

c.  $4x^3 + 5x^2 - 7x - 2 = 0$

3) Solve the following equations (knowledge about the derivative is needed):

a.  $x^3 + x^2 - 5x + 3 = 0$

b.  $x^3 - 3x - 2 = 0$

c.  $x^5 - 3x^4 - 5x^3 + 27x^2 - 32x + 12 = 0$

d.  $x^3 - 3x^2 + 3x - 1 = 0$

4) Answer the following questions:

a. Solve the equation  $x^3 - x^2 - 2x + 2 = 0$ .

b. Find all the zeroes of the polynomial  $p(x) = x^3 - x^2 - 2x + 2$ .

5) Answer the following questions:

a. Solve the equation  $x^3 - 4x^2 + 5x - 2 = 0$ .

b. Find all the zeroes of the polynomial  $f(x) = x^3 - 4x^2 + 5x - 2$ .

6) Solve the equations:

a.  $x^3 - x^2 - x - 2 = 0$

b.  $x^4 - x^3 - 5x^2 + 3x + 6 = 0$

7) For each polynomial, list all its zeros and give their multiplicities:

a.  $p(x) = x^4 + 3x^3 + 3x^2 - x - 6$

b.  $q(x) = x^5 - 5x^4 + 9x^3 - 9x^2 + 8x - 4$

**Answer Key:**

- 1) a.  $x = 2$   $z = 2, \pm i$       b.  $x = -1, 2, -3$       c.  $x = 1, -2, 3$       d.  $x = \pm 1, -2, 3$   
e.  $x = -1, 2, \pm 3$       f.  $x = 1, 2, 4$       g.  $x = 1, 3, -5$
- 2) a.  $x = 1, 3, -\frac{1}{2}$       b.  $x = -1, -\frac{1}{2}$       c.  $x = -\frac{1}{4}, -2$
- 3) a.  $x = 1, -3$       b.  $x = -1, 2$       c.  $x = 1, 2, -3$       d.  $x = 1$
- 4) a.  $x = 1, \pm\sqrt{2}$       b.  $x = 1, \pm\sqrt{2}$
- 5) a.  $x = 1, 2$       b.  $x = 1, 2$
- 6) a.  $x = 2$  ,  $z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$       b.  $x = -1, 2, \pm\sqrt{3}$
- 7) a.  $x = 1, -2$  ,  $z = -1 \pm \sqrt{2}i$       b.  $x = 1, 2$  ,  $z = \pm i$

## Partial Fractions

### Questions:

Determine the partial fractions decomposition of the following expressions:

1)  $\frac{1}{x^2 - 4}$

2)  $\frac{5 - x}{x^2 + 5x}$

3)  $\frac{x}{x^2 + 5x + 6}$

4)  $\frac{8x - 1}{2x^2 - 3x - 2}$

5)  $\frac{x + 4}{(x - 1)^2}$

6)  $\frac{6 - x}{x^2 + 8x + 16}$

7)  $\frac{x^2 + x - 1}{x^3 - x}$

8)  $\frac{10x}{x^4 - 13x^2 + 36}$

9)  $\frac{8x}{(x - 2)^2(x + 2)}$

10)  $\frac{5 - x}{x^3 + x^2}$

11)  $\frac{9x + 36}{x^3 + 6x^2 + 9x}$

12)  $\frac{1}{(x^2 - 2x + 1)(x^2 - 4x + 4)}$

13)  $\frac{x + 4}{(x - 1)^3}$

14)  $\frac{6x^2 - 4x + 1}{(x - 1)^3}$

15)  $\frac{2x^2 + 2x + 1}{(x^2 + 1)(x + 2)}$

16)  $\frac{2x^2 + x - 1}{(x^2 + 1)(x - 3)}$

17)  $\frac{3}{(x^2 + 1)(x^2 + 4)}$

18)  $\frac{1}{x(x^2 + 1)^2}$

### Answer Key:

1)  $\frac{1}{4} \frac{1}{x - 2} - \frac{1}{4} \frac{1}{x + 2}$

2)  $\frac{1}{x} - \frac{2}{x + 5}$

3)  $\frac{3}{x + 3} - \frac{2}{x + 2}$

4)  $\frac{2}{2x + 1} + \frac{3}{x - 2}$

5)  $\frac{1}{x - 1} + \frac{5}{(x - 1)^2}$

6)  $-\frac{1}{x + 4} + \frac{10}{(x + 4)^2}$

7)  $\frac{1}{x} + \frac{1/2}{x - 1} - \frac{1/2}{x + 1}$

8)  $\frac{1}{x + 3} + \frac{1}{x - 3} - \frac{1}{x + 2} - \frac{1}{x - 2}$

9)  $\frac{1}{x - 2} + \frac{4}{(x - 2)^2} - \frac{1}{x + 2}$

10)  $-\frac{6}{x} + \frac{5}{x^2} + \frac{6}{x + 1}$

11)  $\frac{4}{x} - \frac{4}{x + 3} - \frac{3}{(x + 3)^2}$

12)  $\frac{2}{x - 1} + \frac{1}{(x - 1)^2} - \frac{2}{x - 2} + \frac{1}{(x - 2)^2}$

13)  $\frac{1}{(x - 1)^2} + \frac{5}{(x - 1)^3}$

14)  $\frac{6}{x - 1} + \frac{8}{(x - 1)^2} + \frac{3}{(x - 1)^3}$

15)  $\frac{x}{x^2 + 1} + \frac{1}{x + 2}$

16)  $\frac{1}{x^2 + 1} + \frac{2}{x - 3}$

17)  $\frac{1}{x^2 + 1} - \frac{1}{x^2 + 4}$

18)  $\frac{1}{x} - \frac{x}{x^2 + 1} - \frac{x}{(x^2 + 1)^2}$