

Workbook



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Power Series

Region \ Radius of Convergence

Questions:

1) Find the region of convergence of the series:

$$a. \sum_{n=1}^{\infty} \frac{1}{4n+1} \left(\frac{1-x}{1+x} \right)^n$$

$$b. \sum_{n=1}^{\infty} \frac{2^n}{n!(x-5)^n}$$

$$c. \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)10^n (x-4)^n}$$

$$d. \sum_{n=1}^{\infty} \frac{1}{n \ln^4 nx}$$

$$e. \sum_{n=1}^{\infty} \frac{(-1)^n}{n^x}$$

$$f. \sum_{n=1}^{\infty} \frac{1}{(x+n)(x+n-1)}$$

2) Check the uniform convergence of the series:

$$a. \sum_{n=1}^{\infty} \frac{\cos nx}{n^2} \quad -\infty < x < \infty$$

$$b. \sum_{n=1}^{\infty} \frac{x^n}{n^{\frac{3}{2}}} \quad -1 \leq x \leq 1$$

$$c. \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n+x^2}} \quad -\infty < x < \infty$$

$$d. \sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n!}} (x^n + x^{-n}) \quad \frac{1}{4} \leq x \leq 4$$

$$e. \sum_{n=2}^{\infty} \ln \left(1 + \frac{x^2}{n \ln^2 n} \right) \quad -a \leq x \leq a$$

$$f. \sum_{n=1}^{\infty} \frac{n^2 x}{1+n^7 x^2} \quad -\infty < x < \infty$$

3) Find the range and radius of convergence of the series:

a.
$$\sum_{n=0}^{\infty} \frac{x^n}{n+1}$$

b.
$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$$

c.
$$\sum_{n=1}^{\infty} \frac{5^n}{n^2} x^n$$

d.
$$\sum_{n=1}^{\infty} x^n \sin^2 \frac{1}{n}$$

e.
$$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x+2)^n}{\sqrt{n}}$$

f.
$$\sum_{n=1}^{\infty} \frac{(n+1)^5}{(2n+1)} x^{2n}$$

g.
$$\sum_{n=0}^{\infty} \frac{n!}{3^n} (x-1)^n$$

h.
$$\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{(2n-2)!} x^n$$

i.
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x+1)^n}{n \cdot 4^n}$$

j.
$$\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n (x+5)^n$$

k.
$$\sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{n^4 \cdot 100^n}$$

l.
$$\sum_{n=1}^{\infty} \frac{(x+5)^{2n+1}}{n \cdot 2^{2n+1}}$$

4) Expand the functions as a power series and find the range and radius of convergence:

a.
$$f(x) = \frac{1}{1+x}$$

b.
$$f(x) = \frac{3}{1-x^4}$$

c.
$$f(x) = \frac{1}{1+9x^2}$$

d.
$$f(x) = \frac{1}{x-5}$$

e.
$$f(x) = \frac{x}{4x+1}$$

Answer Key:

- 1) a. Converges absolutely
c. Converges for $x \geq 4\frac{1}{10}$ or $x < 3\frac{9}{10}$
e. Converges for $x > 0$
- b. Converges absolutely
d. Converges
f. Converges to $\frac{1}{x}$ for $x \neq 0, -1, -2, -5, \dots$
- 2) a. Yes b. Yes c. Yes d. Yes e. Yes f. Yes
- 3) a. 1 , $-1 \leq x < 1$ b. ∞ , $-\infty < x < \infty$ c. $\frac{1}{5}$, $-\frac{1}{5} \leq x \leq \frac{1}{5}$
d. 1 , $-1 \leq x \leq 1$ e. 1 , $-3 \leq x \leq -1$ f. 1 , $-1 \leq x \leq 1$
g. 0 , $x = 1$ h. ∞ , $-\infty \leq x \leq \infty$ i. 4 , $-5 < x \leq 3$
j. $\frac{4}{3}$, $-6\frac{1}{5} \leq x \leq -3\frac{2}{3}$ k. 10 , $-9 \leq x \leq 11$ l. 2 , $-7 \leq x \leq -3$
- 4) a. $|x| < 1$ b. $|x| < 1$ c. $-\frac{1}{3} < x < \frac{1}{3}$ d. $-5 < x < 5$
e. $-\frac{1}{4} < x < \frac{1}{4}$