proprep

Workbook

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The Derivative of a Function

Basic Derivatives of Functions

Questions

1) Find the first derivative:

a. $f(x)=4$ b. $g(x)=\frac{e+\sqrt{2}}{2}$ c. $h(x)=x^4$

d. $y=\frac{1}{x^2}$ e. $f(x)=\sqrt{x}$ f. $y=\frac{1}{\sqrt{x}}$

g. $y=4x^{10}+\frac{1}{x}$ h. $y=x+x^2$ i. $y=4x^2+8x^3-5$

j. $y(x)=\frac{2}{x}-ex$ k. $y=\sqrt{2}x^2+2ex$ l. $y(t)=\frac{4}{t}+\sqrt[3]{t}$

2) Find the first derivative:

a. $y=\left(x^2+3\right)(x-1)$ b. $y=(4x+10)\left(\sqrt{x}-1\right)$

c. $y=(x-1)(x-1)(x-2)$ d. $y=\frac{4x+10}{x^2-x}$

e. $y=\frac{x^2+4x-1}{2x-3}$ f. $y=\frac{ex+1}{ex-1}$

g. $y=(4x+10)^3$ h. $y=\left(x^2+1\right)^5$

i. $y=\sqrt{\left(x^2+x+1\right)^3}$ j. $y=(2x+1)^3(4x-5)^4$

k. $y=\frac{(2x+3)^4}{(x-5)^3}$ l. $y=\frac{1}{\sqrt[3]{4x+1}}$

Answer Key

1) a. $f'(x)=0$ b. $g'(x)=0$ c. $h'(x)=4x$

d. $y'(x)=\frac{-2}{x^3}$ e. $f'(x)=\frac{1}{2\sqrt{x}}$ f. $z'(x)=-\frac{1}{2x^{1.5}}$

g. $y'(x)=40x^9-\frac{1}{x^2}$ h. $y'(x)=1+2x$ i. $y'(x)=8x+24x^2$

j. $y'(x)=-\frac{2}{x^2}-e$ k. $y'(x)=2\sqrt{2}x+2e$ l. $y'(t)=-\frac{4}{t^2}+\frac{1}{3\sqrt[3]{t^2}}$

2) a. $y'(x)=3x^2-2x+3$ b. $y'(x)=6\sqrt{x}-4+\frac{5\sqrt{x}}{x}$

c. $y'(x)=3x^2-8x+5$ d. $y'(x)=\frac{2\left(2x^2+10x-5\right)}{x^2(x-1)^2}$

e. $y'(x)=\frac{2\left(x^2-35-5\right)}{(3-2x)^2}$ f. $y'(x)=\frac{-2e}{(ex-1)^2}$

g. $y'(x)=12(4x+10)^2$ h. $y'(x)=10x\left(x^2+1\right)^4$

i. $y'(x)=\left(3x+\frac{3}{2}\right)\sqrt{x^2+x+1}$ j. $y'(x)=14(2x+1)^2(4x-5)^3(2x-1)$

k. $y'(x)=\frac{(2x+3)^3(2x-49)}{(x-5)^4}$ l. $y'(x)=-\frac{4}{3(4x+1)^{\frac{4}{3}}}$

Derivative of Exponents and Logarithmic Functions

Questions

1) Find the first derivative:

a. $y = e^x$

b. $y = 4e^x + 2x^3$

c. $y = e^x\left(x^2 + x + 4\right)$

d. $y = \dfrac{e^x}{x^2 - x}$

e. $y = e^{4x-1} + e^{2x}$

f. $y = e^{-x}\left(x + 1\right)$

g. $y = \dfrac{e^{2x} - x}{e^{4+x}}$

h. $y = e^{\sqrt{x}}$

i. $y = \dfrac{e^{\pi x}}{x - 2}$

j. $y = \dfrac{1}{\sqrt{e^{4x} + 1}}$

k. $y = \sqrt[3]{e^{x^2+1} + 1}$

l. $y = \dfrac{e^{-x^2}}{x}$

2) Find the first derivative and second derivatives:

a. $y = \dfrac{\ln x}{x}$

b. $y = \ln^2 x + \dfrac{1}{\ln x}$

c. $y = e^{2x}\ln\left(x^2 + 4\right)$

d. $y = \dfrac{\ln x}{e^x}$

Answer Key

1) a. $y'(x)=e^x$

b. $y'(x)=4e^x+6x^2$

c. $y'(x)=e^x\left(x^2+3x+5\right)$

d. $y'(x)=\dfrac{e^x\left(x^2-3x+1\right)}{\left(x^2-x\right)^2}$

e. $y'(x)=4e^{4x-1}+2e^{2x}$

f. $y'(x)=-xe^{-x}$

g. $y'(x)=e^{-x-4}\left(x+e^{2x}-1\right)$

h. $y'(x)=\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

i. $y'(x)=\dfrac{e^{\pi x}\left(\pi x-(2\pi+1)\right)}{(x-2)^2}$

j. $y'(x)=\dfrac{-2e^{4x}}{\left(e^{4x}+1\right)\sqrt{e^{4x}+1}}$

k. $y'(x)=\dfrac{2e^{x^2}x}{3\sqrt[3]{\left(e^{x^2}+1\right)^2}}$

l. $y'(x)=\dfrac{e^{-x^2}\left(2x^2+1\right)}{x^2}$

2) a. $y'(x)=\dfrac{1-\ln(x)}{x^2}$, $y''(x)=\dfrac{2\ln(x)-3}{x^3}$

b. $y'(x)=\dfrac{2\ln^2(x)-1}{x\ln^2(x)}$, $y''(x)=\dfrac{2\ln^4(x)+2\ln^3(x)+\ln(x)+2}{x^2\ln^3(x)}$

c. $y'(x)=2e^{2x}\left(\ln\left(x^2+4\right)+\dfrac{x}{x^2+4}\right)$, $y''(x)=\dfrac{2e^{2x}\left(4x^3-x^2+2\left(x^2+4\right)^2\ln\left(x^2+4\right)+16x+4\right)}{\left(x^2+4\right)^2}$

d. $y'(x)=\dfrac{\left(\dfrac{1}{x}-\ln(x)\right)}{e^x}$, $y''(x)=\dfrac{e^x\left(-x^2\ln(x)-2x-1\right)}{x^2}$

Trigonometric Derivatives

Questions

1) Find the first derivative:

a. $f(x)=\sin x$ b. $g(x)=\sin 4x$ c. $y=\cos(0.5x)$

d. $y=\sin^2 x$ e. $f(x)=\cos^4(5x)$ f. $z(x)=\sqrt{\sin 2x}$

g. $y=\sin x\cos 3x$ h. $y=\dfrac{\sin x-1}{\cos 2x+2}$ i. $y=x^3\sin 4x$

j. $y(x)=\ln(\cos(x))$ k. $y=e^{\sin 2x}\ln x$ l. $y(t)=\sin(\cos(x))$

Answer key

1) a. $f'(x)=\cos(x)$ b. $g'(x)=4\cos(4x)$

c. $y'(x)=-0.5\sin(0.5x)$ d. $y'(x)=2\sin(x)\cos(x)$

e. $f'(x)=-500\sin^3(x)$ f. $f'(x)=\dfrac{\cos(2x)}{\sqrt{\sin(2x)}}$

g. $y'(x)=\cos(x)\cos(3x)-\sin(x)\sin(3x)-3\sin(x)$

h. $y'(x)=\dfrac{\cos(x)\cos(2x)+2\cos(x)+2\sin(x)\sin(2x)-2\sin(2x)}{(\cos 2x+2)^2}$

i. $y'(x)=3x^2\sin(4x)+4x^3\cos(4x)$ j. $y'(x)=-\tan(x)$

k. $y'(x)=e^{\sin 2x}\left(2\cos(2x)\ln(x)+\dfrac{1}{x}\right)$ l. $y'(t)=-\sin(t)\cdot\cos(\cos(t))$

Derivative of Power Functions

Questions

1) Find the first derivative:

a. $y = x^{2x}$ b. $y = x^{\ln x}$ c. $y = (\ln x)^{x}$

d. $y = \left(x^2 + 1\right)^{4x}$ e. $y = x^{x^2+1}$ f. $y = \left(\sqrt{x}\right)^{\sqrt{2x}}$

g. $y = x^{e^x}$ h. $y = \left(x^{x^x}\right)$ i. $y = (\sin x)^{x}$

j. $y = x^{\cos 2x}$ k. $y = (\tan x)^{2x}$ l. $y = (\sin x)^{\ln x}$

Answer key

1) a. $y'(x) = 2x^{2x}\left(\ln(x)+1\right)$

b. $y'(x) = \dfrac{2e^{\ln^2(x)}\ln(x)}{x}$

c. $y'(x) = \ln(x)^{x}\cdot\left(\ln\left(\ln(x)\right)+\dfrac{1}{x}\right)$

d. $y'(x) = 4\left(x^2+1\right)^{4x}\cdot\left(\ln\left(x^2+1\right)+\dfrac{2x^2}{x^2+1}\right)$

e. $y'(x) = x^{x^2+1}\left(2x\ln(x)+\dfrac{x^2+1}{x}\right)$

f. $y'(x) = \sqrt{x}^{\sqrt{2x}}\cdot\dfrac{1}{2}\left(\dfrac{\ln(x)}{\sqrt{2x}}+\dfrac{\sqrt{2x}}{x}\right)$

g. $y'(x) = x^{e^x}\cdot{}^{x}\left(\ln(x)+\dfrac{1}{x}\right)$

h. $y'(x) = x^{x^x+x}\left(\ln^2(x)+\ln(x)+x^{x-1}\right)$

i. $y'(x) = (\sin(x))^{x}\left[\ln(\sin(x))+x\cot(x)\right]$

j. $y'(x) = x^{\cos(2x)}\left[-\sin(2x)2\ln(x)+\dfrac{1}{x}\cos(2x)\right]$

k. $y'(x) = (\tan(x))^{2x}\left[2\left(\ln(\tan(x))+\dfrac{x\sin(x)}{\cos^2 x}\right)\right]$

l. $y'(x) = \sin(x)^{\ln(x)}\left[\dfrac{\ln(\sin(x))}{x}+\ln(x)\cot(x)\right]$

Implicit Differentiation

Questions

1) Find y':

a. $x^2+y^2=1$

b. $x^2y^3=x+y^2$

c. $\dfrac{y^2+x}{y^3-4x}=1$

d. $\sqrt{y}+\sqrt{x}=1$

e. $(y+2)^3=xy$

f. $e^x+e^y=1$

g. $\ln x+\ln y=y$

h. $(\ln y)^2+y\ln x=1$

i. $\sin y+\cos x=y^2$

j. $x\tan y=\sqrt{y}$

k. $x^y+y^x=1$

l. $y^{\ln x}+x^{\ln y}=4$

2) Find the first and the second derivatives:

a. $x^2+y^3=1$

b. $x^2y^3=x+y$

c. $\ln x+\ln y=1$

d. $\sin x+\sin y=x$

Answer Key

1) a. $y'(x) = -\frac{x}{y}$ b. $y'(x) = \frac{1-2xy^3}{3xy^2-2y}$ c. $y'(x) = \frac{-y^3-4y^2}{-y^4-8xy-3xy^2}$

d. $y'(x) = -\sqrt{\frac{y}{x}}$ e. $y'(x) = \frac{y}{3(y+2)^2-x}$ f. $y'(x) = -e^{x-y}$

g. $y'(x) = \frac{y}{x(y-1)}$ h. $y'(x) = -\frac{y^2}{z(2\ln y + y\ln x)}$ i. $y'(x) = \frac{\sin x}{\cos y - 2y}$

j. $y'(x) = \frac{\tan y}{\frac{1}{3\sqrt[3]{y^2}} - \frac{x}{\cos^2 y}}$ k. $y'(x) = -\frac{y^x(\ln y + 1)}{x^y\left(\ln x + \frac{x}{y}\right)}$ l. $y'(x) = -\frac{y\ln y}{x\ln x}$

2) a. $y'(x) = \frac{-2x}{3y^2}$, $y''(x) = \frac{2y^2+8x^2}{9y^6}$

b. $y'(x) = \frac{1-2xy^3}{3x^2y^2-1}$, $y''(x) = -2\left[y^3 + 3xy^2\frac{1-2xy^3}{3x^2y^2-1}\right]\left[3x^2y^2-1\right] - 6\left(1-2xy^3\right)\left(xy^2 + 2x^2y\frac{1-2xy^2}{3x^2y^2-1}\right)$

c. $y'(x) = -\frac{y}{x}$, $y''(x) = \frac{2y}{x^2}$

d. $y'(x) = \frac{1-\cos(x)}{\cos(y)}$, $y''(x) = \frac{(1+\sin(x))\cos(y)}{\sin(y)(\cos(x)-1)}$

Calculations Using the Definition of Derivative

Questions

1) Find the first derivative of the given function using the definition of the derivative:

a. $f(x)=x^2$ b. $g(x)=x^2+4x+1$ c. $f(x)=x^3$ d. $y=\frac{1}{x}$

e. $f(x)=\sqrt{x}$ f. $z(x)=\ln x$ g. $y=e^x$ h. $y=\sin 2x$

Answer Key

1) a. $y'(x)=2x$ b. $y'(x)=2x+4$ c. $y'(x)=3x^2$

d. $y'(x)=\frac{-1}{x^2}$ e. $y'(x)=\frac{1}{2\sqrt{x}}$ f. $y'(x)=\frac{1}{x}$

g. $y'(x)=e^x$ h. $y'(x)=2\cos(x)$

The Derivative of an Inverse of a Function

Questions

1) Prove that $(\ln x)' = \frac{1}{x}$. Use the rule of derivative of the inverse function.

2) Prove that $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$. Use the rule of derivative of the inverse function.

3) Prove that $(\arcsin x)' = \frac{1}{\sqrt{1+x^2}}$. Use the rule of derivative of the inverse function.

4) Prove that $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$. Use the rule of derivative of the inverse function.

5) Supposed that f^{-1} is the inverse function of a differentiable function f and that: $f(2)=7$, $f'(2)=\sqrt{7}$. Find $\left(f^{-1}\right)=(7)$.

Answer Key

1-4) To view the answers to the exercises, please refer to the appropriate videos on site.

5) $\frac{1}{\sqrt{7}}$

Logarithmic Differentiation

Questions

1) Given the followinf function: $y=\sqrt[4]{\frac{10x-1}{x+1}}\cdot\sqrt[10]{(2x+1)^7}$, find y'.

2) Given the followinf function: $y=\left(\sqrt[4]{10x+1}\right)^{2x}$, find y'.

3) Find the equation of the line that is tanfent to the curve: $f(x)=x^3-4x^2+2x-5$
At the point on the curve where $x=1$.
Does the line intersect the curve at any other point?

Answer Key

1) $y'=y\left(\frac{1}{4}\frac{10}{10x-1}-\frac{1}{4}\times\frac{1}{x+1}+\frac{7}{10}\frac{2}{2x+1}\right)$

2) $y'=\frac{1}{4}2^x y\left[\ln 2\ln(10x+1)+\frac{10}{10x+1}\right]$

3) $y=-3x-3$, no.