

Workbook



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Questions

- 1) Given the transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ $T[x, y, z] = [x + y, y + z, z - x]$
and given two bases of \mathbb{R}^3 : $B_1 = \{[1, 1, 0], [0, 1, 0], [0, 1, 1]\}$, $B_2 = \{[1, 0, 1], [0, 1, 1], [0, 0, 1]\}$
1. Find the matrix that represents the transformation in basis B_1 .
 2. Find the matrix that represents the transformation in basis B_2 .

Remark on Notation: we'll often use row notation for convenience.

More precisely we should write:
$$T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x + y \\ y + z \\ z - x \end{bmatrix}$$

- 2) For each of the following linear transformations, find its representation as a matrix relative to the standard basis of the appropriate \mathbb{R}^n :
- a. $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$, $T[x, y] = [x + y, y, -x]$
 - b. $T: \mathbb{R}^4 \rightarrow \mathbb{R}^2$, $T[x, y, z, t] = [4x - y - z + t, x + y + 4z + t]$

Remark on Notation: we'll often use row notation for convenience.

More precisely: a. $T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + y \\ y \\ -x \end{bmatrix}$; b. $T \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 4x - y - z + t \\ x + y + 4z + t \end{bmatrix}$.

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- 3) Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear transformation defined by $T[x, y, z] = [4x + y - z, x - y + z]$.

Compute the representation matrix of T relative to the bases

$$B_1 = \{(1, 1, 0), (0, 1, 1), (0, 0, 1)\} \text{ of } \mathbb{R}^3 \text{ and } B_2 = \{(1, 4), (1, 5)\} \text{ of } \mathbb{R}^2. \text{ I.e., find } [T]_{B_1}^{B_2}.$$

Remark on Notation: we often use row notation for convenience.

$$\text{More precisely: } T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4x + y - z \\ x - y + z \end{bmatrix}.$$

- 4) Find the matrix that represents the linear transformation

$$D: P_4[\mathbb{R}] \rightarrow P_3[\mathbb{R}], \quad D(p(x)) = p'(x) \quad [D \text{ for Derivative}],$$

relative to the standard bases of $P_4[\mathbb{R}]$ and $P_3[\mathbb{R}]$.

- 5) Find the matrix that represents the linear transformation

$$T: M_2[\mathbb{R}] \rightarrow M_2[\mathbb{R}], \quad T(A) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} A$$

$$\text{relative to the basis } B = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}.$$

- 6) Let B_1 and B_2 be two bases of the space \mathbb{R}^3 and let T be a linear operator on \mathbb{R}^3 .

$$\text{Given that: } [M]_{B_1}^{B_2} = \begin{bmatrix} -1 & -9 & 6 \\ 1 & 6 & -4 \\ 1 & 5 & -2 \end{bmatrix}, \quad [T]_{B_1} = \begin{bmatrix} -29 & -45 & 6 \\ 20 & 31 & -4 \\ 13 & 19 & -1 \end{bmatrix},$$

$$\text{compute: } [M]_{B_2}^{B_1}, \text{ and } [T]_{B_2}.$$

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Answer key

1) For the solution see the video.

$$2) \text{ a. } [T] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 0 \end{bmatrix} = [T]_{B_1}^{B_2} \quad \text{b. } [T] = \begin{bmatrix} 4 & -1 & -1 & 1 \\ 1 & 1 & 4 & 1 \end{bmatrix} = [T]_{B_1}^{B_2}$$

$$3) [T] = \begin{bmatrix} 25 & 0 & -6 \\ -20 & 0 & 5 \end{bmatrix} = [T]_{B_1}^{B_2}$$

$$4) [D]_{E_4}^{E_3} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

$$5) [T]_B = \begin{bmatrix} 1 & 2 & 2 & 0 \\ 0 & -1 & 0 & 2 \\ 3 & 5 & 4 & -2 \\ 0 & -2 & 0 & 6 \end{bmatrix}$$

$$6) [M]_{B_2}^{B_1} = \begin{bmatrix} 2 & 3 & 0 \\ -0.5 & -1 & 0.5 \\ -0.25 & -1 & 0.75 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & -0.5 & 1 \\ -0.75 & 2.75 & 0.5 \end{bmatrix} = [T]_{B_2}$$