

Workbook



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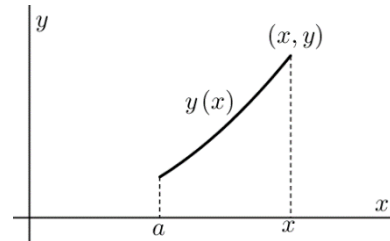
Word Problems

Word Problems

Questions

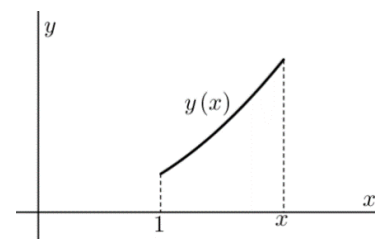
- 1) For a given curve, the slope of the tangent at each point (x, y) on the curve is equal to $-\frac{x}{y}$.
Find the equation of the curve.
- 2) Given a curve, in the first quadrant, which goes through the point $(1, 3)$,
and that the slope of its tangent at the point (x, y) equals $-\left(1 + \frac{y}{x}\right)$.
Find the equation of the curve.
- 3) Find the equation of the curve, whose normal at each point passes through the origin.
- 4) Find the equation of the curve, the slope of whose tangent at each point is equal to half
the slope of the segment from the origin to the point.
- 5) Find the equation of the curve which passes through the point $(1, 2)$ and, for each point
 (x, y) on it, the slope of the normal is $\frac{2xy}{y^2 - x^2}$.
- 6) Given a curve in the first quadrant, passing through the point $(2, 4)$, and that for each point
 $A(x, y)$ on it, the difference between the slope of the tangent to the curve at A and between
the slope of the line connecting A with the origin, is equal to the y -coordinate of A.
Find the equation of the curve.
- 7) Find the equation of the curve that passes through the origin, and which is perpendicular
to each line connecting a point on the curve to the point $(3, 4)$.

- 8) The area S is bounded the curve $y = y(x)$, the x -axis and the lines $x = a$, $x = x$ (variable); see diagram. It is known that the area S is proportional to the arc length between the points $(a, y(a))$ and $(x, y(x))$. Find the equation of the curve.

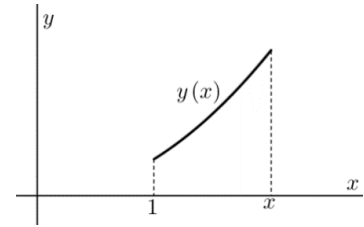


- 9) Find the family of curves orthogonal to the family $\{x + 2y = c\}$.
- 10) Find the family of curves orthogonal to the family $\{xy = c\}$.
- 11) Answer the following questions:
- Find the family of curves orthogonal to the family $\{x^2 + 2y^2 = c\}$.
 - Find the curve orthogonal to the curve $x^2 + 2y^2 = 9$ at the point $(1, 2)$ on it.
- 12) Find the family of curves orthogonal to the family $\{x^2 + y^2 = cx\}$.
- 13) Find the family of curves which form a 45° angle with the family $\{x^2 + y^2 = c\}$.
- 14) At each point on a curve the segment of the normal between the point and the x -axis is bisected by the y -axis. Find the equation of the curve.
- 15) Find the equation of the curve passing through the point $(0, 1)$, such that the triangle bounded by the y -axis, the tangent to the curve at any point $M(x, y)$ on it, and the segment OM from the origin O to M , is an isosceles triangle whose base is the segment MN , where N is the intersection of the tangent with the y -axis. Illustrate the problem with a sketch in the first quadrant.

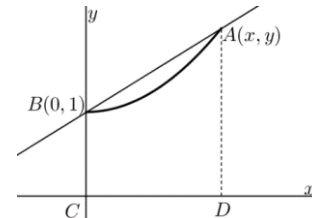
- 16) The area S is bounded the curve $y = y(x)$ the x -axis and the lines $x = 1$, $x = x$ (variable); See diagram. It is known that $y(1) = 2$. Does such a curve exist, so that the area of S equals $2y(x)$?



- 17) The area S is bounded the curve $y = y(x)$, the x -axis and the lines $x = 1$, $x = x$ (variable); See diagram.
It is known that $y(1) = 2$. Does such a curve exist such that the area of S equals $y(x) - 2$?



- 18) Given a curve passing through the point $B(0,1)$.
At each point A on the curve, the slope is equal to the area of the trapezoid $ABCD$ as shown in the diagram.
What is the equation of the curve?



- 19) A quantity $y(t)$ grows [decays] exponentially; i.e. at each instant the rate of growth [decay] is proportional to its value. Suppose that at the start time $t = 0$ the quantity is y_0 and that the constant of proportionality is k .
Find a formula for the quantity at any time t .
- 20) The population of the earth is increasing at a rate of 2% per year.
It was found to be 4 billion in 1980.
- What will the population of the earth be in 2010?
 - What was the population of the earth in 1974?
 - When will a population of 50 billion be reached?
(Assume that the population is growing exponentially; i.e. at each instant the rate of growth is proportional to its value).
- 21) The population in a certain city grows exponentially.
In a certain year there were 400 thousand residents and 4 years later there were 440 thousand.
- Find the annual growth rate (as a %).
 - After how many years (from that certain year) were there 550 thousand residents?
- 22) A man deposited money in the bank at an interest rate of 4% compounded annually.
After 5 years he had accumulated \$5,000.
- How much did he initially deposit?
 - After how many years will he have accumulated \$7,000?
- 23) The number of wild animals at a nature reserve grows exponentially.
There were 1000 animals at the initial count.
At a second count, 20 months later, there were 1400 wild animals.
How many months after the initial count will the reserve have 2000 animals?

- 24)** The radioactive isotope carbon-14 has a half-life of 5,750 years.
At any given moment its rate of decay is proportion to the amount present.
- How many grams of this isotope will survive after 1,000 years, if there were 100 grams initially?
 - After how many years will there remain just 10 grams of the initial 100 grams?
- 25)** In a certain pool there are 240 tons of fish, and the quantity of fish in it increases by 4% each week. In a second pool there are 200 tons of fish, and the quantity of fish in it increases by 10% each week.
- After how many weeks will both pools have the same quantity of fish?
 - After how many weeks will the second pool have twice the quantity of fish as the first pool?
- 26)** At time $t = 0$ a tank contains 4 kg of salt dissolved in 200 liters of water. Salt water, at a concentration of 0.2 kg per liter of water, is flowing into the tank at a rate of 25 liters per minute and, simultaneously, the mixed solution is draining out of the tank at the same rate.
- Compute the amount of salt in the tank after 8 minutes.
 - After how long the amount of salt in the tank will be twice the initial amount?
- 27)** A rowboat is initially towed at a rate of 12 km/h. At time $t = 0$ the cable is released and a man in the boat starts rowing in the direction of the motion and applies a force of 20 Newton to the boat. The mass of the boat & rower is 500 kg and the resistance (newton) is $2v$, where v is the velocity of the boat in meters/sec.
- What is the velocity of the boat after 30 seconds?
 - When the velocity of the boat will be 5 m/sec ?
 - What is the asymptotic velocity of the boat (i.e. as $t \rightarrow \infty$)?
- 28)** Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and temperature of its surroundings. A substance with a temperature of 150°C is in a container which has the surrounding temperature of the air, a constant 30°C . The substance cools in accordance with Newton's Law of Cooling and, after half an hour, its temperature drops to 70°C .
- What is its temperature after an hour?
 - After how long its temperature will be 40°C ?
- 29)** A spring of negligible weight is suspended vertically. A mass m is connected to its free end. If the mass is moving at a velocity v_0 m/sec when the spring is not extended, find the velocity v (in m/sec) as a function of the spring's extension (in m).

30) Consider the dynamical system described by the following two differential equations:

$$\frac{dN_1}{dt} = \frac{\alpha N_1 N_2}{N} - \lambda N_1, \quad \frac{dN_2}{dt} = \frac{-\alpha N_1 N_2}{N} + \lambda N_1.$$

Using the fact that $N_1 + N_2 = N$ at all times, write $\frac{dN_1}{dt}$ as a function of N_1 only.

Answer Key

1) $y^2 + x^2 = k$

3) $x^2 + y^2 = k$

5) $x^3 - 3y^2x = 11$

7) $y = 4 \pm \sqrt{25 - (x-3)^2}$

9) $y = 2x + k$

11) a. $y = ax^2$ b. $y = 2x^2$

12) $y = m(x-c)^2, y > 0$

14) $2x^2 + y^2 = k$

16) $4e^{-0.5}e^{0.5x} \Big|_1^x \neq 4e^{-0.5}e^{0.5x}$

18) $y = 2e^{x^2/4} - 1$

20) a. 7.28 b. 4.51 c. 126, year 2016.

21) a. $400e^{0.02t}$ b. 15.92 years

22) a. $y(t) = 4093.65 \cdot e^{0.04t}$ b. 13.41 years.

23) 40.77 months.

24) a. 88.69 gr. b. 19,188 years.

25) a. 3.04 weeks. b. 14.6 weeks.

26) a. 26.75_{kg} b. 0.942 min.

27) 10 m/sec.

28) a. $\left(43\frac{1}{3}\right)^\circ$ b. 1.13 hours.

29) $v = \pm \sqrt{2gx - \frac{kx^2}{m} + v_0^2}$

30) $\frac{dN_1}{dt} = aN_1 - bN_1^2$

2) $2yx + x^2 = 7$

4) $y^2 = ax$

6) $y = 2e^{-2}xe^x = 2xe^{x-2}$

8) $y = k \cosh\left(\pm \frac{1}{k}x + C\right)$

10) $y^2 - x^2 = k$

13) $\ln|x| + \frac{1}{2} \ln\left(\left(\frac{y}{x}\right)^2 + 1\right) = -\arctan\left(\frac{y}{x}\right) + c$

15) $2 = y + \sqrt{y^2 + x^2}$

17) $2e^{x-1} - 2 = 2e^{x-1} - 2$

19) $y(t) = y_0e^{kt}$