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Workbook

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Confidence Intervals for Proportions

Constructing Confidence Intervals

Our goal is to estimate a population proportion, denoted by p.
We do this by using the sample proportion and building a confidence interval around that sample proportion.

The standard error of the sample proportion is distributed normally, $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$,

and we use this to estimate the population standard error.

We create a confidence interval for p using the sample proportion, like this: $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

Conditions:

1) The sample needs to be a random sample.
2) There must be at least 10 successes and 10 failures so that the sampling distribution of the sample proportion will be approximately normal.
3) The individual trials must be independent.

Example (solution in the recording)

In order to estimate the unemployment rate in New York, 200 adults are sampled, and it is found that 24 are unemployed.

a. Construct a 95% confidence interval for the state unemployment rate.
b. What is the standard error of the sample proportion?

Questions

1) 200 apartments in Orlando were randomly sampled, 48 of which have security systems.

a. Construct a 95% confidence interval for the proportion of apartments in Orlando with security systems. Explain what it means.

b. Suppose that there are 80,000 apartments in Orlando.
Construct a 95% confidence interval for the number of apartments with security systems.

2) 300 employees were randomly selected at a large national company to be polled, and 180 of them prefer a flex work schedule.

a. Construct a 95% confidence interval for the proportion of all employees that prefer a flex work schedule.

b. How would the interval length change if the confidence level were lower?

c. How would the length of the interval change if the sample size were increased?

3) The following confidence interval for the proportion of drivers who need vision correction was derived from a random sample of 400 drivers: $0.08 < p < 0.18$.

a. How many drivers in the sample need vision correction?

b. What level of confidence was used to create this confidence interval?

4) During election season, 840 people were asked whether they planned to vote for Candidate A and 546 people answered in the affirmative.
The survey reported an error estimate of $\pm 3\%$.
What's the confidence level of this maximum error?

5) In a sample of 300 women aged 35-40 in Italy, 140 were married, 80 were divorced, 60 were single, and 20 were widows.

a. Find a 90% confidence interval for the proportion of divorced women among women aged 35-40 in Italy.

b. Find a 99% confidence interval for the probability that a 35-40 year old woman in Italy is unmarried (divorced, single or widowed).

6) A random sample is taken from a certain population.
The rate of success in the samples was 10%, and a 95% confidence interval was constructed.
The length of the interval is 8.3156%.
What was the sample size?

Answer Key

1) a. $(0.18081,\ 0.29919)$ b. $(14,465,\ 23,935)$

2) a. $(0.54456,\ 0.65544)$ b. The interval will be shorter, because Z^* will be smaller.

c. The Length of the confidence interval will be smaller, because the standard error of the sample proportion will be smaller.

3) a. 52 b. 99.7%

4) 93.17%

5) a. $(0.22467,\ 0.30866)$ b. $(0.45914,\ 0.60753)$

6) 200

Determining the Sample Size in Estimating a Proportion

Theory

This chapter discusses how to determine the sample size when estimating a proportion in a given population. The researcher determines the confidence level in advance and the maximum statistical error and uses this to plan for the sample size.

We use $\hat{p} \pm z^{*}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, and the fact that the maximum value of $\hat{p}\left(1-\hat{p}\right)$, will occur when $\hat{p} = 0.5$.

Example (solution in the recording)

We want to estimate the unemployment rate in Gainsville at a 90% level of confidence and with an estimation error of at most 4%.
What sample size is needed?

Questions

1) Every month, the White House estimates the percent of support for the president. What sample size is necessary to obtain a 95% level of certainty that the estimate will not deviate from the actual support rate by more than 3%?

2) The Department of Labor wants to know the percentage of households with a broadband internet connection.
How many households should be sampled, if we want a 90% level of certainty for a confidence interval no more than 8% long?

3) A television station wishes to estimate its prime time viewing rate.
The goal is a 95% level of certainty that the maximum difference between the estimator and the real rating will not be greater than 4%.
What sample size is necessary?

4) A public health organization is examining the chances of a vaccinated person getting influenza. It wants a 98% confidence level and for its estimation error to not exceed 3%.

a. How many vaccinated people should be sampled?
b. Suppose they sampled using the sample size from part a and found that 15% of the sample got the flu. Construct a 98% confidence interval for the probability that a vaccinated person gets the flu.
c. What is the maximum estimation error for part b? Why is it less than 3%?

Answer Key

1) 1068

2) 423

3) 601

4) a. 1503 b. $(0.12892,\ 0.17181)$ c. 0.02181