

# Workbook



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# Sampling Distributions

## Properties of Sampling Distributions

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### Basics

The average of all possible sample means (or “the mean of the sampling distribution of the mean”) equals the population mean:  $E(\bar{x}) = \mu_{\bar{x}} = \mu$ .

The variance of all possible sample means equals the population variance divided by  $n$ :

$$V(\bar{x}) = \sigma_x^2 = \frac{\sigma^2}{n}$$

The standard deviation of all possible sample means is also called the standard error:

$$\sigma(\bar{x}) = \sqrt{\frac{\sigma^2}{n}} = \frac{\sigma}{\sqrt{n}}$$

### Example (Solution in the recording)

The average monthly salary in a large company is \$9,000 with a standard deviation of \$4,000. 25 employees are randomly sampled.

- a. What is the study’s population? What is the variable?
- b. What are the study’s parameters?
- c. What is the expected value and standard deviation of the sampling distribution of the mean?

## The Central Limit Theorem

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### Theory

The Central Limit Theorem tells us that if you take a sufficiently large sample (sample size at least 30), from *any* distribution, with mean  $\mu$  and standard deviation  $\sigma$ , the sampling distribution of the sample mean will be approximately normal, with mean  $\mu$  and variance  $\frac{\sigma_x}{\sqrt{n}}$ .

**Examples** (solutions in the recording)

**1) Sampling with Large Sample**

The average weight of a chocolate bar on a production line is 100 grams with a standard deviation of 4 grams. We sample 36 random chocolate bars.

What is the probability that the average weight of the sample will be less than 102 grams?

**2) Sampling with a Normally Distributed Variable**

Birth weight is a variable that is distributed normally, with an average of 3.4kg and a standard deviation of 0.4kg.

What is the probability that a random sample of 4 babies will have an average birth weight of less than 3.5kg?

Questions

- 1) The average grade of all students in Intro Stats U was 78, with a standard deviation of 15. Two students are sampled from among those students.
- What is the expected value of the sample mean?
  - What is the standard error?

- 2) Families in Alphaville have flat screen TV's according to this distribution:

- Construct the probability distribution of  $X$ .
- Calculate the expected value, variance, and standard deviation of  $X$ .
- If we sample four families from the community (with replacement), what are the expected value, variance, and standard deviation of the sample mean?

#Families	$X = \text{\#TV's}$
500	0
2500	1
3500	2
3000	3
500	4
<b>Total: 1000</b>	

- 3) A die is rolled twice, and we take the average of the two rolls. What is the expected value and standard deviation of this average?
- 4) A baby's birth weight follows a normal probability distribution with an average weight of 3.4 kg grams and a standard deviation of 0.4 kg. What is the probability that a randomly selected baby weighs less than 3.8 kg at birth?
- 5) A baby's birth weight follows a normal probability distribution, with an average of 3.4 kg and a standard deviation of 0.4 kg. Assume that four babies are born on a given day.
- What is the probability that their average weight is more than 4 kg?
  - What is the probability that their average weight is less than 2.5 kg?
  - What is the probability that their average weight is within 50 grams of the expected average?
  - Explain, without calculations, how the answer to the preceding question would change, if the sample size were bigger than four babies.
- 6) Height of male American college students is distributed normally, with a mean of 175 cm (which is about 5'9"), and a standard deviation of 10 cm (which is about 4 inches). We focus on a random sample of 16 male students.
- What is the probability that the average height of the sample will be at least 190 cm?
  - What is the probability that the average height of the sample will be exactly 180 cm?
  - What is the probability that the average height of the sample will be within 5 cm of 175 cm?
  - What is the probability that the average height of the sample will be less than the 90<sup>th</sup> percentile for height?

- 7) The volume of wine in a bottle has a normal probability distribution with a mean of 750 ccm and a standard deviation of 10 ccm. A box contains 4 wine bottles.
- What is the probability that the average volume per bottle in the box is exactly 755 ccm ?
  - What is the probability that the average volume per bottle in the box is more than 755 ccm ?
  - What is the probability that the average volume per bottle in the box is less than 755 ccm ?
  - A box's 4 bottles are poured into a 3-liter bowl (note that 1000 ccm = 1 liter). What is the probability that the wine will overflow?
- 8) A die is rolled 50 times and each result is recorded. What is the probability that the average is at least 3.72?
- 9) The average length of a pipe produced in a factory is 70 cm with a standard deviation of 10 cm.
- 100 pipes are randomly selected. What is the probability that the average pipe length will be between 68 cm and 78 cm ?
  - Two buildings must be connected by pipes. The distance between the two buildings is 7,200 cm. What is the probability that 100 pipes will be enough for the job?
  - Suppose that we want to choose a sample size so that the probability that the sample average is less than 69 cm is at most 5%. What is the minimum sample size required? Use the Central Limit Theorem.
- 10) The following table displays the probability function of a discrete random variable:

$X$	8	6	4	2
$P(X)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

A sample of 50 observations from this probability distribution is taken. What are the chances that the sample average will be less than 5?

- 11) The weight of a chocolate bar on production line averages 100 grams. The chocolate bars are packaged in cartons, where each carton contains 36 randomly selected chocolate bars. The probability that the average weight of a chocolate bar is more than 99 grams is 0.9932.
- What is the standard deviation of the weight of an individual chocolate bar?
  - What are the chances that exactly one out of four selected cartons have an average chocolate bar weight of less than 100 grams?

**Answer Key**

- 1) a. 78                      b. 10.6
- 2) a.  $E(x) = 2.05$ ;               $\sigma(x) = 0.9734$ ;               $V(x) = 0.9475$   
b.  $E(\text{sample mean}) = 2.05$ ;       $\sigma(\text{sample mean}) = 0.4867$ ;       $V(\text{sample mean}) = 0.2369$
- 3)  $E(\text{sample mean}) = 3.5$ ;               $\sigma(\text{sample mean}) = 1.2076$
- 4) 84%
- 5) a. 13%;                      b. Approx. 0;                      c. 2%
- 6) a. 0;                      b. 0;                      c. 95%;                      d. 178.2 cm
- 7) a. 0;                      b. 15.87%;                      c. 84.13%                      d. 50%
- 8) 18.12%
- 9) a. 97.725%;              b. 2.275%;                      c.  $n = 271$
- 10) 5.69%
- 11) a. 2.4314 g;              b. 25%

