proprep™

Workbook

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Stokes Theorem

Stokes Theorem

Questions:

In each of the exercises 1-2 verify Stokes' Theorem: $\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_S (\text{curl}\mathbf{F})\cdot \mathbf{n}\, dS$.

(See remark on notation below)

1) $\mathbf{F} = 2z\mathbf{i} + 3x\mathbf{j} + 5y\mathbf{k}$; S is the part of the paraboloid $z = 4 - x^2 - y^2$ where $z \geq 0$.

2) $\mathbf{F} = (x^2 + y - 4)\mathbf{i} + (-3xy)\mathbf{j} + (2xz + z^2)\mathbf{k}$; S is the half of the sphere centered at the origin with radius 4, that lies above the xy plane.

3) Compute the integral $\oint_C x^2 dx + 4xy^3 dy + y^2 xdz$ where C is the rectangle-shaped curve from $(0,0,0)$ to $(0,3,3)$ to $(1,3,3)$ to $(1,0,0)$.

4) Compute the integral $\oint_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F} = (x + y^2)\mathbf{i} + (y + z^2)\mathbf{j} + (z + x^2)\mathbf{k}$ and C is the perimeter of the triangle with vertices $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ and in the anticlockwise direction (as seen above from the positive z-axis).

5) Compute $\iint_S (\nabla \times \mathbf{F})\cdot \mathbf{n}\, dS$ where $\mathbf{F} = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ inside the cylinder $x^2 + y^2 = 1$ and above the xy plane.

6) Compute $\iint_S (\nabla \times \mathbf{F})\cdot \mathbf{n}\, dS$ where $\mathbf{F} = (x - z)\mathbf{i} + (x^3 + yz)\mathbf{j} - 3xy^2\mathbf{k}$ and S is the part of the cone $z = 2 - \sqrt{x^2 + y^2}$ lying above the xy plane.

Remark on Notation:

Stokes' Theorem states that if $\mathbf{F}(x,y,z) = f(x,y,z)\mathbf{i} + g(x,y,z)\mathbf{j} + h(x,y,z)\mathbf{k}$ is a vector-field then we have the equality: $\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\text{curl}\mathbf{F}) \cdot \mathbf{n} dS$

There are various other formulations of the Divergence Theorem, such as:

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\text{curl}\mathbf{F}) \cdot \mathbf{n} dS$$
$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\text{Rot}\mathbf{F}) \cdot \mathbf{n} dS$$
$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS$$
$$\oint_C f dx + g dy + h dz = \iint_S \left((h_y - g_z)\mathbf{i} + (f_z - h_x)\mathbf{j} + (g_x - f_y)\mathbf{k} \right) \cdot \mathbf{n} dS$$

Answer Key:

1) The common value is 12π

2) The common value is -16π

3) -90

4) -1

5) 0

6) 12π