proprep™

Workbook

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Surface Integrals

General Calculations with Surface Integrals

Questions:

1) Compute $\iint_S x^2 yz\,dS$ where S is the part of the plane $z=1+2x+3y$ above the rectangle $R=[0,3]\times[0,2]$.

2) Compute $\iint_S x\,dS$ where S is the surface $0\le x\le 2$, $0\le z\le 2$, $y=x^2+4z$.

3) Compute $\iint_S yz\,dS$ where S is the part of the plane $z=y+3$ inside the cylinder $x^2+y^2=1$.

4) Compute $\iint_S \left(x^2 z+y^{2z}\right)dS$, where S is the hemisphere $x^2+y^2+z^2=4,\quad z\ge 0$.

5) Compute $\iint_S xyz\,dS$, where S is the part of the cone $\mathbf{r}(u,v)=u\cos v\mathbf{i}+u\sin v\mathbf{j}+3u\mathbf{k}$ satisfying $1\le u\le 2$, $0\le v\le \frac{\pi}{2}$.

6) Compute the surface area of a sphere with radius R, $\iint_S \left(x^2 z+y^2 z\right)dS$, where S is the hemisphere $x^2+y^2+z^2=4,\quad z\ge 0$.

7) The thin sheet S is the part of the paraboloid $z=x^2+y^2$ below the plane $z=1$ and it has a constant density $\delta(x,y,z)=\delta_0$. Compute the mass of the sheet.

In each of the exercises 8-12 compute $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{n}$ is the outward unit normal to S.

8) $\mathbf{F}=\left(\dfrac{x^2y}{1+y^2}+6yz^2\right)\mathbf{i}+\arctan y\,\mathbf{j}-\dfrac{2xz(1+y)+1+y^2}{1+y^2}\mathbf{k}$;
S is the open surface $z=4-x^2-y^2$, $z\geq 0$.

9) $\mathbf{F}=x\mathbf{i}-2y\mathbf{j}+3z\mathbf{k}$; S is the sphere $x^2+y^2+z^2=1$.

10) $\mathbf{F}=(2xy+z)\mathbf{i}+y^2\mathbf{j}-(x+3y)\mathbf{k}$; S is the surface of the pyramid determined by the planes: $2x+2y+z=6,\ x=0,\ y=0,\ z=0$.

11) $\mathbf{F}=5\mathbf{i}+2\mathbf{j}+3\mathbf{k}$; S is the part of the paraboloid $z=4-x^2-y^2$ where $z\geq 0$.

12) $\mathbf{F}=0\mathbf{i}-2z\mathbf{j}+(-3y-1)\mathbf{k}$; S is the hemisphere centered at the origin, with radius 4, and above the xy-plane.

Answer Key:

1) $171\sqrt{14}$ **2)** $\dfrac{33\sqrt{33}-17\sqrt{17}}{6}$ **3)** $\dfrac{\pi\sqrt{2}}{4}$

4) 16π **5)** $\dfrac{93}{\sqrt{10}}$ **6)** $4\pi R^2$

7) $\dfrac{\pi\delta_0}{6}\left(5\sqrt{5}-1\right)$ **8)** -4π **9)** $\dfrac{8\pi}{3}$

10) $37\dfrac{1}{2}$ **11)** 12π **12)** -16π