

# Workbook



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# Taylor and Maclaurin Series

## Basic Exercises with Maclaurin Series

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### Questions:

- 1) Find the Taylor series of  $f(x) = \sin 2x$  around  $x = 0$ .  
(Use the table of Maclaurin expansions in the Appendix)
- 2) Find the Taylor series of  $f(x) = x^2 e^{-4x}$  around  $x = 0$ .  
(Use the table of Maclaurin expansions in the Appendix)
- 3) Find the Taylor series of  $f(x) = \sinh x$  around  $x = 0$ .  
(Use the table of Maclaurin expansions in the Appendix)
- 4) Find the Taylor series of  $f(x) = \sin^2 x$  around  $x = 0$ .  
(Use the table of Maclaurin expansions in the Appendix)
- 5) Find the Taylor series of  $f(x) = \cos^2 x$  around  $x = 0$ .  
(Use the table of Maclaurin expansions in the Appendix)
- 6) Find the Taylor series of  $f(x) = 2^x$  around  $x = 0$ .  
(Use the table of Maclaurin expansions in the Appendix)
- 7) Find the Taylor series of  $f(x) = x \cos(4x^2)$  around  $x = 0$ .  
(Use the table of Maclaurin expansions in the Appendix)
- 8) Find the Taylor series of  $f(x) = \ln(2 - 3x + x^2)$  around  $x = 0$ .  
(Use the table of Maclaurin expansions in the Appendix)
- 9) Find the Taylor series of  $f(x) = \arcsin x$  around  $x = 0$ .  
(Use the table of Maclaurin expansions in the Appendix).

## Expansions about General Point

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### Questions:

- 10) Find the Taylor series of  $f(x) = \ln x$  expanded around  $x = 1$ .
- 11) Find the Taylor series of  $f(x) = \frac{1}{x}$  expanded around  $x = 2$ .
- 12) Find the Taylor series of  $f(x) = \sin x$  expanded around  $x = \frac{\pi}{2}$ .

## Finding Nonzero Terms in Expansions

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### Questions:

- 13) Find the first four nonzero terms of the Maclaurin series of  $f(x) = e^{-x^2} \cos x$ .
- 14) Find the first four nonzero terms of the Maclaurin series of  $f(x) = \tan x$ .
- 15) Find the first four nonzero terms of the Maclaurin series of  $f(x) = \frac{\sin x}{e^x}$ .

## Sum of Series Using Taylor and Maclaurin Expansions Questions:

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- 16) Compute the sum of the following series:

a.  $\sum_{n=0}^{\infty} \frac{1}{n!}$

b.  $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!}$

c.  $\sum_{n=0}^{\infty} \frac{1}{2^n \cdot n!}$

- 17) Compute the sum of the following series:  $\sum_{n=0}^{\infty} \frac{n+1}{n!}$ .

- 18) Compute the sum of the following series:  $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$ .

- 19) Compute the sum of the following series:  $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$ .

- 20) Compute the sum of the following series:  $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$ .

21) Compute the sum of the following series:  $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ .

22) Compute the sum of the following series:  $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}(n+1)}$ .

## Finding Limits Using Expansions

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### Questions:

23) Compute the value of the following limit:

a.  $\lim_{x \rightarrow 0} \frac{x - \arctan x}{x^3}$

b.  $\lim_{x \rightarrow 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$

c.  $\lim_{x \rightarrow 0} \frac{e^x \sin x - x(1+x)}{x^3}$

## Computations with Taylor Series

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### Questions:

24) Answer the following questions:

- Compute  $1/\sqrt[3]{e}$  with an error of less than 0.001.
- Compute  $\sin 3^\circ$  with an error of less than 0.001.
- Compute  $\arctan 0.25$  with an error of less than 0.001.

25) Answer the following questions:

- Evaluate  $\frac{1}{\sqrt{e}}$  using the first three nonzero elements of maclaurin series and estimate the error.
- Evaluate  $\cos 4^\circ$  using the first three nonzero elements of maclaurin series and estimate the error.
- Evaluate  $\ln 1.5$  using the first three nonzero elements of maclaurin series and estimate the error.

26) Answer the following questions:

- What is the maximum error in approximating  $\sin x \cong x - \frac{x^3}{3!}$  for  $|x| \leq \frac{\pi}{6}$ ?
- What is the maximum error in approximating  $\ln(1+x) \cong x$  for  $|x| < 0.01$ ?
- What is the maximum error in approximating  $\cos x \cong 1 - \frac{x^2}{2!}$  for  $|x| \leq 0.2$ ?

27) Answer the following questions:

- For which values of  $x$  is  $\sin x \cong x - \frac{x^3}{3!}$  with error less than 0.001?
- For which values of  $x$  is  $\arctan x \cong x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7}$  with error less than 0.01?

28) Answer the following questions:

- Approximate  $\int_0^{0.2} \frac{\sin x}{x} dx$  with an error less than 0.0001.
- Approximate  $\int_0^{0.1} \frac{\ln(1+x)}{x} dx$  with an error less than 0.001.
- Approximate  $\int_0^{0.5} \frac{1-\cos x}{x^2} dx$  with an error less than 0.001.

Answer Key:

- 1)  $\sum_{n=0}^{\infty} (-1)^n 2^{2n+1} \frac{x^{2n+1}}{(2n+2)!}$     2)  $\sum_{n=0}^{\infty} (-1)^n \frac{x^{n+2}}{n!}$     3)  $\sinh(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$  all  $x$
- 4)  $\sum_{n=0}^{\infty} (-1)^{n+1} 2^{2n-1} \frac{x^{2n}}{2n!}$     5)  $\frac{1}{2} + \sum_{n=0}^{\infty} (-1)^n 2^{2n-1} \frac{x^{2n}}{(2n)!}$     6)  $\sum_{n=0}^{\infty} (\ln 2)^n \frac{x^n}{n!}$
- 7)  $\sum_{n=0}^{\infty} (-1)^n 4^{2n} \frac{x^{4n+1}}{(2n)!}$  all  $x$     8)  $\ln 2 - \sum_{n=0}^{\infty} \left(1 + \frac{1}{2^{n+1}}\right) \frac{x^{n+1}}{n+1}$   $-1 \leq x < 1$
- 9)  $x + \sum_{n=0}^{\infty} (-1)^n \frac{\frac{1}{2} \left(\frac{1}{2} - 1\right) \dots \left(\frac{1}{2} - n + 1\right)}{n!} \frac{x^{2n+1}}{2n+1}$   $-1 \leq x \leq 1$
- 10)  $\sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{n+1}}{n+1}$     11)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} (x-2)^n$   $0 < x < 4$
- 12)  $\sum_{n=0}^{\infty} (-1)^n \frac{\left(x - \frac{\pi}{2}\right)^{2n}}{(2n)!}$  all  $x$     13)  $1 - \frac{3}{2}x^2 + \frac{25}{24}x^4 - \frac{331}{720}x^6$
- 14)  $x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7$   $-\frac{\pi}{2} < x < \frac{\pi}{2}$     15)  $x - x^2 + \frac{1}{3}x^3 - \frac{1}{30}x^5$
- 16) 1.  $e$     2.  $\frac{1}{e^2}$     3.  $\sqrt{e}$     17)  $2e$
- 18)  $\frac{\pi}{4}$     19)  $\sin 1^\circ$     20)  $\cos 1^\circ$     21)  $\ln 2$
- 22)  $\ln 1.5$     23) a.  $\frac{1}{3}$     b.  $\frac{1}{120}$     c.  $\frac{1}{3}$
- 24) a.  $\frac{58}{81}$     b.  $\frac{\pi}{60}$     c.  $\frac{47}{192}$
- 25) a.  $\frac{5}{8}$     b. 1    c.  $\frac{77}{192}$
- 26) a.  $\frac{\left(\frac{\pi}{6}\right)^5}{5!}$     b. 0.00005    c.  $\frac{0.2^6}{720}$
- 27) a.  $|x| < \sqrt[3]{0.12}$     b.  $|x| < \sqrt[3]{0.09}$
- 28) a.  $\frac{449}{2259}$     b.  $\frac{39}{400}$     c.  $\frac{143}{576}$