

Workbook

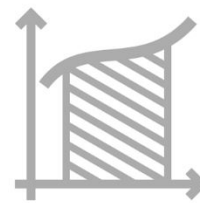


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The Shape of a Function – Curve Sketching

Extrema, Increase, Decrease

Questions:

Find all local extrema of the following functions, and give the interval where the function is increasing and decreasing:

1) $f(x) = x^2 - 2x + 5$

2) $f(x) = x^3 - 3x$

3) $f(x) = 2x^3 - 15x^2 + 24x - 1$

4) $f(x) = x^4 - 2x^3$

5) $f(x) = 3x^5 - 20x^3$

6) $f(x) = \frac{x}{x^2 + 3}$

7) $f(x) = \frac{x}{x^2 + x + 1}$

8) $f(x) = \frac{x-1}{x^2}$

9) $f(x) = \frac{2x^2}{(x+1)^2}$

10) $f(x) = \frac{x^3}{(x+1)^2}$

11) $f(x) = \left(\frac{x+1}{x-1}\right)^3$

12) $f(x) = \frac{x-1}{x^3}$

13) $f(x) = x - e^x$

14) $f(x) = e^{\frac{1}{x}}$

15) $f(x) = xe^{-2x^2}$

16) $f(x) = (x+2)e^{\frac{1}{x}}$

17) $f(x) = \frac{\ln x}{x}$

18) $f(x) = x \ln x$

19) $f(x) = \ln^2 x + 2 \ln x - 3$

20) $f(x) = \frac{1}{\sqrt{x^2 + 1}}$

21) $f(x) = \frac{x}{\sqrt{x^2 + 1}}$

22) $f(x) = x\sqrt{x^2 + 4}$

23) $f(x) = \sqrt[3]{x^2} \cdot (1-x)$

24) $f(x) = \left(\sqrt[3]{x^2} - 1\right)^2$

25) $f(x) = \sqrt[3]{x^2 - 1}$

26) $f(x) = \frac{|x-3|}{x-2}$

27) $f(x) = x - 2 \arctan x$

28) $f(x) = 8 \cos x + 2 \cos 2x - 3, \quad 0 \leq x \leq 2\pi$

29) $f(x) = e^{-x} \sin x, \quad -\pi \leq x \leq 0$

30) $f(x) = 2 \cos^2 x - \sin 2x, \quad x \in [0, \pi]$

Answer Key:

- 1) Extrema: $\text{Min}(1, 4)$, Increase: $x > 1$, Decrease: $x < 1$
- 2) Extrema: $\text{Max}(-1, 4)$, $\text{Min}(1, -2)$, Increase: $x < -1, x > 1$, Decrease: $-1 > x < 1$
- 3) Extrema: $\text{Max}(1, 10)$, $\text{Min}(4, -17)$, Increase: $x < 1, x > 4$, Decrease: $1 < x < 4$
- 4) Extrema: $\text{Min}(1.5, -1.6875)$, Increase: $x > 1.5$, Decrease: $x < 1.5$
- 5) Extrema: $\text{Max}(-2, 64)$, $\text{Min}(2, -64)$, Increase: $x < -2, x > 2$, Decrease: $-2 < x < 2$
- 6) Extrema: $\text{Max}\left(\sqrt{3}, \frac{\sqrt{3}}{6}\right)$, $\text{Min}\left(-\sqrt{3}, -\frac{\sqrt{3}}{6}\right)$, Increase: $-\sqrt{3} < x < \sqrt{3}$,
Decrease: $x < -\sqrt{3}, x > \sqrt{3}$.
- 7) Extrema: $\text{Max}\left(1, \frac{1}{3}\right)$, $\text{Min}(-1, 1)$, Increase: $-1 < x < 1$, Decrease: $x < -1, x > 1$
- 8) Extrema: $\text{Max}\left(2, \frac{1}{4}\right)$, Increase: $0 < x < 2$, Decrease: $x < 0, x > 2$
- 9) Extrema: $\text{Min}(0, 0)$, Increase: $x < -1, x > 0$, Decrease: $-1 < x < 0$
- 10) Extrema: $\text{Max}\left(0, \frac{-27}{4}\right)$, Increase: $x < -3, x > -1$ Decrease: $-3 < x < -1$
- 11) Extrema: None, Increase: None, Decrease: $x \neq 1$

- 12) Extrema: $\text{Max}\left(\frac{3}{2}, \frac{4}{27}\right)$, Increase: $x < 0$, $0 < x < 1.5$ Decrease: $x > 1.5$
- 13) Extrema: $\text{Max}(0, -1)$, Increase: $x < 0$, Decrease: $x > 0$
- 14) Extrema: None, Increase: None, Decrease: $x < 0$, $x > 0$
- 15) Extrema: $\text{Max}(0.5, 0.5e^{-0.5})$, $\text{Min}(-0.5, -0.5e^{-0.5})$, Increase: $-0.5 < x < 0.5$,
Decrease: $x < -0.5$, $x > 0.5$
- 16) Extrema: $\text{Max}\left(-1, \frac{1}{e}\right)$, $\text{Min}(2, 4\sqrt{e})$, Increase: $x < -1$, $x > 2$,
Decrease: $-1 < x < 0$, $0 < x < 2$
- 17) Extrema: $\text{Max}\left(e, \frac{1}{e}\right)$, Increase: $0 < x < e$, Decrease: $x > e$
- 18) Extrema: $\text{Min}\left(\frac{1}{e}, -\frac{1}{e}\right)$, Increase: $x > \frac{1}{e}$, Decrease: $0 < x < \frac{1}{e}$
- 19) Extrema: $\text{Min}\left(\frac{1}{e}, -4\right)$, Increase: $x > \frac{1}{e}$, Decrease: $0 < x < \frac{1}{e}$
- 20) Extrema: $\text{Max}(0, 1)$, Increase: $x < 0$, Decrease: $x > 0$
- 21) Extrema: None, Increase: All x , Decrease: None
- 22) Extrema: None, Increase: All x , Decrease: None
- 23) Extrema: $\text{Max}\left(\frac{2}{5}, \frac{3}{5}\sqrt{\left(\frac{2}{5}\right)^3}\right)$, Increase: $0 < x < \frac{2}{5}$, Decrease: $x < 0$, $x > \frac{2}{5}$
- 24) Extrema: $\text{Max}(1, 0)$, $\text{Min}(-1, 0)$, Increase: $-1 < x < 0$, $x > 1$, Decrease: $x < -1$, $0 < x < 1$
- 25) Extrema: $\text{Min}(0, -1)$, Increase: $x > 0$, Decrease: $x < 0$
- 26) Extrema: $\text{Min}(3, 0)$, Increase: $x > 3$, Decrease: $x < 2$, $2 < x < 3$

27) Extrema: $\text{Max}\left(-1, -1 + \frac{\pi}{2}\right)$, $\text{Min}\left(1, -1 + \frac{\pi}{2}\right)$, Increase: $x < -1$, $x > 1$, Decrease:
 $-1 < x < 1$

28) Extrema: $\text{Max}(0, 7)$, $\text{Min}(\pi, -9)$, $\text{Max}(2\pi, 7)$, Increase: $\pi < x < 2\pi$, Decrease: $0 \leq x < \pi$

29) Extrema: $\text{Max}(-\pi, 0)$, $\text{Max}(0, 0)$, $\text{Min}\left(\frac{-3\pi}{4}, \frac{-e^{\frac{3\pi}{4}}}{\sqrt{2}}\right)$, Increase: $-\frac{3\pi}{4} < x \leq 0$,
Decrease: $-\pi < x < \frac{-3\pi}{4}$

30) Extrema: $\text{Max}\left(\frac{7\pi}{8}, 1 + \sqrt{2}\right)$, $\text{Min}\left(\frac{3\pi}{8}, 1 - \sqrt{2}\right)$, Increase: $\frac{3\pi}{8} < x < \frac{7\pi}{8}$,
Decrease: $0 \leq x < \frac{3\pi}{8}$, $\frac{7\pi}{8} < x < \pi$

Inflection, Convex, Concave

Questions:

Find all inflection points of the following functions, and give the intervals where the function is concave up and concave down:

1) $f(x) = x^2 - 2x + 5$

3) $f(x) = 2x^3 - 15x^2 + 24x - 1$

5) $f(x) = 3x^5 - 20x^3$

7) $f(x) = \frac{x}{x^2 + x + 1}$

9) $f(x) = \frac{2x^2}{(x+1)^2}$

11) $f(x) = \left(\frac{x+1}{x-1}\right)^3$

13) $f(x) = x - e^x$

15) $f(x) = xe^{-2x^2}$

17) $f(x) = \frac{\ln x}{x}$

19) $f(x) = \ln^2 x + 2 \ln x - 3$

21) $f(x) = \frac{x}{\sqrt{x^2 + 1}}$

23) $f(x) = \sqrt[3]{x^2} \cdot (1-x)$

25) $f(x) = \sqrt[3]{x^2 - 1}$

27) $f(x) = x - 2 \arctan x$

29) $f(x) = e^{-x} \sin x, \quad -\pi \leq x \leq 0$

2) $f(x) = x^3 - 3x$

4) $f(x) = x^4 - 2x^3$

6) $f(x) = \frac{x}{x^2 + 3}$

8) $f(x) = \frac{x-1}{x^2}$

10) $f(x) = \frac{x^3}{(x+1)^2}$

12) $f(x) = \frac{x-1}{x^3}$

14) $f(x) = e^{\frac{1}{x}}$

16) $f(x) = (x+2)e^{\frac{1}{x}}$

18) $f(x) = x \ln x$

20) $f(x) = \frac{1}{\sqrt{x^2 + 1}}$

22) $f(x) = x\sqrt{x^2 + 4}$

24) $f(x) = \left(\sqrt[3]{x^2} - 1\right)^2$

26) $f(x) = \frac{|x-3|}{x-2}$

28) $f(x) = 8 \cos x + 2 \cos 2x - 3, \quad 0 \leq x \leq 2\pi$

Answer Key:

- 1) Inflection: None, Convex: All x , Concave: None
- 2) Inflection: $(0,0)$, Convex: $x > 0$, Concave: $x < 0$
- 3) Inflection: $(2.5, -3.5)$, Convex: $x > 2.5$, Concave: $x < 2.5$
- 4) Inflection: $(0,0), (1,-1)$, Convex: $x < 0, x > 1$, Concave: $0 < x < 1$
- 5) Inflection: $(-1,17), (0,0), (1,-17)$, Convex: $-1 < x < 0, x > 1$, Concave: $x < -1, 0 < x < 1$
- 6) Inflection: $\left(-3, -\frac{1}{4}\right), (0,0), \left(3, \frac{1}{4}\right)$, Convex: $-3 < x < 0, x > 3$, Concave: $x < -3, 0 < x < 3$
- 7) Inflection: $\left(\frac{1}{2}, \frac{2}{9}\right)$, Convex: $x < -1, -1 < x < \frac{1}{2}$, Concave: $x > \frac{1}{2}$
- 8) Inflection: $\left(3, \frac{2}{9}\right)$, Convex: $x > 3$, Concave: $x < 0, 0 < x < 3$
- 9) Inflection: $\left(\frac{1}{2}, \frac{2}{9}\right)$, Convex: $x < -1, -1 < x < \frac{1}{2}$, Concave: $x > \frac{1}{2}$
- 10) Inflection: $(0,0)$, Convex: $x > 0$, Concave: $x < -1, -1 < x < 0$
- 11) Inflection: $\left(-3, \frac{1}{8}\right), (-1,0)$, Convex: $-3 < x < -1, x > 1$, Concave: $x < -3, -1 < x < 1$
- 12) Inflection: $\left(2, \frac{1}{8}\right)$, Convex: $x < 0, x > 2$, Concave: $0 < x < 2$
- 13) Inflection: None, Convex: None, Concave: All x
- 14) Inflection: $\left(-\frac{1}{2}, \frac{1}{e^2}\right)$, Convex: $-\frac{1}{2} < x < 0, x > 0$, Concave: $x < -\frac{1}{2}$
- 15) Inflection: $\left(\frac{-\sqrt{3}}{2}, \frac{-\sqrt{2}}{2}e^{-\frac{3}{2}}\right), (0,0), \left(\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}e^{-\frac{3}{2}}\right)$, Convex: $-\frac{\sqrt{3}}{2} < x < 0, x > \frac{\sqrt{3}}{2}$,
 Concave: $x < -\frac{\sqrt{3}}{2}, 0 < x < \frac{\sqrt{3}}{2}$

16) Inflection: $\left(\frac{-2}{5}, \frac{8}{5}e^{-\frac{5}{2}}\right)$, Convex: $-\frac{2}{5} < x < 0$, $x > 0$, Concave: $x < -\frac{2}{5}$

17) Inflection: $\left(e^{\frac{3}{2}}, \frac{3}{2}e^{-\frac{3}{2}}\right)$, Convex: $x > e^{\frac{3}{2}}$, Concave: $0 < x < e^{\frac{3}{2}}$

18) Inflection: None, Convex: $x > 0$, Concave: None

19) Inflection: $(1, -3)$, Convex: $0 < x < 1$, Concave: $x > 1$

20) Inflection: $\left(-\sqrt{0.5}, \frac{1}{\sqrt{1.5}}\right), \left(\sqrt{0.5}, \frac{1}{\sqrt{1.5}}\right)$, Convex: $x < -\sqrt{0.5}$, $x > \sqrt{0.5}$,
Concave: $-\sqrt{0.5} < x < \sqrt{0.5}$

21) Inflection: $(0, 0)$, Convex: $x < 0$, Concave: $x > 0$

22) Inflection: $(0, 0)$, Convex: $x > 0$, Concave: $x < 0$

23) Inflection: $(-0.2, 0.8\sqrt[3]{0.04})$, Convex: $x < 0.2$, Concave: $-0.2 < x < 0$, $x > 0$

24) Inflection: None, Convex: $x < 0$, $x > 0$, Concave: None

25) Inflection: $(-1, 0), (1, 0)$, Convex: $-1 < x < 1$, Concave: $x < -1$, $x > 1$

26) Inflection: $(3, 0)$, Convex: $2 < x < 3$, Concave: $x < 2$, $x > 3$

27) Inflection: $(0, 0)$, Convex: $x > 0$, Concave: $x < 0$

28) Inflection: $\left(\frac{\pi}{3}, -5\sqrt{3} - 3\right), \left(\frac{5\pi}{3}, -5\sqrt{3} - 3\right)$, Convex: $\frac{\pi}{3} < x < \frac{5\pi}{3}$,
Concave: $0 \leq x < \frac{\pi}{3}$, $\frac{5\pi}{3} < x \leq 2\pi$

29) Inflection: $\left(\frac{-\pi}{2}, -e^{\frac{\pi}{2}}\right)$, Convex: $-\pi < x < -\frac{\pi}{2}$, Concave: $\frac{-\pi}{2} < x \leq 0$

Vertical Asymptotes

Questions:

Find the vertical asymptotes of the graph of the following functions:

1) $f(x) = x^2 - 2x + 5$

2) $f(x) = x^3 - 3x$

3) $f(x) = 2x^3 - 15x^2 + 24x - 1$

4) $f(x) = x^4 - 2x^3$

5) $f(x) = 3x^5 - 20x^3$

6) $f(x) = \frac{x}{x^2 + 3}$

7) $f(x) = \frac{x}{x^2 + x + 1}$

8) $f(x) = \frac{x-1}{x^2}$

9) $f(x) = \frac{2x^2}{(x+1)^2}$

10) $f(x) = \frac{x^3}{(x+1)^2}$

11) $f(x) = \left(\frac{x+1}{x-1}\right)^3$

12) $f(x) = \frac{x-1}{x^3}$

13) $f(x) = x - e^x$

14) $f(x) = e^{\frac{1}{x}}$

15) $f(x) = xe^{-2x^2}$

16) $f(x) = (x+2)e^{\frac{1}{x}}$

17) $f(x) = \frac{\ln x}{x}$

18) $f(x) = x \ln x$

19) $f(x) = \ln^2 x + 2 \ln x - 3$

20) $f(x) = \frac{1}{\sqrt{x^2 + 1}}$

21) $f(x) = \frac{x}{\sqrt{x^2 + 1}}$

22) $f(x) = x\sqrt{x^2 + 4}$

23) $f(x) = \sqrt[3]{x^2} \cdot (1-x)$

24) $f(x) = \left(\sqrt[3]{x^2} - 1\right)^2$

25) $f(x) = \sqrt[3]{x^2 - 1}$

26) $f(x) = \frac{|x-3|}{x-2}$

27) $f(x) = x - 2 \arctan x$

28) $f(x) = \frac{1}{1 + e^x}$

29) $f(x) = \frac{x^2 - 1}{x + 1}$

31) $f(x) = \ln(x - 1)$

33) $f(x) = \frac{1}{\sqrt{x - 4}}$

30) $f(x) = \frac{x^2 + x - 2}{x^2 - 3x + 2}$

32) $f(x) = \ln(4 - x)$

34) $f(x) = \sqrt{x - 1}$

Answer Key:

1) None

2) None

3) None

4) None

5) None

6) None

7) None

8) $x = 0$

9) $x = -1$

10) $x = -1$

11) $x = 1$

12) $x = 0$

13) None

14) $x = 0$

15) None

16) $x = 0$

17) $x = 0$

18) None

19) $x = 0$

20) None

21) None

22) None

23) None

24) None

25) None

26) $x = 2$

27) None

28) None

29) None

30) $x = 2$

31) $x = 1$

32) $x = 1$

33) $x = 4$

34) None

Horizontal Asymptotes

Questions:

Find the horizontal asymptotes of the graph of the following functions:

1) $f(x) = x^2 - 2x + 5$

2) $f(x) = x^3 - 3x$

3) $f(x) = 2x^3 - 15x^2 + 24x - 1$

4) $f(x) = x^4 - 2x^3$

5) $f(x) = 3x^5 - 20x^3$

6) $f(x) = \frac{x}{x^2 + 3}$

7) $f(x) = \frac{x}{x^2 + x + 1}$

8) $f(x) = \frac{x-1}{x^2}$

9) $f(x) = \frac{2x^2}{(x+1)^2}$

10) $f(x) = \frac{x^3}{(x+1)^2}$

11) $f(x) = \left(\frac{x+1}{x-1}\right)^3$

12) $f(x) = \frac{x-1}{x^3}$

13) $f(x) = x - e^x$

14) $f(x) = e^{\frac{1}{x}}$

15) $f(x) = xe^{-2x^2}$

16) $f(x) = (x+2)e^{\frac{1}{x}}$

17) $f(x) = \frac{\ln x}{x}$

18) $f(x) = x \ln x$

19) $f(x) = \ln^2 x + 2 \ln x - 3$

20) $f(x) = \frac{1}{\sqrt{x^2 + 1}}$

21) $f(x) = \frac{x}{\sqrt{x^2 + 1}}$

22) $f(x) = x\sqrt{x^2 + 4}$

23) $f(x) = \sqrt[3]{x^2} \cdot (1-x)$

24) $f(x) = \left(\sqrt[3]{x^2} - 1\right)^2$

25) $f(x) = \sqrt[3]{x^2 - 1}$

26) $f(x) = \frac{|x-3|}{x-2}$

27) $f(x) = x - 2 \arctan x$

28) $f(x) = \frac{1}{1 + e^{\frac{1}{x}}}$

29) $f(x) = \frac{x^2 - 1}{x + 1}$

30) $f(x) = \frac{x^2 + x - 2}{x^2 - 3x + 2}$

31) $f(x) = \ln(x-1)$

32) $f(x) = \ln(4-x)$

33) $f(x) = \frac{1}{\sqrt{x-4}}$

34) $f(x) = \sqrt{x-1}$

35) $f(x) = \frac{1}{1 + e^x}$

36) $f(x) = \cos\left(\frac{1}{x}\right)$

37) $f(x) = x \sin\left(\frac{1}{x}\right)$

Answer Key:

- | | |
|---|---|
| 1) None | 2) None |
| 3) None | 4) None |
| 5) None | 6) $y=0$ |
| 7) $y=0$ | 8) $y=0$ |
| 9) $y=2$ | 10) None |
| 11) $y=1$ | 12) $y=0$ |
| 13) None | 14) $y=1$ |
| 15) $y=0$ | 16) None |
| 17) $y=0$ | 18) None |
| 19) None | 20) $y=0$ |
| 21) $y=1$ as $x \rightarrow -\infty$; $y=-1$ as $x \rightarrow \infty$ | 22) None |
| 23) None | 24) None |
| 25) None | 26) $y=1$ as $x \rightarrow \infty$; $y=-1$ as $x \rightarrow -\infty$ |
| 27) None | 28) $y=\frac{1}{2}$ |
| 29) None | 30) $y=1$ |
| 31) None | 32) None |
| 33) $y=0$ | 34) None |
| 35) $y=0$ as $x \rightarrow \infty$; $y=1$ as $x \rightarrow -\infty$ | 36) $y=1$ |
| 37) $y=1$ | |

Oblique Asymptotes

Questions:

Find the oblique asymptotes of the graph of the following functions:

1) $f(x) = x^2 - 2x + 5$

2) $f(x) = x^3 - 3x$

3) $f(x) = 2x^3 - 15x^2 + 24x - 1$

4) $f(x) = x^4 - 2x^3$

5) $f(x) = 3x^5 - 20x^3$

6) $f(x) = \frac{x}{x^2 + 3}$

7) $f(x) = \frac{x}{x^2 + x + 1}$

8) $f(x) = \frac{x-1}{x^2}$

9) $f(x) = \frac{2x^2}{(x+1)^2}$

10) $f(x) = \frac{x^3}{(x+1)^2}$

11) $f(x) = \left(\frac{x+1}{x-1}\right)^3$

12) $f(x) = \frac{x-1}{x^3}$

13) $f(x) = x - e^x$

14) $f(x) = e^{\frac{1}{x}}$

15) $f(x) = xe^{-2x^2}$

16) $f(x) = (x+2)e^{\frac{1}{x}}$

17) $f(x) = \frac{\ln x}{x}$

18) $f(x) = x \ln x$

19) $f(x) = \ln^2 x + 2 \ln x - 3$

20) $f(x) = \frac{1}{\sqrt{x^2 + 1}}$

21) $f(x) = \frac{x}{\sqrt{x^2 + 1}}$

22) $f(x) = x\sqrt{x^2 + 4}$

23) $f(x) = \sqrt[3]{x^2} \cdot (1-x)$

24) $f(x) = \left(\sqrt[3]{x^2} - 1\right)^2$

25) $f(x) = \sqrt[3]{x^2 - 1}$

26) $f(x) = \frac{|x-3|}{x-2}$

27) $f(x) = x - 2 \arctan x$

28) $f(x) = \frac{x^3 - 1}{x^2 + x + 1}$

29) $f(x) = e^{-x}$

30) $f(x) = \frac{x^2 + x - 2}{x^2 - 3x + 2}$

31) $f(x) = \ln(x-1)$

32) $f(x) = \ln(4-x)$

33) $f(x) = \frac{1}{\sqrt{x-4}}$

34) $f(x) = \sqrt{x-1}$

35) $f(x) = \sqrt{x^2 + 1}$

36) $f(x) = \sqrt{4x^2 + 1} + 2x$

Answer Key:

- 1) None 2) None 3) None 4) None 5) None
6) $y=0$ 7) $y=0$ 8) $y=0$ 9) $y=2$ 10) $y=x-2$
11) $y=1$ 12) $y=0$ 13) $y=x$ 14) $y=1$ 15) $y=0$
16) $y=x+3$ 17) $y=0$ 18) None 19) None 20) $y=0$
21) $y=1$ 22) None 23) None 24) None 25) None
26) $y=1$ as $x \rightarrow \infty$; $y=-1$ as $x \rightarrow -\infty$
27) $y=x-\pi$ as $x \rightarrow \infty$; $y=x+\pi$ as $x \rightarrow -\infty$ 28) $y=x-1$
29) $y=0$ as $x \rightarrow \infty$; None as $x \rightarrow -\infty$ 30) $y=1$ 31) None
32) None 33) $y=0$ 34) None
35) $y=x$ as $x \rightarrow \infty$; $y=-x$ as $x \rightarrow -\infty$
36) $y=4x$ as $x \rightarrow \infty$; $y=0$ as $x \rightarrow -\infty$

Curve Plotting

Questions:

1) Sketch $f(x) = x^3 - 3x$ having the following characteristics:

a.

x		-1		1	
$f'(x)$	+	0	-	0	+
$f(x)$	Z	max]	min	Z
y		2		-2	

b.

x		0		
$f''(x)$	-	0	+	
$f(x)$	\cap	inf'	\cup	
y		0		

2) Sketch $f(x) = x^4 - 2x^3$ having the following characteristics:

a.

x		0		1.5	
$f'(x)$	-	0	-	0	+
$f(x)$]]	min	Z
y		0		$-27\sqrt{16}$	

b.

x		0		1	
$f''(x)$	+	0	-	0	+
$f(x)$	\cup	inf'	\cap	inf'	\cup
y		0		-1	

3) Sketch $f(x) = \frac{x}{x^2 + 3}$ having the following characteristics:

a.

x		$-\sqrt{3}$		$\sqrt{3}$	
$f'(x)$	-	0	+	0	-
$f(x)$]	min	Z	max]
y		$-\frac{\sqrt{3}}{6}$		$\frac{\sqrt{3}}{6}$	

b.

x		-3		0		3	
$f''(x)$	-	0	+	0	-	0	+
$f(x)$	\cap	inf'	\cup	inf'	\cap	inf'	\cup
y		$-\frac{1}{4}$		0		$\frac{1}{4}$	

4) Sketch $f(x) = \frac{x-1}{x^2}$ having the following characteristics:

a.

x		0		2	
$f'(x)$	-	N	+	0	-
$f(x)$]]	N	Z	max]]
y		N		$\frac{1}{4}$	

3. Horizontal asymptote: $y=0$

b.

x		0		3	
$f''(x)$	-	N	-	0	+
$f(x)$	\cap	N	\cap	inf'	\cup
y		N		$\frac{2}{9}$	

4. vertical asymptote: $x=0$

5) Sketch $f(x) = \frac{2x^2}{(x+1)^2}$ having the following characteristics:

a.

x		-1		0	
$f'(x)$	+	N	-	0	+
$f(x)$	Z	N]]	min	Z
y		N		0	

c. Horizontal asymptote: $y=2$

b.

x		0		3	
$f''(x)$	-	N	-	0	+
$f(x)$	\cap	N	\cap	inf'	\cup
y		N		$\frac{2}{9}$	

d. vertical asymptote: $x=-1$

6) Sketch $f(x) = \frac{x^3}{(x+1)^2}$ having the following characteristics:

a.

x		-3		-1		0	
$f'(x)$	+	0	-	N	+	0	+
$f(x)$	Z	max]]	N	Z		Z
y		-27/4		N		0	

b.

x		-1		0	
$f''(x)$	-	N	-	0	+
$f(x)$	\cap	N	\cap	inf'	\cup
y		N		0	

c. Horizontal asymptote: $y = x - 2$

d. vertical asymptote: $x = -1$

7) Sketch $f(x) = \left(\frac{x+1}{x-1}\right)^3$ having the following characteristics:

a.

x		-1		1	
$f'(x)$	-	0	-	N	-
$f(x)$]]]]	N]]
y				N	

b.

x		-3		-1		1	
$f''(x)$	-	0	+	0	-	N	+
$f(x)$	\cap	inf'	\cup	inf'	\cap	N	\cup
y		1/8		0		N	

c. Horizontal asymptote: $y = 1$

d. vertical asymptote: $x = 1$

8) Sketch $f(x) = \frac{x-1}{x^3}$ having the following characteristics:

a.

x		0		1.5	
$f'(x)$	+	N	+	0	-
$f(x)$	Z	N	Z	max]
y		N		0.148	

b.

x		0		2	
$f''(x)$	+	N	-	0	+
$f(x)$	∪	N	∩	inf	∪
y		N		1/8	

c. Horizontal asymptote: $y=0$

d. vertical asymptote: $x=0$

9) Sketch $f(x) = x - e^x$ having the following characteristics:

1.

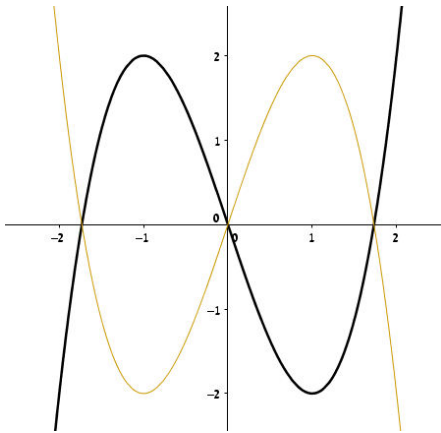
x		0	
$f'(x)$	+	0	-
$f(x)$	Z	max]
y		-1	

b. concave for all x .

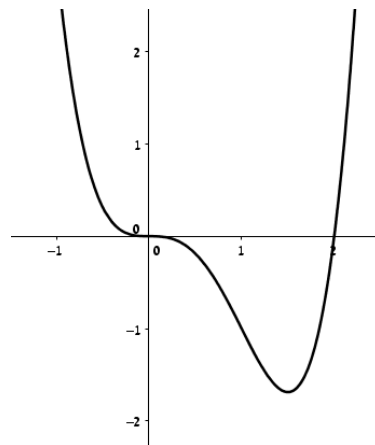
c. oblique asymptote: $y = x$ at $-\infty$

Answer Key:

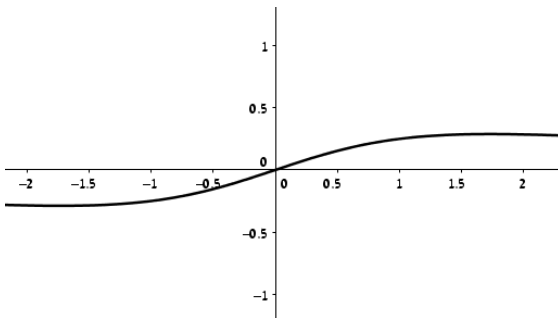
1)



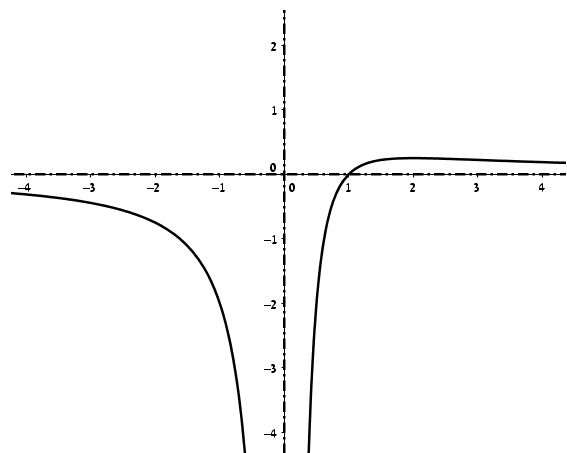
2)



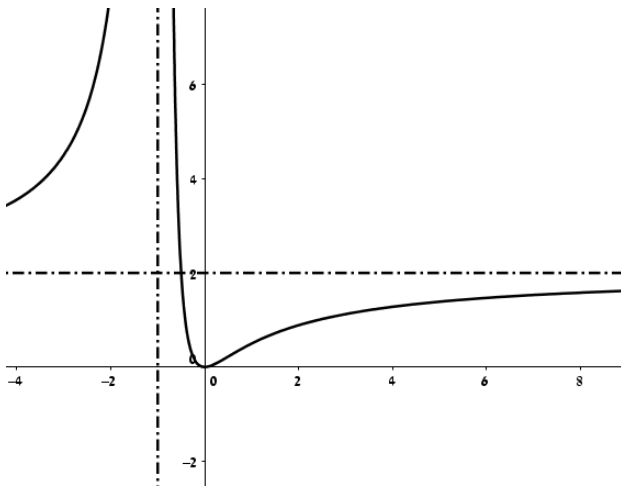
3)



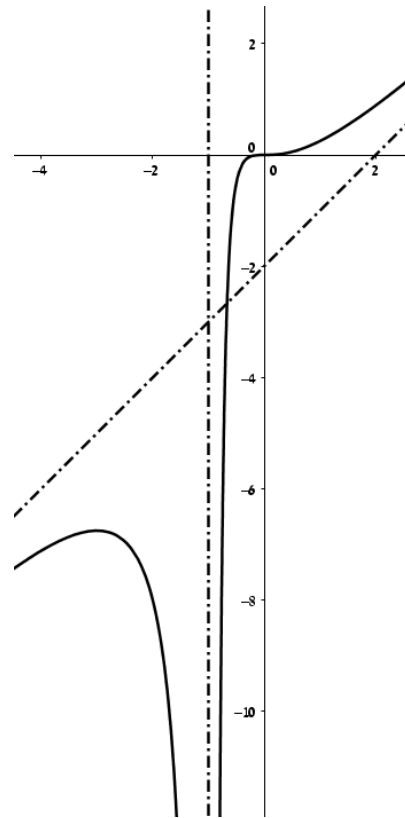
4)



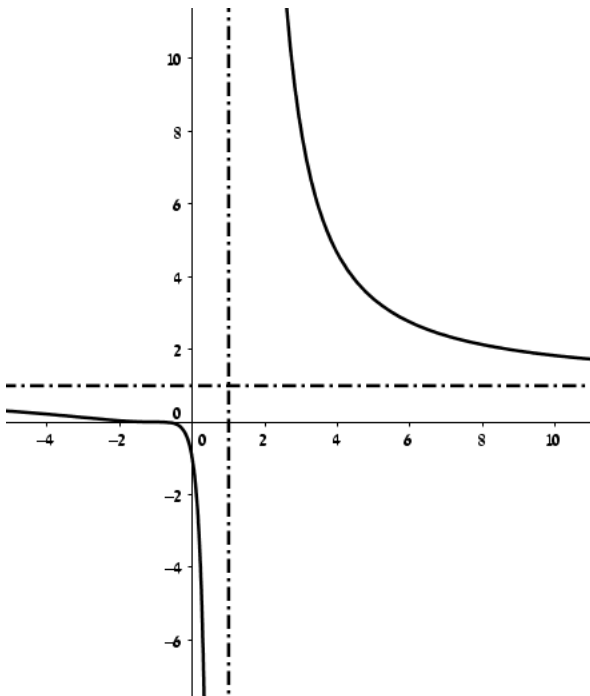
5)



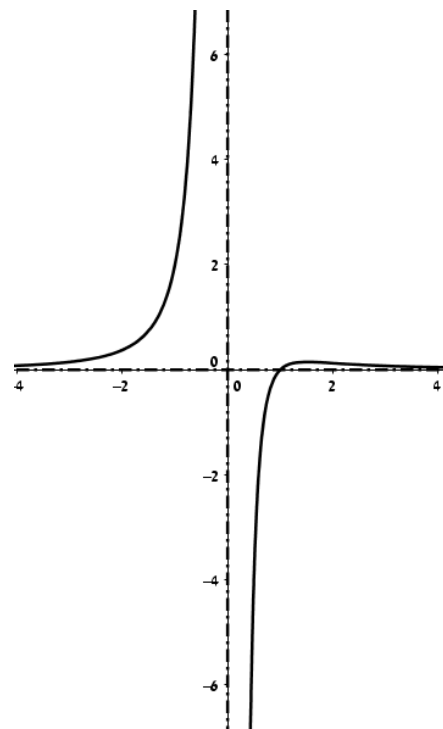
6)



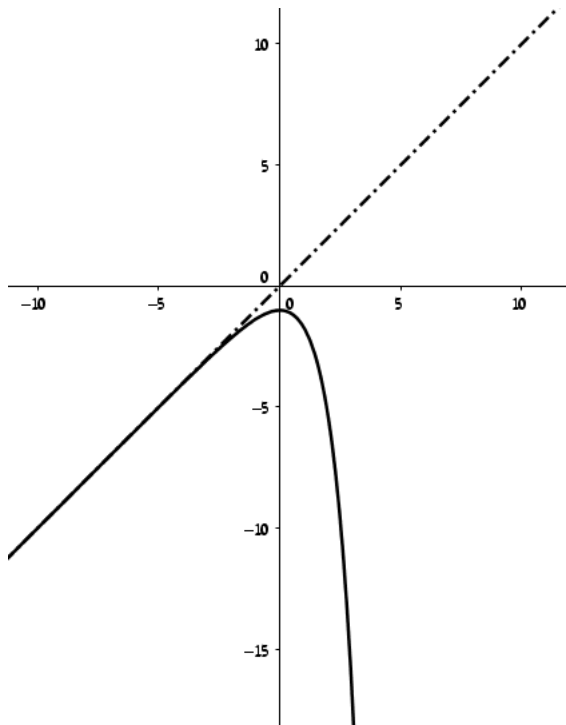
7)



8)



9)



Global Extrema

Questions:

- 1) Find the global extrema of the function $f(x) = x^3 - 3x^2 + 3x$ on the interval $[-1, 3]$.
- 2) Find the global extrema of the function $f(x) = 3x^4 + 4x^3 - 12x^2$ on the interval $[-3, 2]$.
- 3) Find the global extrema of the function $f(x) = \sqrt{-x^2 + 4x + 5}$.
- 4) Find the global extrema of the function $f(x) = x^{\frac{2}{3}}(20 - x)$ on the interval $[-1, 20]$.
- 5) Find the global extrema of the function: $f(x) = \begin{cases} 4x - 2 & x < 1 \\ (x - 2)(x - 3) & x \geq 1 \end{cases}$ on the interval $[0.5, 3.5]$.
- 6) Find the global extrema of the function $f(x) = 1 + |9 - x^2|$ on the interval $[-5, 1]$.
- 7) Find the global extrema of the function $f(x) = \frac{x^2}{x + 1}$ on the interval $[-5, -1]$.
- 8) Find the global extrema of the function $f(x) = x^3 - 9x + 1$ on the interval $[-\infty, \infty]$.
- 9) Prove that $x^3 e^{-x} \leq \frac{27}{e^3}$ for all $x \in (-\infty, \infty)$.
- 10) Prove that $x e^{-\sqrt{x}} \leq 1$ for $x > 0$.
- 11) Prove that $0 \leq x^2 e^{x-1} \leq 1$ for $x \leq 1$.
- 12) Prove that $-\frac{1}{4} \leq \frac{x}{x^2 + 4} \leq \frac{1}{4}$ for all x .
- 13) Prove that $\frac{\ln x}{x} \leq e^{-1}$ for $x > 0$.

Answer Key:

- 1) Global Min: -7 , Global Max: -1
- 2) Global Min: -32, Global Max: 32
- 3) Global Min: 0, Global Max: 3
- 4) Global Min: 0, Global Max: 48
- 5) Global Min: -0.25, Global Max: 2
- 6) Global Min: 1, Global Max: 17
- 7) Global Min: None, Global Max: -4
- 8) None
- 9-13) Refer to the videos.