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# Workbook

# Table of Contents

# Early Transcendentals – 14th Edition

# Derivatives

## Tangent Lines and the Derivative at a Point

### Questions

#### Calculations Using the Definition of Derivative

**1)** Find the first derivative of the given functions, using the definition of the derivative:

a. $f(x)=x^2$ b. $g(x)=x^2+4x+1$ c. $f(x)=x^3$

d. $y=\frac{1}{x}$ e. $f(x)=\sqrt{x}$

### Answer Key

**1)** a. $y'(x)=2x$ b. $y'(x)=2x+4$ c. $y'(x)=3x^2$

d. $y'(x)=\frac{-1}{x^2}$ e. $y'(x)=\frac{1}{2\sqrt{x}}$

# Differentiation Rules

## Questions

### Basic Derivatives of Functions

**1)** Find the first derivative:

a. $f(x)=4$

b. $g(x)=\frac{e+\sqrt{2}}{2}$

c. $h(x)=x^4$

d. $y=\frac{1}{x^2}$

e. $f(x)=\sqrt{x}$

f. $y=\frac{1}{\sqrt{x}}$

g. $y=4x^{10}+\frac{1}{x}$

h. $y=x+x^2$

i. $y=4x^2+8x^3-5$

j. $y(x)=\frac{2}{x}-ex$

k. $y=\sqrt{2}x^2+2ex$

l. $y(t)=\frac{4}{t}+\sqrt[3]{t}$

**2)** Find the first derivative:

a. $y=\left(x^2+3\right)(x-1)$

b. $y=(4x+10)\left(\sqrt{x}-1\right)$

c. $y=(x-1)(x-1)(x-2)$

d. $y=\frac{4x+10}{x^2-x}$

e. $y=\frac{x^2+4x-1}{2x-3}$

f. $y=\frac{ex+1}{ex-1}$

g. $y=(4x+10)^3$

h. $y=\left(x^2+1\right)^5$

i. $y=\sqrt{\left(x^2+x+1\right)^3}$

j. $y=(2x+1)^3(4x-5)^4$

k. $y=\frac{(2x+3)^4}{(x-5)^3}$

l. $y=\frac{1}{\sqrt[3]{4x+1}}$

**3)** Find the first derivative of the function using the definition of the derivative: $y=e^x$.

**4)** Find the first derivative:

a. $y=e^x$

b. $y=4e^x+2x^3$

c. $y=e^x\left(x^2+x+4\right)$

d. $y=\frac{e^x}{x^2-x}$

e. $y=e^{-x}(x+1)$

f. $y=\frac{e^{\pi x}}{x-2}$

## Answer Key

**1)** a. $f'(x)=0$ b. $g'(x)=0$ c. $h'(x)=4x$

d. $y'(x)=\frac{-2}{x^3}$ e. $f'(x)=\frac{1}{2\sqrt{x}}$ f. $z'(x)=-\frac{1}{2x^{1.5}}$

g. $y'(x)=40x^9-\frac{1}{x^2}$ h. $y'(x)=1+2x$ i. $y'(x)=8x+24x^2$

j. $y'(x)=-\frac{2}{x^2}-e$ k. $y'(x)=2\sqrt{2}x+2e$ l. $y'(t)=-\frac{4}{t^2}+\frac{1}{3\sqrt[3]{t^2}}$

**2)** a. $y'(x)=3x^2-2x+3$ b. $y'(x)=6\sqrt{x}-4+\frac{5\sqrt{x}}{x}$

c. $y'(x)=3x^2-8x+5$ d. $y'(x)=\frac{2\left(2x^2+10x-5\right)}{x^2\left(x-1\right)^2}$

e. $y'(x)=\frac{2\left(x^2-35-5\right)}{\left(3-2x\right)^2}$ f. $y'(x)=\frac{-2e}{\left(ex-1\right)^2}$

g. $y'(x)=12\left(4x+10\right)^2$ h. $y'(x)=10x\left(x^2+1\right)^4$

i. $y'(x)=\left(3x+\frac{3}{2}\right)\sqrt{x^2+x+1}$ j. $y'(x)=14\left(2x+1\right)^2\left(4x-5\right)^3\left(2x-1\right)$

k. $y'(x)=\frac{\left(2x+3\right)^3\left(2x-49\right)}{\left(x-5\right)^4}$ l. $y'(x)=-\frac{4}{3\left(4x+1\right)^{\frac{4}{3}}}$

**3)** $y'(x)=e^x$

**4)** a. $y'(x)=e^x$ b. $y'(x)=4e^x+6x^2$ c. $y'(x)=e^x\left(x^2+3x+5\right)$

d. $y'(x)=\frac{e^x\left(x^2-3x+1\right)}{\left(x^2-x\right)^2}$ e. $y'(x)=-xe^{-x}$ f. $y'(x)=\frac{e^{\pi x}\left(\pi x-\left(2\pi+1\right)\right)}{\left(x-2\right)^2}$

# Derivatives of Trigonometric Functions

## Questions

**1)** Find the first derivative:

a. $f(x) = \sin x$

b. $y = \sin^2 x$

**2)** Find the first derivative of the function using the definition of the derivative: $y = \sin 2x$.

## Answer Key

**1)** a. $f'(x) = 0$

b. $y'(x) = \frac{-2}{x^3}$

**2)** $y'(x) = 2\cos(x)$

# The Chain Rule

## Questions

### Derivative of Exponents and Logarithmic Functions

**1)** Find the first derivative:

a. $y=e^{4x-1}+e^{2x}$  b. $y=\dfrac{e^{2x}-x}{e^{4+x}}$  c. $y=e^{\sqrt{x}}$

d. $y=\dfrac{1}{\sqrt{e^{4x}+1}}$  e. $y=\sqrt[3]{e^{x^2+1}+1}$  f. $y=\dfrac{e^{-x^2}}{x}$

**2)** Find the first derivative:

a. $g(x)=\sin 4x$  b. $y=\cos(0.5x)$  c. $f(x)=\cos^4(5x)$

d. $z(x)=\sqrt{\sin 2x}$  e. $y=\sin x\cos 3x$  f. $y=\dfrac{\sin x-1}{\cos 2x+2}$

g. $y=x^3\sin 4x$  h. $y(t)=\sin(\cos(x))$

## Answer Key

**1)** a. $y'(x)=4e^{4x-1}+2e^{2x}$  b. $y'(x)=e^{-x-4}\left(x+e^{2x}-1\right)$  c. $y'(x)=\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

d. $y'(x)=\dfrac{-2e^{4x}}{\left(e^{4x}+1\right)\sqrt{e^{4x}+1}}$  e. $y'(x)=\dfrac{2e^{x^2}x}{3\sqrt[3]{\left(e^{x^2}+1\right)^2}}$  f. $y'(x)=\dfrac{e^{-x^2}\left(2x^2+1\right)}{x^2}$

**2)** a. $g'(x)=4\cos(4x)$  b. $y'(x)=-0.5\sin(0.5x)$  c. $f'(x)=-500\sin^3(x)$

d. $f'(x)=\dfrac{\cos(2x)}{\sqrt{\sin(2x)}}$  e. $y'(x)=\cos(x)\cos(3x)-\sin(x)\sin(3x)-3\sin(x)$

f. $y'(x)=\dfrac{\cos(x)\cos(2x)+2\cos(x)+2\sin(x)\sin(2x)-2\sin(2x)}{\left(\cos 2x+2\right)^2}$

g. $y'(x)=3x^2\sin(4x)+4x^3\cos(4x)$  h. $y'(t)=-\sin(t)\cdot\cos(\cos(t))$

## Implicit Differentiation

### Questions

**1)** Find $y'$:

a. $x^2+y^2=1$  b. $x^2y^3=x+y^2$  c. $\frac{y^2+x}{y^3-4x}=1$  d. $\sqrt{y}+\sqrt{x}=1$

e. $(y+2)^3=xy$  f. $e^x+e^y=1$  g. $x\tan y=\sqrt{y}$

**2)** Find the first and the second derivatives:

a. $x^2+y^3=1$  b. $x^2y^3=x+y$  c. $\sin x+\sin y=x$

### Answer Key

**3)** $y'(x)=-\frac{x}{y}$  b. $y'(x)=\frac{1-2xy^3}{3xy^2-2y}$  c. $y'(x)=\frac{-y^3-4y^2}{-y^4-8xy-3xy^2}$

d. $y'(x)=-\sqrt{\frac{y}{x}}$  e. $y'(x)=\frac{y}{3(y+2)^2-x}$  f. $y'(x)=-e^{x-y}$

g. $y'(x)=\frac{\tan y}{\frac{1}{3\sqrt[3]{y^2}}-\frac{x}{\cos^2 y}}$

**4)** a. $y'(x)=\frac{-2x}{3y^2}$ , $y''(x)=\frac{2y^2+8x^2}{9y^6}$

b. $y'(x)=\frac{1-2xy^3}{3x^2y^2-1}$ , $y''(x)=-2\left[y^3+3xy^2\frac{1-2xy^3}{3x^2y^2-1}\right]\left[3x^2y^2-1\right]-6\left(1-2xy^3\right)\left(xy^2+2x^2y\frac{1-2xy^2}{3x^2y^2-1}\right)$

c. $y'(x)=\frac{1-\cos(x)}{\cos(y)}$ , $y''(x)=\frac{(1+\sin(x))\cos(y)}{\sin(y)(\cos(x)-1)}$

## Derivatives of Inverse Functions and Logarithms

### Questions

**1)** Prove that $(\ln x)'=\frac{1}{x}$. Use the rule of derivative of the inverse function.

**2)** Prove that $\left(\sqrt{x}\right)'=\frac{1}{2\sqrt{x}}$. Use the rule of derivative of the inverse function.

**3)** Supposed that $f^{-1}$ is the inverse function of a differentiable function $f$, and that:
$f(2)=7$ , $f'(2)=\sqrt{7}$.
Find $\left(f^{-1}\right)=(7)$.

**4)** Find the first derivative of the function using the definition of the derivative $z(x)=\ln x$.

**5)** Find the first derivative and second derivatives:

a. $y=\frac{\ln x}{x}$ b. $y=\ln^2 x+\frac{1}{\ln x}$ c. $y=e^{2x}\ln\left(x^2+4\right)$ d. $y=\frac{\ln x}{e^x}$

**6)** Find the first derivative:

a. $y(x)=\ln\left(\cos(x)\right)$ b. $y=e^{\sin 2x}\ln x$

**7)** Find $y'$:

a. $\ln x+\ln y=y$ b. $(\ln y)^2+y\ln x=1$

c. $x^y+y^x=1$ d. $y^{\ln x}+x^{\ln y}=4$

**8)** Given the followinf function: $y=\sqrt[4]{\frac{10x-1}{x+1}}\cdot\sqrt[10]{(2x+1)^7}$ , find $y'$.

**9)** Given the followinf function: $y=\left(\sqrt[4]{10x+1}\right)^{2x}$ , find $y'$.

**10)** Find the equation of the line that is tanfent to the curve: $f(x)=x^3-4x^2+2x-5$
At the point on the curve where $x=1$.
Does the line intersect the curve at any other point?

**11)** Find the first derivative:

a. $y=x^{2x}$

b. $y=x^{\ln x}$

c. $y=(\ln x)^{x}$

d. $y=\left(x^2+1\right)^{4x}$

e. $y=x^{x^2+1}$

f. $y=\left(\sqrt{x}\right)^{\sqrt{2x}}$

g. $y=x^{e^x}$

h. $y=\left(x^{x^x}\right)$

i. $y=(\sin x)^{x}$

j. $y=x^{\cos 2x}$

k. $y=(\tan x)^{2x}$

l. $y=(\sin x)^{\ln x}$

## Answer Key

**1)** Solution in the recording.

**2)** Solution in the recording.

**3)** $\frac{1}{\sqrt{7}}$

**4)** $y'(x)=\frac{1}{x}$

**5)** a. $y'(x)=\frac{1-\ln(x)}{x^2}$ , $y''(x)=\frac{2\ln(x)-3}{x^3}$

b. $y'(x)=\frac{2\ln^2(x)-1}{x\ln^2(x)}$ , $y''(x)=\frac{2\ln^4(x)+2\ln^3(x)+\ln(x)+2}{x^2\ln^3(x)}$

c. $y'(x)=2e^{2x}\left(\ln\left(x^2+4\right)+\frac{x}{x^2+4}\right)$ , $y''(x)=\frac{2e^{2x}\left(4x^3-x^2+2\left(x^2+4\right)^2\ln\left(x^2+4\right)+16x+4\right)}{\left(x^2+4\right)^2}$

d. $y'(x)=\frac{\left(\frac{1}{x}-\ln(x)\right)}{e^x}$ , $y''(x)=\frac{e^x\left(-x^2\ln(x)-2x-1\right)}{x^2}$

**6)** a. $y'(x)=-\tan(x)$ b. $y'(x)=e^{\sin 2x}\left(2\cos(2x)\ln(x)+\frac{1}{x}\right)$

**7)** a. $y'(x)=\frac{y}{x(y-1)}$ b. $y'(x)=-\frac{y^2}{z(2\ln y+y\ln x)}$

c. $y'(x)=-\frac{y^x(\ln y+1)}{x^y\left(\ln x+\frac{x}{y}\right)}$ d. $y'(x)=-\frac{y\ln y}{x\ln x}$

**8)** $y'=y\left(\frac{1}{4}\frac{10}{10x-1}-\frac{1}{4}\times\frac{1}{x+1}+\frac{7}{10}\frac{2}{2x+1}\right)$

**9)** $y'=\frac{1}{4}2^x y\left[\ln 2\ln(10x+1)+\frac{10}{10x+1}\right]$

**10)** $y=-3x-3$, no.

**11)** a. $y'(x)=2x^{2x}(\ln(x)+1)$ b. $y'(x)=\frac{2e^{\ln^2(x)}\ln(x)}{x}$

c. $y'(x)=\ln(x)^x\cdot\left(\ln(\ln(x))+\frac{1}{x}\right)$ d. $y'(x)=4(x^2+1)^{4x}\cdot\left(\ln(x^2+1)+\frac{2x^2}{x^2+1}\right)$

e. $y'(x)=x^{x^2+1}\left(2x\ln(x)+\frac{x^2+1}{x}\right)$ f. $y'(x)=\sqrt{x}^{\sqrt{2x}}\cdot\frac{1}{2}\left(\frac{\ln(x)}{\sqrt{2x}}+\frac{\sqrt{2x}}{x}\right)$

g. $y'(x)=x^{e^x}\cdot^x\left(\ln(x)+\frac{1}{x}\right)$ h. $y'(x)=x^{x^x+x}\left(\ln^2(x)+\ln(x)+x^{x-1}\right)$

i. $y'(x)=(\sin(x))^x\left[\ln(\sin(x))+x\cot(x)\right]$

j. $y'(x)=x^{\cos(2x)}\left[-\sin(2x)2\ln(x)+\frac{1}{x}\cos(2x)\right]$

k. $y'(x)=(\tan(x))^{2x}\left[2\left(\ln(\tan(x))+\frac{x\sin(x)}{\cos^2 x}\right)\right]$

l. $y'(x)=\sin(x)^{\ln(x)}\left[\frac{\ln(\sin(x))}{x}+\ln(x)\cot(x)\right]$

# Inverse Trigonometric Functions

## Questions

**1)** Prove that $\left(\arcsin x\right)' = \frac{1}{\sqrt{1+x^2}}$. Use the rule of derivative of the inverse function.

**2)** Prove that $\left(\arcsin x\right)' = \frac{1}{\sqrt{1-x^2}}$. Use the rule of derivative of the inverse function.

## Answer Key

**1)** Solution in the recording.
**2)** Solution in the recording.

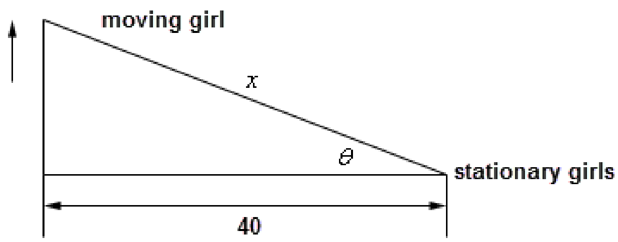
# Related Rates

## Questions

**1)** Determine the rate at which the radius of a spherical balloon is increasing when the diameter of the balloon is $40\text{cm}$, if it is known that Air is being pumped into the balloon at a rate of $10\text{cm}^3/\text{min}$.

**2)** A 20-foot ladder is resting against the wall. The bottom is initially $15 \text{ ft.}$ away fro the wall and is being pushed towards the wall at a rate of $\frac{1}{2}$ ft/sec.
How fast is the top of the ladder moving up the wall 16 seconds after we start pushing?

**3)** Two girls are 40 ft apart, one of them starts walking north at a rate so that the angle shown in the diagram below is changing at a constant rate of $0.02$ rad/min.
At what rate is the distance between the two girls changing when $\theta = 0.5$ radians?




**4)** A tank of **Gasoline** in the shape of a cone is leaking at a constant rate of $3 \text{ ft}^3/\text{hour}$.
The base radius of the tank is $4$ ft and the height of the tank is $13$ ft.

a. At what rate is the depth of the gasoline in the tank changing when the depth of the gasoline is $5$ ft?
b. At what rate is the radius of the top of the water in the tank changing when the depth of the water is $5$ ft?

**5)** A trough of water is 9 meters deep and its ends are in the shape of isosceles triangles whose 6 meters width and 3 meters height. If water is being pumped in at a constant rate of $6 \text{ m}^3/\text{sec}$.
At what rate is the height of the water changing when the water has a height of $110$ cm?

**6)** A light is on top of a $13$ ft. tall pole and a $5$ ft. tall girl is running away from the pole at a rate of $3$ ft/sec.

a. At what rate is the tip of the shadow moving away from the pole when the girl is $30$ ft. from the pole?

b. At what rate is the tip of the shadow moving away from the girl when the girl is $30$ft. from the pole?

**7)** A spot light is on the ground $24$ ft. away from a wall and a $4$ ft. tall girl is walking towards the wall at a rate of $2$ ft/sec.
How fast the height of the shadow change, when the person is $10$ ft. from the wall?
Is the shadow's height increasing or decreasing at this time?

**8)** Brad and Angelina are riding on bikes, separated by 500 meters. Brad starts riding north at a rate of $4$ m/sec and 6 minutes later Angelina starts riding south at $4$ m/sec.
At what rate the distance separating Brad and Angelina will change, 25 minutes after Brad starts riding?

## Answer Key

**1)** $\frac{1}{480\pi}$ cm/sec

**2)** $\frac{7}{2\sqrt{351}}$ ft/sec

**3)** 0.498 ft/sec

**4)** a. -0.403 ft/hour b. -0.124 ft/hour

**5)** 0.341 m/sec

**6)** a. 4.875 ft/sec b. 1.875 ft/sec

**7)** -0.98 ft/sec, The shadow is decreasing

**8)** 7.991 m/sec

## Linearization and Differentials

### Questions

**1)** Use linear approximation to approximate the value of $\sqrt[5]{33}$.

**2)** Use linear approximation to approximate the value of $\sqrt[4]{15}$.

**3)** Use linear approximation to approximate the value of $\sin 3°$.

**4)** Use linear approximation to approximate the value of $\arctan 0.25$.

**5)** Use linear approximation to approximate the value of $\frac{1}{e}$.

**6)** Determine points of non-differentiability of the following functions, and find $f'(2)$, if possible.

a. $f(x)=\begin{cases} x^2-4x & x\ge 2 \\ x^3-14 & x<2 \end{cases}$

b. $f(x)=\begin{cases} x^2-5x & x\ge 2 \\ x^3-14 & x<2 \end{cases}$

c. $f(x)=\begin{cases} x^2+8x & x\ge 2 \\ x^3+12 & x<2 \end{cases}$

**7)** Determine points of non-differentiability of the following functions, and write the formula for $f'(x)$:

a. $f(x)=\begin{cases} \ln(1+2x) & -0.5<x<0 \\ x^2+2x & x\ge 0 \end{cases}$

b. $f(x)=2+4|x-1|$

c. $f(x)=3x^2+x|x|+1$

**8)** Determine points of non-differentiability of the following functions, and find $f'(0)$ if possible:

a. $f(x)=\begin{cases} x\cdot\sin\frac{1}{x} & x>0 \\ 0 & x\le 0 \end{cases}$

b. $f(x)=\begin{cases} x^2\cdot\sin\frac{1}{x} & x>0 \\ 0 & x\le 0 \end{cases}$

**9)** For what value or values of the constants $a$ and $b$, is $f$ differentiable for all values of $x$:

a. $f(x)=\begin{cases} x^2+ax & x\ge 2 \\ x^3+b & x<2 \end{cases}$

b. $f(x)=\begin{cases} e^x & x\le 0 \\ ax+b & x>0 \end{cases}$

c. $f(x)=\begin{cases} \ln^3 x & x\ge e \\ ax+b & x<e \end{cases}$

## Answer Key

**1)** 2.0125 **2)** 1.96875 **3)** 0.05236 **4)** 0.25 **5)** 0

**6)** a. $x=2$ b. $x=2$ c. $f(x)$ is differentiable for all $x$.

**7)** a. $f'(x)=\begin{cases} \dfrac{2}{1+2x} & -0.5<x<0 \\ 2 & x=0 \\ 2x+2 & x>0 \end{cases}$ b. $x=1$

c. $f(x)$ is differentiable for all $x$, $f'(x)=\begin{cases} 8x & x>0 \\ 0 & x=0 \\ 4x & x<0 \end{cases}$

**8)** a. $f'(x)=\begin{cases} \sin\dfrac{1}{x}-\dfrac{1}{x^2}x\cos\dfrac{1}{x} & x>0 \\ \text{not exist} & x=0 \\ 0 & x<0 \end{cases}$ b. $f'(x)=\begin{cases} 2x\sin\dfrac{1}{x}-\cos\dfrac{1}{x} & x>0 \\ 0 & x=0 \\ 0 & x<0 \end{cases}$

**9)** a. $a=8$, $b=12$ b. $b=1$ c. $a=\dfrac{3}{e}$, $b=-2$