proprep™

Workbook

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Early Transcendentals – 14th Edition

Infinite Sequences and Series

Infinite Series

Questions

Infinite Geometric Series

1) For the following series determine if the series converges or diverges.
If the series converges give its value.

a. $\sum_{n=1}^{\infty}(0.44)^n$

b. $\sum_{n=0}^{\infty}\frac{4^n}{7^{n+1}}$

c. $\sum_{n=1}^{\infty}(-1)^n\frac{5^n}{4^{n+2}}$

d. $\sum_{n=0}^{\infty}(-4)\left(\frac{3}{4}\right)^{2n}$

e. $\sum_{n=1}^{\infty}\frac{4^n+(-5)^n}{7^n}$

f. $\sum_{n=4}^{\infty}2^{3n+4}3^{1-2n}$

g. $\sum_{n=3}^{\infty}\frac{(-5)^{3-n}}{8^{2-n}}$

h. $\sum_{n=2}^{\infty}2^{3n+4}5^{1-n}$

i. $2^{-2}+2^{-4}+2^{-6}+\ldots$

2) For the following series determine if the series converges or diverges.
If the series converges give its value.

a. $\sum_{n=1}^{\infty}\frac{1}{(n+1)(n+2)}$

b. $\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\ldots$

c. $\sum_{n=1}^{\infty}\frac{1}{16n^2+8n-3}$

d. $\ln\left(1+\frac{1}{1}\right)+\ln\left(1+\frac{1}{2}\right)+\ln\left(1+\frac{1}{3}\right)+\ln\left(1+\frac{1}{4}\right)+\ldots$

The Divergence Test

For each of the following series determine if the series converges or diverges:

3) $\sum_{n=1}^{\infty}\frac{4n+5}{7n+8}$

4) $\sum_{n=1}^{\infty}1$

5) $\sum_{n=1}^{\infty}\cos(\ln n)$

6) $\sum_{n=1}^{\infty}n$

7) $\sum_{n=1}^{\infty}\frac{e^n}{n^3}$

8) $\sum_{n=1}^{\infty}\frac{n^n}{n!}$

Answer Key:

1) a. Converges to $\frac{11}{14} = 0.7857$. b. Converge to $\frac{1}{3} = 0.333$. c. Diverges.

d. Converge to $-\frac{64}{7} = -9.14$ e. Converge to $\frac{11}{12}$ f. Converge to $269\frac{169}{243}$.

g. Diverges. h. Diverges. i. Converge to $\frac{1}{3} = 0.333$.

2) a. Converge to 0.5. b. Converge to $\frac{3}{4}$. c. Converge to $\frac{1}{12}$.

d. Diverges.

3) Diverges.

4) Diverges.

5) Diverges.

6) Diverges.

7) Diverges.

8) Diverges.

The Integral Test

Questions

For the following series determine if the series converges or diverges:

1) $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$

2) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+5}}$

3) $\sum_{n=1}^{\infty} ne^{-n}$

4) $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

5) $\sum_{n=2}^{\infty} \frac{1}{n \ln^3 n}$

6) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

The Harmonic Series and the P-Series

For each of the following series determine if the series converges or diverges:

7) $\sum_{n=1}^{\infty} \frac{3}{5n}$

8) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

9) $\sum_{n=1}^{\infty} \frac{1}{n^4}$

10) $\sum_{n=1}^{\infty} \frac{1}{n^e}$

11) $\sum_{n=1}^{\infty} \frac{10}{\sqrt[3]{n^4}}$

12) $\sum_{n=10}^{\infty} n^{-2/3}$

Answer Key

1) Diverges.
2) Diverges.
3) Converges.
4) Converges.
5) Converges.
6) Diverges.
7) Diverges.
8) Diverges.
9) Converges.
10) Converges.
11) Converges.
12) Diverges.

Comparison Tests

Questions

The Comparison Test \ The Limit Comparison Test

For the following series determine if the series converges or diverges:

1) $\sum_{n=1}^{\infty} \frac{5n^2+4n+8}{14n^5+10n^3+4n^2+10n+1}$

2) $\sum_{n=1}^{\infty} \frac{1\cdot 2\cdot 3\cdot \ldots \cdot n}{n\cdot n\cdot n\cdot \ldots \cdot n}$

3) $\sum_{n=1}^{\infty} \frac{2n^3+n^2+4n+1}{\sqrt{n^{10}+4n^4+n^2+n+1}}$

4) $\sum_{n=1}^{\infty} \frac{4n+5}{\sqrt{n^4+2n^3+n^2+4n+1}}$

5) $\sum_{n=1}^{\infty} \frac{2^n-2}{3^n+2n}$

6) $\sum_{n=1}^{\infty} \frac{5\sin^2 n}{n!}$

7) $\sum_{n=1}^{\infty} \left(\sqrt{n^2+1}-n\right)$

8) $\sum_{n=1}^{\infty} \left(1-\cos\frac{1}{n}\right)$

9) $\sum_{n=1}^{\infty} \frac{\sqrt{n}\ln n}{n^2+1}$

10) $\sum_{n=1}^{\infty} \frac{5n^2+4n+8}{14n^5+10n^3+4n^2+10n+1}$

11) $\sum_{n=1}^{\infty} \frac{1\cdot 2\cdot 3\cdot \ldots \cdot n}{n\cdot n\cdot n\cdot \ldots \cdot n}$

12) $\sum_{n=1}^{\infty} \frac{2n^3+n^2+4n+1}{\sqrt{n^{10}+4n^4+n^2+n+1}}$

13) $\sum_{n=1}^{\infty} \frac{4n+5}{\sqrt{n^4+2n^3+n^2+4n+1}}$

14) $\sum_{n=1}^{\infty} \frac{2^n-2}{3^n+2n}$

15) $\sum_{n=1}^{\infty} \frac{5\sin^2 n}{n!}$

16) $\sum_{n=1}^{\infty} \left(\sqrt{n^2+1}-n\right)$

17) $\sum_{n=1}^{\infty} \left(1-\cos\frac{1}{n}\right)$

18) $\sum_{n=1}^{\infty} \frac{\sqrt{n}\ln n}{n^2+1}$

Answer Key

1) Converges.	**2)** Converges.	**3)** Converges.
4) Diverges.	**5)** Converges.	**6)** Converges.
7) Diverges.	**8)** Converges.	**9)** Converges.
10) Converges.	**11)** Converges.	**12)** Converges.
13) Diverges.	**14)** Converges.	**15)** Converges.
16) Diverges.	**17)** Converges.	**18)** Converges.

Absolute Convergence; The Ratio and Root Tests

Questions

Questions:

For the following series determine if the series converges or diverges:

1) $\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$

2) $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

3) $\sum_{n=1}^{\infty} \frac{(n+3)!}{n! \cdot 3^n}$

4) $\sum_{n=1}^{\infty} \frac{(2n)!}{n!(2n)^n}$

5) $\sum_{n=1}^{\infty} \frac{3^n (1+n^2)}{n!}$

6) $\sum_{n=1}^{\infty} n^{1000} e^{-n}$

7) $\sum_{n=2}^{\infty} \frac{4^n (n^2+4n+5)}{3^n \ln n}$

8) $\sum_{n=1}^{\infty} \frac{(4n^2+5n+1)^n}{4^n n^{2n}}$

9) $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

Answer Key

1) Converges.

2) Converges.

3) Converges.

4) Converges.

5) Converges.

6) Converges.

7) Diverges.

8) Inconclusive.

9) Converges.

Alternating Series and Conditional Convergence

Questions

For the following series determine if the series converges or diverges:

1) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{4n+1}}$

2) $\sum_{n=1}^{\infty}(-1)^n \frac{\ln n}{n}$

3) $\sum_{n=1}^{\infty}(-1)^n \frac{n+2}{n^2+n}$

Absolute and Conditional Convergence of Series

Determine if the following series is absolutely convergent, conditionally convergent or divergent:

4) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n+1}}$

5) $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1}{n^2+1}$

6) $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{\cos n}{n^2}$

7) $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n}$

8) $\sum_{n=1}^{\infty}(-1)^n \frac{\ln n}{n}$

9) $\sum_{n=1}^{\infty}\left(\frac{-1}{\ln n}\right)^n$

10) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n(n+1)}}$

Prove or disprove:

11) If $\sum a_n$ converges and $\sum b_n$ diverges, then $\sum(a_n+b_n)$ diverges.

12) If $\sum a_n$ diverges and $\sum b_n$ diverges, then $\sum(a_n+b_n)$ diverges.

13) 1. If $\sum(a_n)^2$ converges, then $\sum a_n$ converges.

2. If $\sum(a_n)^2$ converges and $\sum a_n$ converges, then $\sum a_n$ converges absolutely.

14) If $\sum a_n$ converges and positive, then $\sum \frac{1}{a_n}$ diverges.

15) If $\sum a_n$ converges, then $\sum(a_n)^2$ converges.

Answer Key

1) Converges. **2)** Converges. **3)** Converges.

4) Conditionally converges. **5)** Converges absolutely.

6) Converges absolutely. **7)** Conditionally converges.

8) Conditionally converges. **9)** Converges absolutely.

10) Conditionally converges.

11) - 15) Solution in the recording.

Power Series

Questions

1) Find the region of convergence of the series:

a. $\sum_{n-1}^{\infty} \frac{1}{4n+1}\left(\frac{1-x}{1+x}\right)^n$

b. $\sum_{n=1}^{\infty} \frac{2^n}{n!(x-5)^n}$

c. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)10^n (x-4)^n}$

d. $\sum_{n=1}^{\infty} \frac{1}{n\ln^4 nx}$

e. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^x}$

f. $\sum_{n=1}^{\infty} \frac{1}{(x+n)(x+n-1)}$

2) Check the uniform convergence of the series:

a. $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2} \qquad -\infty < x < \infty$

b. $\sum_{n=1}^{\infty} \frac{x^n}{n^{\frac{3}{2}}} \qquad -1 \le x \le 1$

c. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}+x^2} \qquad -\infty < x < \infty$

d. $\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n!}}\left(x^n + x^{-n}\right) \qquad \frac{1}{4} \le x \le 4$

e. $\sum_{n=2}^{\infty} \ln\left(1+\frac{x^2}{n\ln^2 n}\right) \qquad -a \le x \le a$

f. $\sum_{n=1}^{\infty} \frac{n^2 x}{1+n^7 x^2} \qquad -\infty < x < \infty$

3) Find the range and radius of convergence of the series:

a. $\sum_{n=0}^{\infty} \frac{x^n}{n+1}$

b. $\sum_{n=0}^{\infty} \frac{(-1)x^n}{n!}$

c. $\sum_{n=1}^{\infty} \frac{5^n}{n^2} x^n$

d. $\sum_{n=1}^{\infty} x^n \sin^2 \frac{1}{n}$

e. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x+2)^n}{\sqrt{n}}$

f. $\sum_{n=1}^{\infty} \frac{(n+1)^5}{(2n+1)} x^{2n}$

g. $\sum_{n=0}^{\infty} \frac{n!}{3^n}(x-1)^n$

h. $\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdot .....\cdot(2n-1)}{(2n-2)!} x^n$

i. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x+1)^n}{n\cdot 4^n}$

j. $\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n (x+5)^n$

k. $\sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{n^4 \cdot 100^n}$

l. $\sum_{n=1}^{\infty} \frac{(x+5)^{2n+1}}{n\cdot 2^{2n+1}}$

4) Expend the functions as a power series and find the range and radius of convergence:

a. $f(x)=\frac{1}{1+x}$ b. $f(x)=\frac{3}{1-x^4}$ c. $f(x)=\frac{1}{1+9x^2}$

d. $f(x)=\frac{1}{x-5}$ e. $f(x)=\frac{x}{4x+1}$ f.

Answer Key

1) a. Converges absolutely.

b. Converges absolutely.

c. Converges for $x \ge 4\frac{1}{10}$ or $x < 3\frac{9}{10}$.

d. Converges.

e. Converges for $x > 0$.

f. Converges to $\frac{1}{x}$ for $x \ne 0,-1,-2,-5....$.

2) a. Yes. b. Yes. c. Yes. d. Yes. e. Yes. f. Yes.

3) a. 1 , $-1 \le x < 1$ b. ∞ , $-\infty < x < \infty$ c. $\frac{1}{5}$, $-\frac{1}{5} \le x \le \frac{1}{5}$

d. 1 , $-1 \le x \le 1$ e. 1 , $-3 \le x \le -1$ f. 1 , $-1 \le x \le 1$

g. 0 , $x = 1$ h. ∞ , $-\infty \le x \le \infty$ i. 4 , $-5 < x \le 3$

j. $\frac{4}{3}$, $-6\frac{1}{5} \le x \le -3\frac{2}{3}$ k. 10 , $-9 \le x \le 11$ l. 2 , $-7 \le x \le -3$

4) a. $|x| < 1$ b. $|x| < 1$ c. $-\frac{1}{3} < x < \frac{1}{3}$ d. $-5 < x < 5$

e. $-\frac{1}{4} < x < \frac{1}{4}$

Taylor and Maclaurin Series & The Convergence of Taylor Series

Questions

For questions **1-9**, use the Commonly Used Taylor Series in the Appendix.

1) Find the Taylor series of $f(x)=\sin 2x$ around $x=0$.

2) Find the Taylor series of $f(x)=x^2e^{-4x}$ around $x=0$.

3) Find the Taylor series of $f(x)=\sinh x$ around $x=0$.

4) Find the Taylor series of $f(x)=\sin^2 x$ around $x=0$.

5) Find the Taylor series of $f(x)=\cos^2 x$ around $x=0$.

6) Find the Taylor series of $f(x)=2^x$ around $x=0$.

7) Find the Taylor series of $f(x)=x\cos\left(4x^2\right)$ around $x=0$.

8) Find the Taylor series of $f(x)=\ln\left(2-3x+x^2\right)$ around $x=0$.

9) Find the Taylor series of $f(x)=\arcsin x$ around $x=0$.

10) Find the Taylor series of $f(x)=\ln x$ expended around $x=1$.

11) Find the Taylor series of $f(x)=\frac{1}{x}$ expended around $x=2$.

12) Find the Taylor series of $f(x)=\sin x$ expended around $x=\frac{\pi}{2}$.

Answer Key

1) $\sum_{n=0}^{\infty}(-1)^n 2^{2n+1}\frac{x^{2n+1}}{(2n+2)!}$

2) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{n+2}}{n!}$

3) $\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}$ all x

4) $\sum_{n=0}^{\infty}(-1)^{n+1} 2^{2n-1}\frac{x^{2n}}{2n!}$

5) $\frac{1}{2}+\sum_{n=0}^{\infty}(-1)^n 2^{2n-1}\frac{x^{2n}}{(2n)!}$

6) $\sum_{n=0}^{\infty}(\ln 2)^n \frac{x^n}{n!}$

7) $\sum_{n=0}^{\infty}(-1)^n 4^{2n}\frac{x^{4n+1}}{(2n)!}$ all x

8) $\ln 2-\sum_{n=0}^{\infty}\left(1+\frac{1}{2^{n+1}}\right)\frac{x^{n+1}}{n+1}$ $-1\leq x<1$

9) $x+\sum_{n=0}^{\infty}(-1)^n \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\ldots\ldots\left(\frac{1}{2}-n+1\right)}{n!}\frac{x^{2n+1}}{2n+1}$ $-1\leq x\leq 1$

10) $\sum_{n=0}^{\infty}(-1)^n \frac{(x-1)^{n+1}}{n+1}$

11) $\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{n+1}}(x-2)^n$ $0<x<4$

12) $\sum_{n=0}^{\infty}(-1)^n \frac{\left(x-\frac{\pi}{2}\right)^{2n}}{(2n)!}$ all x

Applications of Taylor Series

Questions

Finding Nonzero Terms in Expansions

1) Find the first four nonzero terms of the Maclaurin series of $f\left(x\right)=e^{-x^2}\cos x$.

2) Find the first four nonzero terms of the Maclaurin series of $f\left(x\right)=\tan x$.

3) Find the first four nonzero terms of the Maclaurin series of $f\left(x\right)=\frac{\sin x}{e^x}$.

Sum of Series Using Taylor and Maclaurin Expansions Questions

4) Compute the sum of the following series:

a. $\sum_{n=0}^{\infty}\frac{1}{n!}$

b. $\sum_{n=0}^{\infty}\frac{\left(-1\right)^n 2^n}{n!}$

c. $\sum_{n=0}^{\infty}\frac{1}{2^n\cdot n!}$

5) Compute the sum of the following series: $\sum_{n=0}^{\infty}\frac{n+1}{n!}$.

6) Compute the sum of the following series: $\sum_{n=0}^{\infty}\frac{\left(-1\right)^n}{2n+1}$.

7) Compute the sum of the following series: $\sum_{n=0}^{\infty}\frac{\left(-1\right)^n}{\left(2n+1\right)!}$.

8) Compute the sum of the following series: $\sum_{n=0}^{\infty}\frac{\left(-1\right)^n}{\left(2n\right)!}$.

9) Compute the sum of the following series: $\sum_{n=0}^{\infty}\frac{\left(-1\right)^n}{n+1}$.

10) Compute the sum of the following series: $\sum_{n=0}^{\infty}\frac{\left(-1\right)^n}{2^{n+1}\left(n+1\right)}$.

Finding Limits Using Expansions

11) Compute the value of the following limit:

a. $\lim\limits_{x\to 0}\dfrac{x-\arctan x}{x^3}$

b. $\lim\limits_{x\to 0}\dfrac{\sin x - x + \frac{1}{6}x^3}{x^5}$

c. $\lim\limits_{x\to 0}\dfrac{e^x \sin x - x(1+x)}{x^3}$

Computations with Taylor Series

12) Answer the following questions:

a. Compute $1/\sqrt[3]{e}$ with an error of less than 0.001.
b. Compute $\sin 3°$ with an error of less than 0.001.
c. Compute $\arctan 0.25$ with an error of less than 0.001.

13) Answer the following questions:

a. Evaluate $\dfrac{1}{\sqrt{e}}$ using the first three nonzero elements of maclaurin series and estimate the error.
b. Evaluate $\cos 4°$ using the first three nonzero elements of maclaurin series and estimate the error.
c. Evaluate $\ln 1.5$ using the first three nonzero elements of maclaurin series and estimate the error.

14) Answer the following questions:

a. What is the maximum error in approximating $\sin x \cong x - \dfrac{x^3}{3!}$ for $|x| \le \dfrac{\pi}{6}$?
b. What is the maximum error in approximating $\ln(1+x) \cong x$ for $|x| < 0.01$?
c. What is the maximum error in approximating $\cos x \cong 1 - \dfrac{x^2}{2!}$ for $|x| \le 0.2$?

15) Answer the following questions:

a. For which values of x is $\sin x \cong x - \dfrac{x^3}{3!}$ with error less than 0.001?
b. For which values of x is $\arctan x \cong x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \dfrac{x^7}{7}$ with error less than 0.01?

16) Answer the following questions:

a. Approximate $\int_0^{0.2} \frac{\sin x}{x}\,dx$ with an error less than 0.0001.

b. Approximate $\int_0^{0.1} \frac{\ln(1+x)}{x}\,dx$ with an error less than 0.001.

c. Approximate $\int_0^{0.5} \frac{1-\cos x}{x^2}\,dx$ with an error less than 0.001.

Answer Key

1) $1-\frac{3}{2}x^2+\frac{25}{24}x^4-\frac{331}{720}x^6$

2) $x+\frac{1}{3}x^3+\frac{2}{15}x^5+\frac{17}{315}x^7 \quad -\frac{\pi}{2}<x<\frac{\pi}{2}$

3) $x-x^2+\frac{1}{3}x^3-\frac{1}{30}x^5$

4) a. e c. $\frac{1}{e^2}$ b. $\sqrt{e}$

5) $2e$

6) $\frac{\pi}{4}$

7) $\sin 1^\circ$

8) $\cos 1^\circ$

9) $\ln 2$

10) $\ln 1.5$

11) a. $\frac{1}{3}$ b. $\frac{1}{120}$ c. $\frac{1}{3}$

12) a. $\frac{58}{81}$ b. $\frac{\pi}{60}$ c. $\frac{47}{192}$

13) a. $\frac{5}{8}$ b. 1 c. $\frac{77}{192}$

14) a. $\frac{\left(\frac{\pi}{6}\right)^5}{5!}$ b. 0.00005 c. $\frac{0.2^6}{720}$

15) a. $|x|<\sqrt[5]{0.12}$ b. $|x|<\sqrt[9]{0.09}$

16) a. $\frac{449}{2259}$ b. $\frac{39}{400}$ c. $\frac{143}{576}$

Appendix - Commonly Used Taylor Series

Series	When is valid / true
$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+\frac{x^1}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots$	$-\infty<x<\infty$
$\sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\dots$	$-\infty<x<\infty$
$\cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\dots$	$-\infty<x<\infty$
$\ln(1+x)=\sum_{n=0}^{\infty}(-1)^n\frac{x^{n+1}}{n+1}=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\dots$	$-1<x\le 1$
$\arctan x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\dots$	$-1\le x\le 1$
$\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n=1+x^1+x^2+x^3+\dots$	$-1<x<1$
$(1+x)^m=1+\sum_{n=1}^{\infty}\frac{m(m-1)\cdot\ldots\cdot(m-n+1)}{n!}x^n$ $=1+mx+\frac{m(m-1)}{2!}x^2+\frac{m(m-1)(m-2)}{3!}x^3+\dots$	$-1\le x\le 1\ (m>0)$ $-1<x\le 1\ (-1<m<0)$ $-1<x<1\ (m\le -1)$ $m\ne 0,1,2,3,\dots$