proprep™

Workbook

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Early Transcendentals – 14th Edition

Limits and Continuty

Limit of a Function and Limit Laws

Questions

Technique 1 – Substitution

Find the following limits, if possible:

1) $\lim_{x\to 4} x^2 + x + 1$ **2)** $\lim_{x\to 10} \frac{x+1}{x+2}$ **3)** $\lim_{x\to 1^+} \sqrt{x+3}$ **4)** $\lim_{x\to 100} 20$

Technique 2 – Factoring

Find the following limits if possible:

5) $\lim_{x\to 3} \frac{x^2 - x - 6}{x^2 - 9}$ **6)** $\lim_{x\to -5} \frac{2x^2 - 50}{2x^2 + 3x - 35}$

7) $\lim_{x\to 1} \frac{x^7 - x}{x - 1}$ **8)** $\lim_{x\to 1} \frac{x^n - x}{x - 1}$ $n > 1$

Technique 3 – The Sandwich/ Squeeze Theorem

9) $\lim_{x\to 0} x \cdot \sin\left(\frac{1}{x}\right)$ **10)** $\lim_{x\to 0} x^2 \cdot \cos\left(\ln x^2\right)$

Answer Key

1) 21 **2)** $\frac{11}{12}$ **3)** 2 **4)** 20

5) $\frac{5}{6}$ **6)** $\frac{20}{17}$ **7)** 6 **8)** $n-1$

9) 0 **10)** 0

The Precise Definition of a Limit

Questions

1) Use the definition of the limit to prove that $\lim_{x\to 2}(7x+14)=28$.

2) Use the definition of the limit to prove that $\lim_{x\to 3}x^2=9$.

3) Use the definition of the limit to prove that $\lim_{x\to 1}(x^2-1)=0$.

4) Use the definition of the limit to prove that $\lim_{x\to 24}\sqrt{x+1}=5$.

5) Use the definition of the limit to prove that $\lim_{x\to 1}\frac{1}{x}=1$.

6) Use the definition of the limit to prove that $\lim_{x\to \frac{\pi}{4}}\sin x=\sin\frac{\pi}{4}$.

7) Use the definition of the limit to prove that $\lim_{x\to 2}\frac{3+x}{x^2+1}=1$.

Answer Key

Solutions in the recordings.

One-Sided Limits

Questions

Technique 4 – Piecewise Functions

Find the following limits if possible:

1) $\lim\limits_{x\to 0} f(x)\ ;\ f(x)=\begin{cases}\dfrac{\sin 4x}{x} & x>0\\ 4+e^{\frac{1}{x}} & x<0\end{cases}$

2) $\lim\limits_{x\to 1} f(x)\ ;\ f(x)=\begin{cases}\dfrac{x^2+x-2}{x-1} & x>1\\ \dfrac{x-1}{\sqrt{x}-1} & x<1\end{cases}$

3) $\lim\limits_{x\to 0}\dfrac{|x|}{x}$

4) Use the definition of the limit to prove that $\lim\limits_{x\to 4^-}\left(\sqrt{4-x}\right)=0$.

5) Use the definition of the limit to prove that:

a. $\lim\limits_{x\to 0^+}\dfrac{|x|}{x}=1$

b. $\lim\limits_{x\to 0^-}\dfrac{|x|}{x}=-1$

Answer Key

1) 4
2) No limit.
3) No limit.
4) Solution in the recording.
5) Solution in the recording.

Continuity

Questions

Limit from Definition

1) Use the definition of continuity to prove that $f(x)=2x-3$ is continuous at $x=4$.

Trigonometric Limits

Find the following limits, if possible:

2) $\lim\limits_{x\to 0}\dfrac{\sin(3x)}{4x}$

3) $\lim\limits_{x\to 0}\dfrac{\sin(3x)}{\sin(4x)}$

4) $\lim\limits_{x\to 0}\dfrac{x\cos(x)}{\sin(2x)}$

5) $\lim\limits_{x\to 0}\dfrac{1-\cos(x)}{x^2}$

6) $\lim\limits_{x\to 0}\dfrac{\tan(x)\sin(x)}{x}$

7) $\lim\limits_{x\to 0}\dfrac{\sqrt{1-\sin(x)}-\sqrt{\cos(x)}}{x}$

8) $\lim\limits_{x\to 0}\dfrac{1-\cos\left(1-\cos(x)\right)}{x^4}$

9) $\lim\limits_{x\to 0}\dfrac{3\sin(x)-\sin(3x)}{x^3}$

10) $\lim\limits_{x\to 0}\dfrac{1-\sqrt{\cos(x)}}{x^2}$

Definition of Continuity

11) Consider the function: $f(x)=\begin{cases}x & x\geq 1\\ x^2 & x<1\end{cases}$.

Is the function $f(x)$ continuous at $x=1$? Sketch the graph of $f(x)$.

12) Consider the function: $f(x)=\begin{cases}x+1 & x\leq 2\\ 5-x & x>2\end{cases}$.

Is the function $f(x)$ continuous at $x=2$? Sketch the graph of $f(x)$.

13) Is the function $f(x)=\begin{cases}\dfrac{\sin 4x}{x} & x>0\\ 4+e^{\frac{1}{x}} & x<0\end{cases}$ continuous at $x=0$?

14) Is the function $f(x)=\begin{cases}\dfrac{\sin x}{x} & x>0\\ 2 & x=0\\ 1+e^{\frac{1}{x}} & x<0\end{cases}$ continuous at $x=0$?

15) Is the function $f(x)=\begin{cases}\sin x & x<0\\ x^2 & 0\le x<1\\ 2-x & 1\le x<2\\ x-3 & x\ge 2\end{cases}$ continuous at $x=0,1,2$?

16) Is the function $f(x)=\begin{cases}\dfrac{1}{x} & x\le 1\\ |x-2| & 1<x<2\\ 1 & x=2\\ x-2 & x>2\end{cases}$ continuous at $x=1,2$?

17) Find the value of k for which $f(x)=\begin{cases}kx^2+x-2 & x<2\\ 5kx-6 & x>2\end{cases}$ is continuous everywhere.

18) Find k which makes the function $f(x)=\begin{cases}\dfrac{x^2+2x-3}{x-1} & x\neq 1\\ k & x=1\end{cases}$ continuous for every x.

19) Find k which makes the function $f(x)=\begin{cases}\dfrac{\sqrt{x^2+5}-3}{x-2} & x\neq 2\\ k & x=2\end{cases}$ continuous for every x.

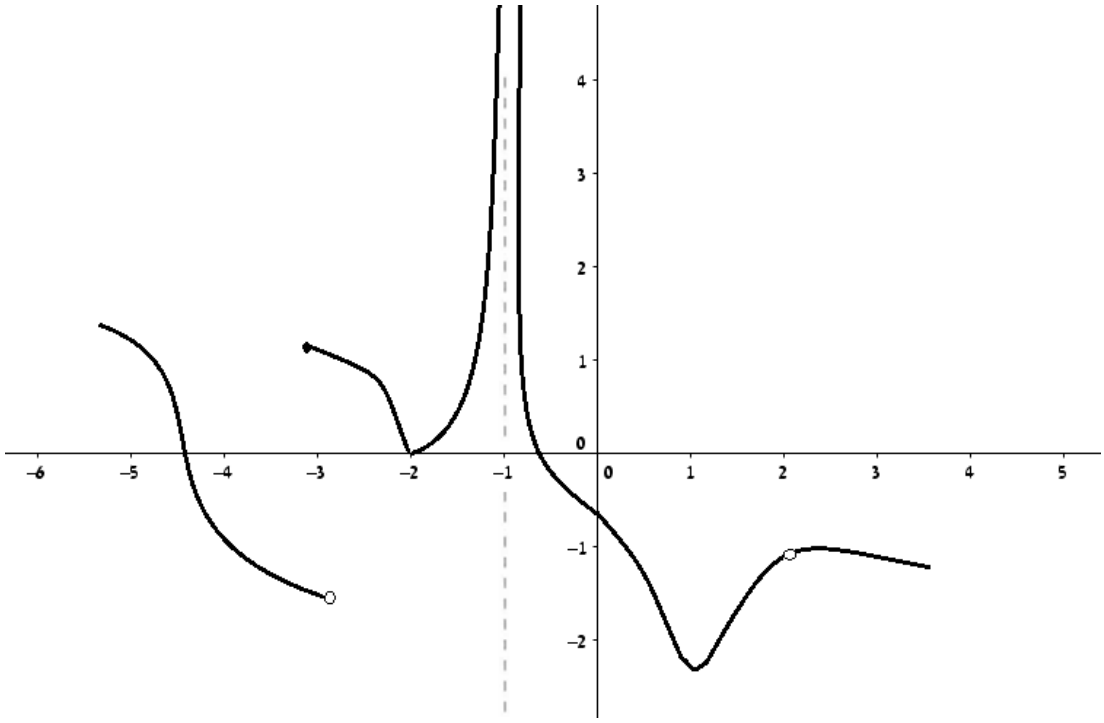
20) Find k which makes the function $f(x)=\begin{cases}2x-k & x\le 0\\ 5kx-6 & x>0\end{cases}$ continuous for every x.

21) Find the values of constants a and b for which function $f(x)=\begin{cases}ax+b & x\le 0\\ \dfrac{\sin x}{2x} & 0<x<\pi\\ a\cos x & x\ge\pi\end{cases}$

is continuous in its domain.

Points of Discontinuity

22) Classify any points of discontinuity of f over the graphed interval:



23) Find and classify any points of discontinuity of the functions below:

a. $f(x)=\dfrac{x+1}{x-1}$

b. $f(x)=\dfrac{x^2+x-2}{x^2-1}$

24) Find and classify any points of discontinuity of the functions below:

a. $f(x)=\begin{cases} x^2 & x<1 \\ 0 & x=1 \\ 2-x & x>1 \end{cases}$

b. $f(x)=\begin{cases} x^2 & x<1 \\ 0 & x=1 \\ -x^2+2x+1 & x>1 \end{cases}$

c. $f(x)=\begin{cases} x^2 & x<1 \\ 0 & x=1 \\ \dfrac{1}{x-1} & x>1 \end{cases}$

The Intermediate Value Theorem

25) Use the intermediate value theorem to show that the equation $\cos x = x$ must have at least one solution.

26) Use the intermediate value theorem to show that the equation $x^3 + 4x = 1$ must have at least one solution.

27) Use the intermediate value theorem to show that the equation $\ln x = -x^2$ must have at least one solution.

28) Use the intermediate value theorem to show that the equation $x^3 + bx^2 + cx + d = 0$ must have at least one solution.

29) Use the intermediate value theorem to show that the equation $4x^3 + 5x = \frac{1}{x}$ must have at least two solutions.

30) Use the intermediate value theorem to show that the equation $e^x = 5x$ must have at least one solution.

Answer Key

1) Solution in the recording.

2) $\frac{3}{4}$ **3)** $\frac{3}{4}$ **4)** $\frac{1}{2}$ **5)** $\frac{1}{2}$ **6)** $\frac{1}{2}$

7) $\frac{1}{2}$ **8)** $\frac{1}{8}$ **9)** 4 **10)** $\frac{1}{4}$

11) Continuous at $x=1$

12) Continuous at $x=2$

13) Not defined at $x=0$

14) Not continuous

15) Continuous at $x=0$, Continuous at $x=1$, Not continuous at $x=2$

16) Continuous at $x=1$, Not continuous at $x=2$

17) $k=1$

18) $k=4$

19) $k=\frac{2}{3}$

20) $k=-1$

21) $a=0,\ b=\frac{1}{2}$

22) $x=-3$, $x=-1$, $x=2$.

23) a. $x=1$, undefined. b. $f(1)$ undefined.

24) a. $x=1$ removable discontinuity. b. Jump discontinuity at $x=1$.
c. Essential discontinuity at $x=1$.

25) - 30) Solution in the recording.

Limits Involving Infinity; Asymptotes of Graphs

Questions

Technique 4 – $f(x)$ Tends to Infinity

Calculate the following limits if possible:

1) $\lim_{x\to 0}\dfrac{x^2+4}{x}$

2) $\lim_{x\to 2}\dfrac{(x-1)^2}{x-2}$

3) $\lim_{x\to 2}\dfrac{x^2-1}{(x-2)(x-5)}$

4) $\lim_{x\to 0^+}\dfrac{\ln x}{x}$

5) $\lim_{x\to 2^-}-\dfrac{1}{2}\ln(2-x)$

6) $\lim_{x\to 0^+}\left((\ln x)^2+x(\ln x-3)\right)$

7) $\lim_{x\to 0}e^{\frac{1}{x}}$

8) $\lim_{x\to 0^+}\dfrac{1}{1+2^{\frac{1}{x}}}$

9) $\lim_{x\to 0^-}\dfrac{1}{1+2^{\frac{1}{x}}}$

10) $\lim_{x\to 0}\dfrac{1}{1+2^{\frac{1}{x}}}$

11) $\lim_{x\to 0^+}\ln x\cdot\cot x$

Technique 5 – x Tends to Infinity

Find the following limits, if possible:

12) $\lim_{x\to\infty}\left(e^{-x}\right)^{\ln x}$

13) $\lim_{x\to-\infty}\arctan(x)+e^x$

14) $\lim_{x\to\infty}\dfrac{4x^2+2}{x^2+1000x}$

15) $\lim_{x\to-\infty}\dfrac{x^4+2x^2+6}{3x^2+10x}$

16) $\lim_{x\to\infty}\dfrac{x^4+2x^2+6}{3x^5+10x}$

17) $\lim_{x\to\infty}\left(\dfrac{x^2-5x+6}{2x+10}-\dfrac{x}{2}\right)$

18) $\lim_{x\to\infty}\dfrac{\sqrt{x^2+1}}{x}$

19) $\lim_{x\to-\infty}\dfrac{\sqrt{x^2+1}}{x}$

20) $\lim_{x\to-\infty}\dfrac{\sqrt{9x^6-5x}}{x^3-2x^2+1}$

21) $\lim_{x\to\infty}\dfrac{\sqrt[3]{x^4+2x^2+6+27x^6}}{\sqrt{3x^3+10x+4x^4}}$

22) $\lim_{x\to\infty}\dfrac{\sqrt{x+2}-\sqrt{3x-3}}{\sqrt{4x+1}-\sqrt{5x-1}}$

23) $\lim_{x\to-\infty}\dfrac{16^x+4^{x+1}}{2^{4x+2}+2^{x+3}}$

24) $\lim_{x\to\infty}\dfrac{4\cdot 9^x+3^{x+1}}{81^{0.5x}+3^{x+3}}$

25) $\lim_{x\to-\infty}\dfrac{4\cdot 9^x+3^{x+1}}{81^{0.5x}+3^{x+3}}$

26) $\lim_{x\to\infty}\sqrt{\dfrac{4x^2+2}{x^2+1000x}}$

27) $\lim_{x\to\infty}\ln\left(\dfrac{3x^3-5x-1}{x^3-2x^2+1}\right)$

28) $\lim_{x\to\infty}e^{\frac{x^4+2x^2+6}{3x^4+10x}}$

29) $\lim_{x\to-\infty}\sin\left(\dfrac{x^4+2x^2+6}{3x^5+10x}\right)$

30) $\lim_{x\to\infty}\left(\sqrt{x^2+kx}-x\right)$

31) $\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-x\right)$

32) $\lim_{x\to-\infty}\left(\sqrt{x^2+x+1}+x\right)$

33) $\lim_{x\to\infty}\left(\sqrt{x^4+x^2+1}-x^2\right)$

34) $\lim_{x\to\infty}\left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right)$

Technique 8 – The Sandwich/Squeeze Theorem

Find the following limits, if possible:

35) $\lim\limits_{x\to\infty}\dfrac{\sin x}{x}$

36) $\lim\limits_{x\to\infty}\dfrac{\cos(2x+1)}{x}$

37) $\lim\limits_{x\to\infty}\dfrac{3x+\sin x}{4x+\cos x}$

38) $\lim\limits_{x\to\infty}\dfrac{3x+\arctan(2x-3)}{4x+\arctan(x-\ln x)}$

39) $\lim\limits_{x\to\infty}\sqrt[x]{2^x+3^x+4^x}$

40) $\lim\limits_{x\to\infty}\dfrac{1}{x}[x]$

Technique 9 – Piecewise Functions

Find the following limits if possible:

41) $\lim\limits_{x\to\infty}\dfrac{|x|}{x}$

42) $\lim\limits_{x\to-\infty}\dfrac{|x|}{x}$

Limit from Definition

43) Use the definition of the limit to prove that $\lim\limits_{x\to2}\dfrac{-5}{(x-2)^2}=-\infty$.

44) Use the definition of the limit to prove that $\lim\limits_{x\to3^-}\dfrac{-2}{x-3}=\infty$.

45) Use the definition of the limit to prove that $\lim\limits_{x\to0^+}\ln x=-\infty$.

46) Use the definition of the limit to prove that $\lim\limits_{x\to\infty}\dfrac{x+7}{x+2}=1$.

47) Use the definition of the limit to prove that $\lim\limits_{x\to\infty}\dfrac{3-4x}{2x+1}=-2$.

48) Use the definition of the limit to prove that $\lim\limits_{x\to\infty}\dfrac{3x^2-1}{x^2+x+1}=3$.

49) Given a function $f(x)$ which satisfies $\lim\limits_{x\to\infty}f(x)=-5$.

Prove that there exists an $M>0$ such that $f(x)<-4$ whenever $x>M$.

50) Given a function $f(x)$ which satisfies $\lim_{x\to-\infty} f(x)=5$.

Prove that there exists an $M<0$ such that $f(x)>4$ whenever $x<M$.

51) Given a positive function $f(x)$ on the interval $[a,\infty)$ which satisfies $\lim_{x\to\infty} f(x)=0$.

Prove [using *epsilon-delta*] that $\lim_{x\to\infty}\sqrt{f(x)}=0$.

52) Given the limit: $\lim_{x\to\infty}\underbrace{\frac{x^2+2x}{x^2+3x+2}}_{f(x)}=\underbrace{1}_{L}$ *(no need to prove)*.

Find a value for $M>0$ such that $|f(x)-L|<\underbrace{0.1}_{\varepsilon}$ whenever $x>M$.

Answer Key

1) No limit. **2)** No limit. **3)** No limit. **4)** $-\infty$
5) ∞ **6)** ∞ **7)** No limit. **8)** 0
9) 1 **10)** No limit. **11)** $-\infty$ **12)** 0
13) $-\frac{\pi}{2}$ **14)** 4 **15)** $-\infty$ **16)** 0
17) -5 **18)** 1 **19)** -1 **20)** -3
21) $\frac{3}{2}$ **22)** $\frac{1-\sqrt{3}}{2-\sqrt{3}}$ **23)** 0 **24)** 4
25) $\frac{1}{9}$ **26)** 2 **27)** $\ln 3$ **28)** $e^{\frac{1}{3}}$
29) 0 **30)** $\frac{k}{2}$ **31)** $\frac{1}{2}$ **32)** $-\frac{1}{2}$
33) $\frac{1}{2}$ **34)** $\frac{a-b}{2}$ **35)** 0 **36)** 0
37) $\frac{3}{4}$ **38)** $\frac{3}{4}$ **39)** 4 **40)** 1
41) 1 **42)** -1

43) - 51) Solutions in the recordings.
52) $x>30$