proprep™

Workbook

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Early Transcendentals – 14th Edition

Multiple Integrals

Double and Iterated Integrals over Rectangles

Questions

1) Compute the double integral $\iint\limits_R \sin\frac{1}{2}(x+y)\cos\frac{1}{2}(x-y)dA$, where the region R is the triangle with vertices: $\mathrm{A}(0,0)$, $\mathrm{B}(2,0)$, $\mathrm{C}(1,1)$.

2) Compute the double integral $\iint\limits_R (4x+8y)dA$, where the region R is the parallelogram with vertices: $\mathrm{A}(\text{-}1,3)$, $\mathrm{B}(1,\text{-}3)$, $\mathrm{C}(3,\text{-}1)$, $\mathrm{D}(1,5)$.

Answer Key

1) $1-\frac{1}{2}\sin 2$

2) 192

Double Integrals over General Regions

Questions

1) Compute the double integral $\iint_R \frac{x-y}{x+y}dA$, where R is the region bounded by the lines: $y=x,\ y=x-1,\ y=1-x,\ y=3-x$.

2) Compute the double integral $\iint_R e^{xy}dA$, where R is the region bounded by the functions: $y=x,\ y=0.5x,\ y=\frac{1}{x},\ y=\frac{2}{x}$.

Answer Key

1) $\frac{1}{4}\ln 3$

2) $\frac{1}{2}\ln 2\left(e^2-e\right)$

Area by Double Integration

Questions

1) Compute the **areas** of the **regions** bounded by the given curves:

a. $x+y=2,\ x^2-4y=4$

b. $xy=a^2,\ x+y=2.5a\ (a>0)$

c. $x^2+y^2=2x,\ y=0,\ y=x\sqrt{3}$

d. $x+y=3,\ y^2=4x$

2) Compute the **volumes** of the **solids** bounded by the given surfaces:

a. $y=0,\ x=0,\ x+y=1,\ z=0, z=1+x+y$

b. $y=x^2,\ y=1,\ z=x^2+y^2,\ z=0$

c. $y=2/x,\ y=2x,\ y=0.5x,\ z=x^2+y,\ z=0\ (x\geq 0)$

d. $2y^2=x,\ \frac{x}{4}+\frac{y}{2}+\frac{z}{4}=1,\ z=0$

e. $z=y,\ x^2+0.25y^2=1\ (z\geq 0)$

f. $y=0,\ x=0,\ z=6,\ z=x+y$

3) A flat triangular board with vertices $\delta(x,y)=xy$, has a density function $(0,0)$, $(1,0)$ and $(0,1)$.

a. Compute the **mass** of the board

b. Compute its center of mass.

4) A flat board with rectangular shape $R=\left\{(x,y)\mid -\frac{b}{2}\leq y\leq \frac{b}{2},\ -\frac{a}{2}\leq x\leq \frac{a}{2}\right\}$ has a constant density function (the board is homogeneous).

Compute the moment of inertia of the board about the z-axis. Express you answer in terms of the mass M of the board.

5) Find the surface area of the part of the cylinder $x^2+z^2=4$ which lies above the rectangle $R=\{(x,y)\mid 0\leq x\leq 1,\ 0\leq y\leq 4\}$ in the xy plane.

Answer Key

1) a. $21\frac{1}{3}$ b. $a^2\left(\frac{15}{8}-2\ln 2\right)$ c. $\frac{\pi}{3}+\frac{\sqrt{3}}{4}$ d. $21\frac{1}{3}$

2) a. $\frac{5}{6}$ b. $\frac{88}{105}$ c. $\frac{17}{6}$ d. $16\frac{1}{5}$

e. $\frac{8}{3}$ f. 36

3) a. $\frac{1}{24}$ b. $\left(\frac{2}{5},\frac{2}{5}\right)$

4) $\frac{M\left(a^2+b^2\right)}{12}$

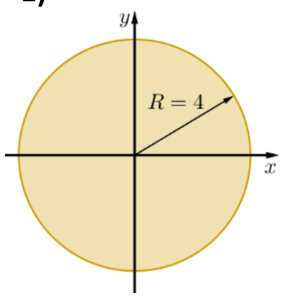
5) $\frac{1}{6}\pi\left(5\sqrt{5}-1\right)$

Double Integrals in Polar Form

Questions

Compute: $\iint_D \sqrt{x^2+y^2}\,dA$, where D is the domain described in the sketch.

1)



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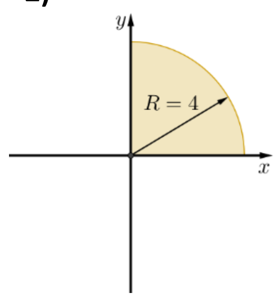

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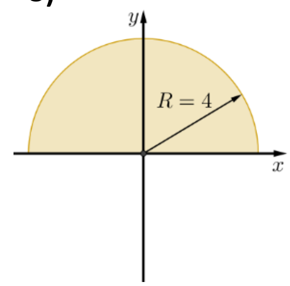

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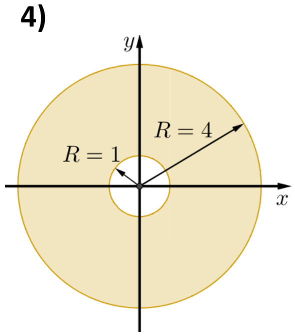

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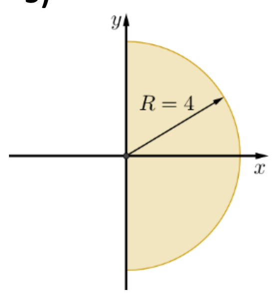

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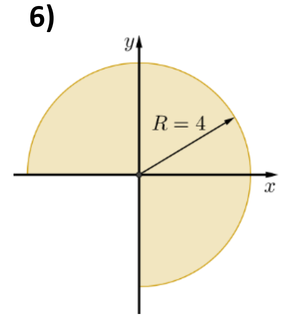

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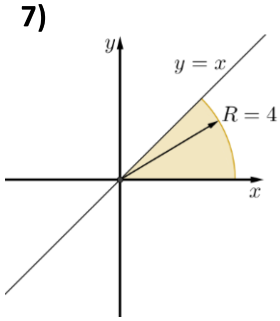

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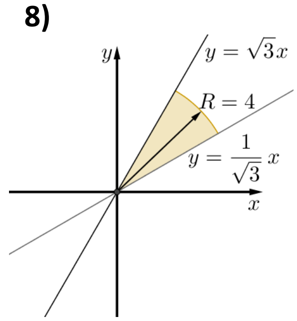

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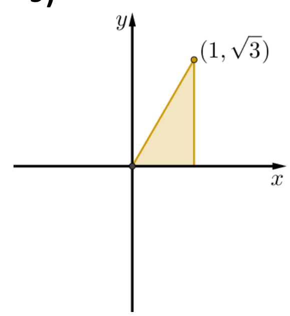

Compute the following integrals by converting to polar coordinates:

10) $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} dydx$

11) $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dydx$

12) $\int_{0}^{1}\int_{0}^{\sqrt{1-y^2}} \left(x^2+y^2\right)dxdy$

13) $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \left(x^2+y^2\right)dxdy$

14) $\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dydx$

15) $\int_{0}^{2}\int_{0}^{\sqrt{4-y^2}} \left(x^2+y^2\right)dxdy$

16) $\int_{0}^{6}\int_{0}^{y} xdxdy$

17) $\int_{0}^{2}\int_{0}^{x} ydydx$

18) $\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}}dydx$

19) $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{0} \frac{4\sqrt{x^2+y^2}}{1+x^2+y^2}dxdy$

20) $\int_{0}^{\ln 2}\int_{0}^{\sqrt{\ln^2 2-y^2}} e^{\sqrt{x^2+y^2}}dxdy$

21) $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}} e^{-(x^2+y^2)}dydx$

22) $\int_{0}^{2}\int_{0}^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2}dydx$

23) $\int_{0}^{2}\int_{-\sqrt{1-(y-1)^2}}^{0} xy^2dxdy$

24) $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln\left(x^2+y^2+1\right)dxdy$

25) $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{\left(1+x^2+y^2\right)^2}dydx$

In each of the following, compute the volume of the described solid:

26) The solid bounded by the sphere $x^2+y^2+z^2=9$ and the cylinder $x^2+y^2=1$.

27) The solid bounded inside the cylinder $x^2+y^2=2y$, below the cone $z=\sqrt{x^2+y^2}$, and above the xy plane.

28) The solid bounded inside the cylinder $x^2+y^2=x$, below the paraboloid $z=1-x^2-y^2$, and above the xy plane.

Answer Key

1) $\frac{128\pi}{3}$

2) $\frac{32\pi}{3}$

3) $\frac{64\pi}{3}$

4) 42π

5) $\frac{64\pi}{3}$

6) 32π

7) $\frac{16\pi}{3}$

8) $\frac{32\pi}{9}$

9) $S=\frac{1}{3}\int_{0}^{\pi/3}\frac{1}{\cos^3\theta}d\theta$

10) $\frac{\pi}{2}$

11) π

12) $\frac{\pi}{8}$

13) $\frac{\pi}{2}$

14) πa^2

15) 2π

16) 36

17) $\frac{4}{3}$

18) $\pi\ln\frac{e}{2}$

19) $\pi(4-\pi)$

20) $\frac{\pi}{2}\ln\frac{4}{e}$

21) $\frac{\pi(e-1)}{4e}$

22) $\frac{\pi}{2}+1$

23) $-\frac{4}{5}$

24) $\pi\ln\frac{4}{e}$

25) π

26) $\frac{\left(108-64\sqrt{2}\right)\pi}{9}$

27) $\frac{32}{9}$

28) $\frac{5\pi}{32}$

Triple Integrals in Rectangular Coordinates

Questions

1) Compute the following integrals:

a. $\int_0^1\int_0^z\int_0^{x+z} 6xz\,dy\,dx\,dz$

b. $\int_0^3\int_0^1\int_0^{\sqrt{1-z^2}} ze^y\,dx\,dz\,dy$

c. $\iiint_B xyz^2\,dV \ , B=\{(x,y,z)|0\le x\le 1, -1\le y\le 2, 0\le z\le 3\}$

d. $\iiint_B 6xy\,dV \ , B=\{(x,y,z)|0\le x\le 1 \ , 0\le y\le \sqrt{x} \ , 0\le z\le 1+x+y\}$

2) Compute the following integrals by changing the order of integration:

a. $\int_0^4\int_0^1\int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}}\,dx\,dy\,dz$

b. $\int_0^1\int_0^1\int_{x^2}^1 12xze^{zy^2}\,dy\,dx\,dz$

c. $\int_0^1\int_{\sqrt[3]{z}}^1\int_0^{\ln 3} \frac{\pi e^{2x}\sin \pi y^2}{y^2}\,dx\,dy\,dz$

d. $\int_0^2\int_0^{4-x^2}\int_0^x \frac{\sin 2z}{4-z}\,dy\,dz\,dx$

3) Compute the volumes of the solids bounded by the given surfaces:

a. $y=0 \ , x=0 \ , \ x+y=1 \ , \ z=0, \ z=1+x+y$

b. $y=x^2 \ , y=1 , \ z=x^2+y^2 \ , \ z=0$

c. $y=\frac{2}{x} \ , y=2x \ , y=0.5x , z=x^2+y \ , \ z=0 \ \ (x\ge 0)$

d. $2y^2=x \ , \ \frac{x}{4}+\frac{y}{2}+\frac{z}{4}=1 \ , \ z=0$

e. $x^2+\frac{y^2}{4}=1, \ z=y \ \ (z\ge 0)$

f. $x=0 \ , \ y=0 \ , z=x+y, \ z=6$

Answer Key

1) a. 1 b. $\frac{1}{3}\left(e^3-1\right)$ c. $\frac{27}{4}$ d. $\frac{65}{28}$

2) a. $2\sin 4$ b. $3e-6$ c. $\frac{27}{4}$ d. $\frac{\sin^2 4}{2}$

3) a. $\frac{5}{6}$ b. $\frac{88}{100}$ c. $\frac{17}{6}$ d. $16\frac{1}{5}$

e. $\frac{8}{3}$ f. 36

Applications

Questions

1) Compute the mass and the center of mass of a cylinder whose height is h and whose base has radius r. It is given that the density function δ at each point is proportional the distance of the point from the base, i.e. $\delta(x,y,z)=kz\ (k>0)$.

2) Compute the moment of inertia of the homogeneous* cube $V=\{(x,y,z)|0\le x\le a\ ,\ 0\le y\le b\ ,\ 0\le z\le c\}$ about the z-axis.

Express your answer in terms of the mass M of the board.

* Homogeneous means having a constant density function.

Answer Key

1) $M=\frac{1}{2}kh^2\pi a^2$, $(\bar{x},\bar{y},\bar{z})=\left(0,0,\frac{2h}{3}\right)$

2) $\frac{2}{3}Ma^2$

Triple Integrals in Cylindrical and Spherical Coordinates

Questions

1) Compute: $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-(x^2+y^2)}^{x^2+y^2} 21xy^2 dzdydx$.

2) Compute: $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{1-x^2} \int_{\sqrt{x^2+y^2}}^{1} dzdydx$.

3) Switch to cylindrical coordinates (but don't compute): $\int_0^2 \int_0^{\sqrt{2x-2^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} dzdydx$.

4) A body is bounded by the cylinder $x^2+y^2=9$, the xy-plane below and the hemisphere $z=\sqrt{25-x^2-y^2}$ above.
Compute the volume of the body and its centroid.

5) Compute the volume and the centroid of the body bounded by the sphere $x^2+y^2+z^2=16$ above and by the cone $z=\sqrt{x^2+y^2}$.

6) Find the volume of the region above the xy - plane bounded by the paraboloid $z=x^2+y^2$ and the cylinder $x^2+y^2=a^2$.

Triple Integrals and the Jacobian

7) Compute $\iiint_B (z-y)^2 xydV$, when B is body bounded by the surfaces $xy=4$, $xy=2$, $z=y+1$, $z=y$, $x=3$, $x=1$.

8) Compute the volume of the ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

9) Compute $\iiint_E x^2 dV$, when E is the ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

10) Compute the volume of the region bounded by the surfaces:
$y=z^2$, $y=4z^2$, $y=4x$, $y=4x-12$, $y=z$, $y=2z$.

Answer Key

1) 4

2) $\frac{1}{3}\pi$

3) $\int_{\theta=0}^{\pi/2}\int_{r=0}^{2\cos\theta}\int_{z=-\sqrt{4-r^2}}^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$

4) $\left(0,0,\frac{1107}{488}\right)$

5) $\left(0,0,\frac{3}{2\left(2-\sqrt{2}\right)}\right)$

6) $\frac{1}{2}a^4\pi$

7) $2\ln 3$

8) $\frac{4}{3}\pi abc$

9) $\frac{4\pi}{15}a^2bc$

10) $\frac{105}{32}$

Substitutions in Multiple Integrals

Questions

1) Compute the double integral $\iint_R \sqrt{16x^2+9y^2}\,dA$, where R is the region bounded by the ellipse: $\frac{x^2}{9}+\frac{y^2}{16}=1$.

2) Compute the double integral $\iint_R y^2 dA$, where R is the region bounded by the curves: $y=\frac{1}{x},\ y=\frac{2}{x},\ xy^2=1,\ xy^2=2$.

3) Compute the double integral $\iint_R e^{x+y} dA$, where $R=\left\{(x,y)\,\middle|\,|x|+|y|\le 1\right\}$.

Answer Key

1) 96π

2) $\frac{3}{4}$

3) $e-\frac{1}{e}$