

# Workbook



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# Early Transcendentals – 14<sup>th</sup> Edition

## Techniques of Integration

### Integration by Parts

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#### Questions

Compute the following integrals:

1)  $\int (xe^x) dx$

2)  $\int (x^4 \ln x) dx$

3)  $\int (x \sin x) dx$

4)  $\int ((x^2 + 2x + 3) \ln x) dx$

5)  $\int x \cos 2x dx$

6)  $\int x^2 \sin 4x dx$

7)  $\int x^2 e^{-4x} dx$

8)  $\int \ln x dx$

9)  $\int \ln \frac{1}{\sqrt[3]{x}} dx$

10)  $\int \arctan x dx$

11)  $\int \arcsin x dx$

12)  $\int x \ln \sqrt[5]{x-2} dx$

13)  $\int \frac{x}{\cos^2 x} dx$

14)  $\int \frac{\ln x}{x^2} dx$

15)  $\int x \arctan x dx$

16)  $\int x^2 \ln(x^2 + 1) dx$

17)  $\int \ln^2 x dx$

18)  $\int \left( \frac{\ln x}{x} \right)^2 dx$

19)  $\int e^x \cos x dx$

20)  $\int e^{2x} \sin 4x dx$

21)  $\int \sqrt{1+x^2} dx$

22)  $\int \frac{xe^x}{(x+1)^2} dx$

23)  $\int x \tan^2 x dx$

24)  $\int (x+1)^4 \cdot \sqrt{x+2} dx$

## Answer Key

- 1)  $x e^x - e^x + C$
- 2)  $\frac{1}{3} x^5 \ln x - \frac{1}{25} x^5 + C$
- 3)  $-x \cos(x) + \sin(x) + C$
- 4)  $\ln(x) \left( \frac{1}{3} x^3 + x^2 + 3x \right) - \left( \frac{1}{9} x^3 + \frac{1}{2} x^2 + 3x \right) + C$
- 5)  $\frac{1}{2} x \sin 2x + \frac{1}{4} \cos(2x) + C$
- 6)  $-\frac{1}{4} x^2 \cos 4x + \frac{1}{8} x \sin 4x + \frac{1}{32} \cos 4x + C$
- 7)  $-\frac{1}{4} x^2 e^{-4x} - \frac{1}{8} x e^{-4x} - \frac{1}{32} e^{-4x} + C$
- 8)  $x \ln x - x + C$
- 9)  $-\frac{1}{3} (x \ln x - x) + C$
- 10)  $x \arctan x - \frac{1}{2} \ln(1+x^2) + C$
- 11)  $x \arcsin x + \sqrt{1-x^2} + C$
- 12)  $\frac{1}{10} x^2 \ln(x-2) - \frac{1}{20} x^2 + \frac{1}{5} x + \frac{2}{5} \ln(x-2) + C$
- 13)  $x \tan x + \ln|\cos x| + C$
- 14)  $-\frac{\ln x + 1}{x} + C$
- 15)  $\frac{1}{2} x^2 \arctan x - \frac{1}{2} (x - \arctan x) + C$
- 16)  $\frac{1}{3} x^3 \ln(x^2 + 1) - \frac{2}{3} \left( \frac{x^3}{3} - x + \arctan x \right) + C$
- 17)  $x \ln^2 x - 2(x \ln x - x) + C$
- 18)  $-\frac{\ln^2 x}{x} + 2 \left( -\frac{\ln x}{x} - \frac{1}{x} \right) + C$
- 19)  $\frac{1}{2} e^x (\sin x + \cos x) + C$
- 20)  $\frac{4}{5} e^{2x} \left( -\frac{1}{4} \cos 4x + \frac{1}{8} \sin 4x \right) + C$
- 21)  $\frac{1}{2} \left[ x \sqrt{1-x^2} + \arcsin x \right] + C$
- 22)  $\frac{e^x}{x+1} + C$
- 23)  $x(\tan x - x) + \ln|\cos x| + \frac{1}{2} x^2 + C$
- 24)  $\frac{2(x+2)^{\frac{3}{2}} (315x^4 + 980x^3 + 1290x^2 + 708x + 211)}{3465} + C$

## Trigonometric Integrals

### Questions

Compute the following integrals:

- 1) a.  $\int (\sin^2 x + \sin x + 2) \cos x dx$     b.  $\int (\cos^3 x + \cos x - 2) \sin x dx$     c.  $\int \cos^3 x dx$
- 2) a.  $\int \sin^2 2x dx$     b.  $\int \sin^4 x \cos^5 x dx$     c.  $\int \sin^5 x \cos^4 x dx$
- 3) a.  $\int \cos^5 x dx$     b.  $\int \tan^5 x dx$     c.  $\int \frac{1}{\cos x} dx$
- 4) a.  $\int \frac{dx}{\sin x}$     b.  $\int \sin 2x \cdot e^{\cos x} dx$     c.  $\int \frac{2 \sin x}{\cos 2x + 4 \cos x + 7} dx$

### Answer Key

- 1) a.  $\frac{\sin^3 x}{3} + \frac{\sin^3 x}{2} + 2 \sin x + C$     b.  $-\frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + 2 \cos x + C$   
 c.  $\sin x - \frac{\sin^3 x}{3} + C$
- 2) a.  $-\frac{1}{2} \cos 2x + \frac{1}{6} \cos^3 2x + C$     b.  $\frac{\sin^2 x}{5} - \frac{2 \sin^7 x}{7} + \frac{\sin^7 x}{9} + C$   
 c.  $\frac{\cos 5x}{5} - \frac{2 \cos^7 x}{7} + \frac{\cos^9 x}{9} + C$
- 3) a.  $\sin x - \frac{2}{3} \sin^3 x + \frac{1}{3} \sin^5 x + C$     b.  $\frac{1}{4 \cos^4 x} - \frac{1}{\cos^2 x} - \ln |\cos(x)| + C$   
 c.  $\frac{1}{2} \ln \left| \frac{1 - \sin x}{1 + \sin x} \right| + C$
- 4) a.  $\frac{1}{2} \ln \left| \frac{\cos x - 1}{\cos x + 1} \right| + C$     b.  $-2e^{\cos x} (\cos x - 1) + C$   
 c.  $-\frac{1}{\sqrt{2}} \arctan \left( \frac{\cos x + 1}{\sqrt{2}} \right) + C$

## Trigonometric Substitutions

### Questions

#### Integration using Trigonometric Substitution

Compute the following integrals:

1)  $\int \frac{dx}{x^2\sqrt{4-x^2}}$

2)  $\int \frac{1}{\sqrt{4+x^2}} dx$

3)  $\int \frac{dx}{x^2\sqrt{x^2-1}}$

4)  $\int \sqrt{4x^2-1} dx$

5)  $\int \frac{x^2}{\sqrt{4-x^2}} dx$

6)  $\int \sqrt{x^2+2x-3} dx$

7)  $\int \sqrt{-6x-x^2} dx$

8)  $\int \frac{1}{(4+x^2)^2} dx$

9)  $\int \frac{1}{(x^2+2x+5)^{1.5}} dx$

10)  $\int \frac{\sqrt{25x^2-4}}{x} dx$

11)  $\int_{\frac{2}{5}}^{\frac{4}{5}} \frac{\sqrt{25x^2-4}}{x} dx$

12)  $\int_{-\frac{4}{5}}^{\frac{2}{5}} \frac{\sqrt{25x^2-4}}{x} dx$

13)  $\int \frac{1}{x^4\sqrt{9-x^2}} dx$

## Trigonometric Integrals using Identities

Compute the following integrals:

- 14) a.  $\int (\sin x + \cos x) dx$       b.  $\int \left( \sin 2x - 4 \cos \frac{x}{3} \right) dx$       c.  $\int \sin 0.5x dx$
- 15) a.  $\int \left( \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} \right) dx$       b.  $\int \frac{1}{\cos^2 4x} dx$       c.  $\int \frac{1}{\sin^2 10x} dx$
- 16) a.  $\int (\sin^2 x - \cos^2 x) dx$       b.  $\int (\cos^4 x - \sin^4 x) dx$       c.  $\int (\sin x + \cos x)^2 dx$
- 17) a.  $\int (\sin x \cos x \cos 2x) dx$       b.  $\int (\tan^2 x) dx$       c.  $\int \frac{1}{(\sin x \cdot \cos x)^2} dx$
- 18) a.  $\int (\sin 7x \cos 5x) dx$       b.  $\int (\cos x \cos 2x + \sin x \sin 2x) dx$   
c.  $\int (\sin^4 x + \cos^4 x) dx$
- 19) a.  $\int (\cos^2 x) dx$       b.  $\int (\sin^4 4x) dx$       c.  $\int \cos^3 x dx$
- 20) a.  $\int (\sin^3 4x) dx$       b.  $\int \cos^4 x dx$       c.  $\int \sin^4 2x dx$
- 21) a.  $\int \frac{1 + \cos 2x}{1 - \cos 2x} dx$       b.  $\int \frac{\sin 5x - \sin x}{\sin 4x - \sin 2x} dx$       c.  $\int \frac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} dx$
- 22) a.  $\int \frac{\sin^3 x}{1 - \cos x} dx$       b.  $\int \frac{1 + \cos^3 x}{\cos^2 \frac{x}{2}} dx$       c.  $\int \sin^2 x \cos^4 x dx$

## Answer Key

1)  $\frac{1}{4} \cot\left(\arcsin \frac{x}{2}\right) + C$

2)  $\frac{1}{2} \ln \left| \frac{1 + \sin\left(\arctan \frac{x}{2}\right)}{1 - \sin\left(\arctan \frac{x}{2}\right)} \right| + C$

3)  $\sin\left(\arccos \frac{1}{x}\right) + C$

4)  $\frac{1}{4} \left[ \frac{\sin\left(\arccos \frac{a}{x}\right)}{\cos^2\left(\arccos \frac{a}{x}\right)} - \ln \left| \frac{1 + \sin\left(\arccos \frac{a}{x}\right)}{\cos\left(\arccos \frac{a}{x}\right)} \right| \right] + C$

5)  $2 \arcsin \frac{x}{2} - \sin\left(2 \arcsin \frac{x}{2}\right) + C$

6)  $2 \left[ \frac{\sin\left(\arccos \frac{a}{x+1}\right)}{\cos^2\left(\arccos \frac{a}{x+1}\right)} - \frac{1}{2} \ln \left| \frac{1 + \sin\left(\arccos \frac{a}{x+1}\right)}{\cos\left(\arccos \frac{a}{x+1}\right)} \right| \right] + C$

7)  $\frac{9}{2} \left[ \arcsin \frac{x+3}{2} + \frac{1}{2} \sin\left(2 \arcsin \frac{x+3}{3}\right) \right] + 3$

8)  $\frac{1}{8} \left[ \sin\left(\arctan \frac{x}{2}\right) - \frac{1}{3} \sin^3\left(\arctan \frac{x}{2}\right) \right] + C$

9)  $\frac{1}{4} \sin\left(\arctan \frac{x+1}{2}\right) + C$

10)  $2 \left[ \tan\left(\arccos \frac{2}{5x}\right) - \arccos \frac{2}{5x} \right] + C$

11)  $2 \left( \sqrt{3} - \frac{\pi}{3} \right)$

12)  $\frac{2}{3} \pi - 2\sqrt{3}$

13)  $-\frac{\sqrt{9-x^2}(2x^2+9)}{243x^3}$



- 14) a.  $-\cos x + \sin x + C$       b.  $-\frac{1}{2}\cos 2x - 3\sin \frac{x}{3} + C$       c.  $-2\cos\left(\frac{1}{2}x\right) + C$
- 15) a.  $-\cot x - \tan x + C$       b.  $\frac{1}{4}\tan 4x + C$       c.  $-\frac{1}{10}\cot 10x + C$
- 16) a.  $-\frac{1}{2}\sin 2x + C$       b.  $\frac{1}{2}\sin 2x + C$       c.  $x - \frac{1}{2}\cos 2x + C$
- 17) a.  $-\frac{1}{16}\cos 4x + C$       b.  $\tan x - x + C$       c.  $-2\cot(2x) + C$
- 18) a.  $-\frac{1}{24}\cos 12x - \frac{1}{4}\cos 2x + C$       b.  $\sin x + C$       c.  $\frac{3}{4}x + \frac{1}{16}\sin 4x + C$
- 19) a.  $\frac{1}{2}x + \frac{1}{4}\sin 2x + C$       b.  $\frac{1}{2}x - \frac{1}{16}\sin 8x + C$       c.  $\frac{3}{4}\sin x + \frac{1}{6}\sin 3x + C$
- 20) a.  $-\frac{3}{16}\cos 4x + \frac{1}{48}\cos 12x + C$       b.  $\frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$   
 c.  $\frac{3}{8}x - \frac{1}{8}\sin 4x + \frac{1}{64}\sin 8x + C$
- 21) a.  $-\cot x - x + C$       b.  $2\sin x + C$       c.  $-\ln|\cos x| + C$
- 22) a.  $-\cos(x) - \frac{1}{4}\cos(2x) + C$       b.  $3x - 2\sin x + \frac{1}{2}\sin 2x + C$   
 c.  $\frac{1}{16} + \frac{1}{64}\sin 2x - \frac{1}{64}\sin 4x - \frac{1}{192}\cos 6x + C$

## Integration of Rational Functions by Partial Fractions

### Questions

Compute the following integrals:

1)  $\int \frac{1}{x^2 - 4} dx$

2)  $\int \frac{5-x}{x^2+5x} dx$

3)  $\int \frac{x}{x^2+5x+6} dx$

4)  $\int \frac{8x-1}{2x^2-3x-2} dx$

5)  $\int \frac{1}{(x-4)^2} dx$

6)  $\int \frac{x+4}{(x-1)^2} dx$

7)  $\int \frac{6-x}{x^2+8x+16} dx$

8)  $\int \frac{2x}{x^2+5} dx$

9)  $\int \frac{1}{x^2+1} = \arctan x + k$

10)  $\int \frac{2x+1}{x^2+1} dx$

11)  $\int \frac{4x+10}{x^2+9} dx$

12)  $\int \frac{5x+6}{x^2+3} dx$

13)  $\int \frac{1}{x^2+2x+5} dx$

14)  $\int \frac{1}{x^2+4x+13} dx$

15)  $\int \frac{1}{x^2+x+1} dx$

16)  $\int \frac{3x-7}{x^2+2x+5} dx$

17)  $\int \frac{5x+14}{x^2+4x+20} dx$

18)  $\int \frac{x^2+x-1}{x^3-x} dx$

19)  $\int \frac{6x^2+4x-6}{x^3-7x-6} dx$

20)  $\int \frac{10x}{x^4-13x^2+36} dx$

21)  $\int \frac{8x}{(x-2)^2(x+2)} dx$

22)  $\int \frac{5-x}{x^3+x^2} dx$

23)  $\int \frac{9x+36}{x^3+6x^2+9x} dx$

24)  $\int \frac{1}{(x^2-2x+1)(x^2-4x+4)} dx$

25)  $\int \frac{x+4}{(x-1)^3} dx$

26)  $\int \frac{6x^2-4x+1}{(x-1)^3} dx$

27)  $\int \frac{x+4}{(x-1)^3} dx$

28)  $\int \frac{x+4}{(x-1)^3} dx$

29)  $\int \frac{2x+3}{(x^2-2x+1)^4} dx$

30)  $\int \frac{2x^2+2x+1}{(x^2+1)(x+2)} dx$

31)  $\int \frac{2x^2+x-1}{(x^2+1)(x-3)} dx$

32)  $\int \frac{3}{(x^2+1)(x^2+4)} dx$

33)  $\int \frac{1}{x(x^2+1)^2} dx$

## Answer Key

- 1)  $\frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + c$
- 2)  $\ln|x| - 2\ln|x+5| + c$
- 3)  $3\ln|x+3| - 2\ln|x+2| + c$
- 4)  $\ln|x+0.5| + 3\ln|x-2| + c$
- 5)  $-\frac{1}{x-4} + c$
- 6)  $\ln|x-1| - \frac{5}{x-1} + c$
- 7)  $-\ln|x+4| - \frac{10}{x+4} + c$
- 8)  $\frac{7}{2} \ln|x^2+2| + c$
- 9) Formula example
- 10)  $\ln(x^2+1) + \arctan(x) + c$
- 11)  $2\ln(x^2+9) + \frac{10}{3} \arctan\left(\frac{x}{3}\right) + c$
- 12)  $\frac{5}{2} \ln(x^2+3) + 6 \frac{1}{\sqrt{3}} \arctan\left(\frac{x}{\sqrt{3}}\right) + c$
- 13)  $\frac{1}{2} \arctan\left(\frac{x+1}{2}\right) + c$
- 14)  $\frac{1}{3} \arctan\left(\frac{x+2}{3}\right) + c$
- 15)  $\frac{1}{\sqrt{3/4}} \arctan\left(\frac{x+0.5}{\sqrt{3/4}}\right) + c$
- 16)  $\frac{3}{2} \ln((x+1)^2+4) - 5 \arctan\left(\frac{x+1}{2}\right) + c$
- 17)  $2.5 \ln((x+2)^2+16) + \arctan\left(\frac{x+2}{4}\right) + c$
- 18)  $\ln|x| + \frac{1}{2} \ln|x-1| - \frac{1}{2} \ln|x+1| + c$
- 19)  $\ln|x+1| + 2\ln|x+2| + 3\ln|x-3| + c$
- 20)  $\ln \left| \frac{x^2-3}{x^2-4} \right| + c$
- 21)  $\ln \left| \frac{x-2}{x+2} \right| - \frac{4}{x-2} + c$
- 22)  $-6\ln|x| + 5 \frac{x^{-1}}{-1} + 6\ln|x+1| + c$
- 23)  $4\ln|x| - 4\ln|x+3| + 3(x+3)^{-1} + c$
- 24)  $2\ln|x-1| - (x-1)^{-1} - 2\ln|x-2| - (x-2)^{-1} + c$
- 25)  $-\frac{1}{x-1} - \frac{5}{2(x-1)^2} + c$
- 26)  $6\ln|x-1| - \frac{8}{x-1} - \frac{3}{2(x-1)^2} + c$
- 27)  $-(x-1)^{-1} - 5 \frac{(x-1)^{-2}}{2} + c$
- 28)  $\frac{-1}{(x-1)} - \frac{5}{2(x-1)^2} + c$
- 29)  $-\frac{1}{3(x-1)^6} - \frac{5}{7(x-1)^7} + c$
- 30)  $\frac{1}{2} \ln|x^2+1| + \ln|x+2| + c$
- 31)  $\arctan x + 2\ln|x-3| + c$
- 32)  $\arctan x - \frac{1}{2} \arctan\left(\frac{x}{2}\right) + c$
- 33)  $\ln|x| - \frac{1}{2} \ln(x^2+1) - \frac{1}{2(x^2+1)} + c$

## Numerical Integration

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### Questions

- 1) Estimate  $\int_0^2 e^{x^2} dx$  using each of the three rules (Midpoint, Trapezoid, Simpson) and  $n = 4$ .
  
- 2) Estimate  $\int_2^4 \frac{x}{x-1} dx$  using each of the three rules with its specified  $n$ .
  - a. Midpoint ( $n = 4$ )
  - b. Trapezoid ( $n = 4$ )
  - c. Simpson ( $n = 8$ )

### Answer Key

- |                          |                        |                      |
|--------------------------|------------------------|----------------------|
| 1) Midpoint: 14.48561253 | Trapezoid: 20.64455905 | Simpson: 17.35362645 |
| 2) a. 3.08975469         | b. 3.11666667          | c. 3.09872535        |

## Improper Integrals

### Questions

Determine whether the following integrals converges or diverges and, if they converges, determine their values:

- |   |  |   |   |
|---|--|---|---|
| 1) $\int_1^{\infty} \frac{1}{x^4} dx$                     | 2) $\int_1^{\infty} \frac{1}{\sqrt{x}} dx$             | 3) $\int_{-\infty}^0 \frac{1}{(2x-5)^5} dx$ | 4) $\int_2^{\infty} \frac{1}{\sqrt[3]{2x+1}} dx$        |
| 5) $\int_0^{\infty} \frac{1}{e^x} dx$                     | 6) $\int_{-\infty}^0 e^{4x} dx$                        | 7) $\int_{-1}^{\infty} \frac{x}{1+x^2} dx$  | 8) $\int_0^{\infty} xe^{-x^2} dx$                       |
| 9) $\int_e^{\infty} \frac{1}{x \ln^4 x} dx$               | 10) $\int_4^{\infty} \frac{1}{x \sqrt{\ln x}} dx$      | 11) $\int_0^{\infty} \frac{1}{1+x^2} dx$    | 12) $\int_1^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$ |
| 13) $\int_0^{\infty} \frac{1}{1+e^x} dx$                  | 14) $\int_0^{\infty} \cos(x) dx$                       | 15) $\int_0^{\infty} x^2 e^{-x} dx$         | 16) $\int_1^{\infty} \frac{x}{(1+x^2)^2} dx$            |
| 17) $\int_1^{\infty} \frac{1}{(1+x)\sqrt{x}} dx$          | 18) $\int_{-\infty}^0 \frac{e^x}{3-2e^x} dx$           | 19) $\int_{-\infty}^{\infty} x^3 dx$        | 20) $\int_{-\infty}^{\infty} \frac{x}{\sqrt{x^2+4}} dx$ |
| 21) $\int_{-\infty}^{\infty} \frac{e^{-x}}{1+e^{-2x}} dx$ | 22) $\int_1^4 \frac{1}{(x-2)^{2/3}} dx$                | 23) $\int_0^2 \frac{1}{(x-1)^2} dx$         | 24) $\int_0^4 \frac{1}{x-2} dx$                         |
| 25) $\int_1^{\infty} \frac{1}{x\sqrt{x^2-1}} dx$          | 26) $\int_0^1 \sin \frac{1}{x} \cdot \frac{1}{x^2} dx$ | 27) $\int_0^1 \frac{1}{x\sqrt{x^2+1}} dx$   | 28) $\int_0^1 \frac{1}{x\sqrt{x}} dx$                   |

29) Find the area under the curve  $y = xe^{-x}$  for  $x \geq 0$ .

30) Find the area under the curve  $y = x^2 e^{-x^3}$  for  $x \geq 0$ .

31) Find the area under the curve  $y = \frac{1}{\sqrt{x}}$  between  $x = 0$  and  $x = 4$ .

32) Find the area under the curve  $y = \frac{\sin x}{\sqrt{\cos x}}$  between  $x = \frac{\pi}{4}$  and  $x = \frac{\pi}{2}$ .

33) Find the area above the curve  $y = \ln x$  between  $x = 0$  and  $x = 1$ .

34) The area between the graph of  $y = -\ln x$  the  $x$ -axis, and the  $y$ -axis is revolved around the  $x$ -axis. Find the volume of the solid it generates.

## Answer Key

- 1) Converges,  $\frac{1}{3}$ .      2) Diverges.      3) Converges,  $-\frac{1}{5000}$ .
- 4) Diverges.      5) Converges, 1.      6) Converges,  $\frac{1}{4}$ .
- 7) Diverges.      8) Converges, 0.5.      9) Converges,  $\frac{1}{3e^3}$ .
- 10) Diverges.      11) Converges,  $\frac{\pi}{2}$ .      12) Converges,  $\frac{2}{e}$ .
- 13) Converges,  $\ln 2$ .      14) Diverges.      15) Converges, 2.
- 16) Converges,  $\frac{1}{4}$ .      17) Converges,  $\frac{1}{3e^3}$ .      18) Converges,  $\frac{1}{2}(\ln 3 - 1)$ .
- 19) Diverges.      20) Diverges.      21) Converges,  $\frac{\sqrt{2}\pi}{4}$ .
- 22) Converges,  $3(1 + \sqrt[3]{2})$ .      23) Diverges.      24) Diverges.
- 25) Converges,  $\frac{\pi}{2}$ .      26) Diverges.      27) Diverges.
- 28) Diverges.      29) 1      30)  $\frac{1}{3}$
- 31) 4      32)  $\frac{2}{\sqrt[4]{2}}$       33) 1      34)  $\pi$

## Probability

### Questions

- 1) Given:  $f(x) = \begin{cases} 3x^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$ .
- Show that  $f(x)$  is a probability density function (PDF).
  - Find the probabilities:  $P\left(\frac{1}{3} \leq X \leq \frac{2}{3}\right)$ ,  $P\left(X \leq \frac{1}{2}\right)$ ,  $P\left(X = \frac{3}{4}\right)$ .
- 2) Find the mean of the PDF:  $f(x) = \begin{cases} 3x^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$ .
- 3) Given:  $f(x) = \begin{cases} 0 & t \leq 0 \\ 0.2e^{-\frac{t}{5}} & t \geq 0 \end{cases}$ , where  $t$  is in minutes.
- Verify that  $f(t)$  is a PDF.
  - Compute the probability:  $P(T \geq 5)$ .
  - Compute the average waiting time.

### Answer Key

- 1) a. Solution in the recording.  
 b.  $P\left(\frac{1}{3} \leq X \leq \frac{2}{3}\right) = \frac{7}{27}$ ,  $P\left(X \leq \frac{1}{2}\right) = \frac{7}{8}$ ,  $P\left(X = \frac{3}{4}\right) = 0$
- 2)  $\frac{3}{4}$
- 3) a. Solution in the recording.      b.  $\frac{1}{e}$       c. 5