proprep™

Workbook

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Early Transcendentals – 14th Edition

Vector-Valued Functions and Motion in Space

Curves in Space and Their Tangents

Questions

1) Find the domain of the following vector functions.

a. $\vec{r}(t)=\left\langle t^3-3t,\sqrt{t-1},\frac{5}{t-3}\right\rangle$ (3D)

b. $\vec{r}(t)=\left\langle \sqrt{t+2},\ln\left(9-t^2\right)\right\rangle$ (2D)

2) Sketch the graph of the following 2D vector functions using the specified method:

a. $\vec{r}(t)=\langle 4t,10-2\text{t}\rangle$ by making a table of points.

b. $\vec{r}(t)=\left\langle t+1,\frac{1}{4}t^2+3\right\rangle$ by obtaining an equation in just x and y.

3) Identify the graphs of the following 3D vector functions (do not sketch):

a. $\vec{r}(t)=\langle 2\cos(3t),\sin(3t),4\rangle$

b. $\vec{r}(t)=\langle 5+2t,3-6t,-4-t\rangle$

4) Find the vector equation (function) of the line segment between the two points:

a. $\text{A}(2,4)$, $\text{B}(-3,5)$ (2D)

b. $\text{P}(3,2,0)$, $\text{Q}(8,-5,1)$ (3D)

5) Parametrize, using Polar coordinates, the straight line through the Cartesian points $(3,2)$ and $(1,4)$. The answer should look something like: $\begin{cases} r=r(t) \\ \theta=\theta(t) \end{cases}$, $0\le t\le 1$.

6) Evaluate the following limits:

a. $\lim\limits_{t\to 2}\left\langle \cos(\pi t), e^{t-2}, \dfrac{t-2}{t^2-4}\right\rangle$

b. $\lim\limits_{t\to 0}\left\langle (t^3+3)\vec{i}-2\vec{j}+\dfrac{1-e^t}{t^2-t}\vec{k}\right\rangle$

c. $\lim\limits_{t\to \infty}\left\langle \dfrac{3t^2}{t^2-t+3}, e^{-t}, \dfrac{2}{t^2}\right\rangle$

7) Differentiate the following vector functions:

a. $\vec{r}(t)=(t^3+3)\vec{i}-\sin(2t)\vec{j}+e^{-3t}\vec{k}$

b. $\vec{r}(t)=\langle \ln(\cos t), te^{3t}, 5\rangle$

c. $\vec{r}(t)=\left\langle \dfrac{\ln t}{t}, \tan(2t), \sin^2 t\right\rangle$

Tangent, Normal and Binormal Vectors

8) Given the vector function $\vec{r}(t)=\langle t^2, \sin 2t, \cos 2t\rangle$.

a. Find the unit tangent vector $\vec{T}(t)$.

b. Find the tangent line, call it $\vec{l}(t)$, at $t=\dfrac{\pi}{2}$.

9) Given the vector function $\vec{r}(t)=\frac{1}{2}e^{2t}\vec{i}-2e^t\vec{j}+2t\vec{k}$.

a. Find the unit tangent vector $\vec{T}(t)$.

b. Find the tangent line, call it $\vec{l}(t)$, at $t=0$.

10) Given $\vec{r}(t)=\langle 1, \sin 3t, \cos 3t\rangle$.

Find the unit normal $\vec{T}(t)$ and the unit binormal $\vec{B}(t)$.

11) Answer the following questions:

a. Compute the frame $\vec{T}(t), \vec{N}(t), \vec{B}(t)$ for the space curve $\vec{r}(t)=(t, 2\sin t, 2\cos t)$

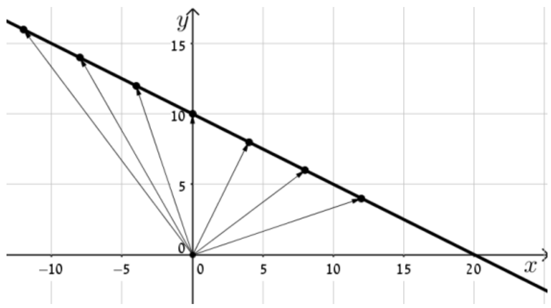
b. Determine the curvature $\kappa(t)$ of $\vec{r}(t)$.

Answer Key

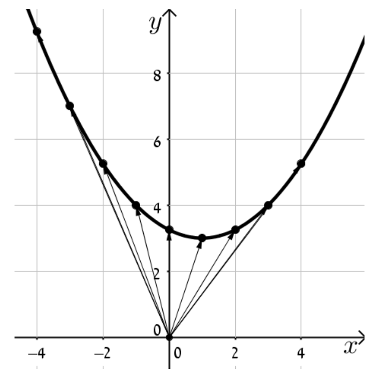
1) a. $t \geq 1\,,\, t \neq 3$ b. $-2 \leq t < 3$

2) Here are the following figures:

a.



b.



3) a. Ellipse suspended four unites above the xy plane. b. line

4) a. $\vec{r}(t) = \langle 2-5t, 4+t \rangle \quad 0 \leq t \leq 1$

b. $\vec{r}(t) = \langle 3+5t, 2-7t, t \rangle \quad 0 \leq t \leq 1$

5) $$\begin{cases} r(t) = \sqrt{8t^2 - 4t + 13} \\ \theta(t) = \arctan \dfrac{2+2t}{3-2t} \end{cases} \quad 0 \leq t \leq 1$$

6) a. $\left\langle 1, 1, \dfrac{1}{4} \right\rangle$ b. $3\vec{i} - 2\vec{j} + \vec{k}$ c. $\langle 3, 0, 0 \rangle$

7) a. $\vec{r}\,'(t) = 3t^2\vec{i} - 2\cos(2t)\,\vec{j} - 3e^{-3t}\vec{k}$ b. $\vec{r}\,'(t) = \langle -\tan t, e^{3t}(1+3t), 0 \rangle$

c. $r'(t) = \left\langle \dfrac{1-\ln t}{t^2}, \dfrac{2}{\cos^2(2t)}, 2\sin t \cos t \right\rangle$

8) a. $\vec{T}(t) = \left\langle \dfrac{t}{\sqrt{t^2+1}}, \dfrac{\cos 2t}{\sqrt{t^2+1}}, \dfrac{-\sin 2t}{\sqrt{t^2+1}} \right\rangle$ b. $\left\langle \dfrac{\pi^2}{4} + \dfrac{\pi}{2}t, -t, -2 \right\rangle$

9) a. $\vec{T}(t) = \dfrac{e^{2t}}{e^{2t}+2}\vec{i} - \dfrac{2e^2}{e^{2t}+2}\vec{j} + \dfrac{2}{e^{2t}+2}\vec{k}$

b. $\vec{l}(t) = \left(\dfrac{1}{2} + t\right)\vec{i} - (2+2t)\,\vec{j} + 2t\vec{k}$

10) Unit normal: $\vec{N}(t) = \langle 0, -\sin 3t, -\cos 3t \rangle$; Unit binormal: $\vec{B}(t) = -\vec{i}$

11) a. $\vec{N}(t) = (0, -\sin t, -\cos t)$, $\vec{T}(t) = \dfrac{1}{\sqrt{5}}(1, 2\cos t, -2\sin t)$, $\vec{B}(t) = \left(\dfrac{2}{\sqrt{5}}, \dfrac{1}{\sqrt{5}}\cos t, \dfrac{-1}{\sqrt{5}}\sin t\right)$

b. $\kappa(t) = \dfrac{2}{5}$

Integrals of Vector Functions; Projectile Motion

Questions

12) Evaluate the following integrals:

a. $\int \vec{r}(t)\,dt$ where $\vec{r}(t) = 3t^2\vec{i} - \tan(2t)\vec{j} + \frac{3t^2}{t^3-1}\vec{k}$.

b. $\int_1^4 \vec{r}(t)\,dt$ where $\vec{r}(t) = \langle 6te^{3t}, 4t-3t^2, 5\rangle$

Answer Key

1) a. $t^3\vec{i} + \frac{1}{2}\ln|\cos(2t)|\vec{j} + \ln|t^3-1|\vec{k} + \vec{c}$ b. $\left\langle \frac{1}{3}\left(22e^{12} - 4e^3\right), -33, 20 \right\rangle$

Arc Length in Space

Questions

1) Find the length of the curve $\vec{r}(t) = \langle 2t-7, 3-5t, -6+4t\rangle$, for: $-2 \le t \le 5$.

2) Find the length of the curve $\vec{r}(t) = \sqrt{2}t^2\vec{i} + \frac{1}{3}t^3\vec{j} - 4t\vec{k}$, for: $0 \le t \le 3$.

3) Find the arc length function $s(t)$, for: $\vec{r}(t) = \langle 1+3t^2, 4+2t^3\rangle$.

4) Answer the following questions:

a. Find the arc length function $s(t)$, for: $\vec{r}(t) = e^{2t}\cos 2t\vec{i} + 2\vec{j} + e^{2t}\sin 2t\vec{k}$.

b. Where on the curve are we, after a travel distance of 10?

Answer Key

1) $21\sqrt{5}$

2) 21

3) $2\left[\left(1+t^2\right)^{\frac{2}{3}} - 1\right]$

4) a. $S(t) = \sqrt{2}\left(e^{2t} - 1\right)$ b. $r(t) = e^{2t}\cos 2t\vec{i} + 2\vec{j} + e^{2t}\sin 2t\vec{k}$

Curvature and Normal Vectors of a Curve

Questions

1) Find the curvature of the curve: $\vec{r}(t)=\left\langle t,\frac{t^2}{2},t^2\right\rangle$.

2) Find the curvature of the curve: $\vec{r}(t)=3t\,\vec{i}+4\sin t\,\vec{j}+4\cos t\,\vec{k}$.

3) What is the length of the curve: $\vec{r}(t)=\left\langle 2t,\frac{1}{2}t^2,2\ln t\right\rangle$, $1\le t\le e$?

Compute the curvature $\kappa(t)$ of $\vec{r}(t)$.

Answer Key

1) $\kappa=\frac{\sqrt{5}}{\left(1+5t^2\right)^{\frac{2}{3}}}$

2) $\frac{4}{25}$

3) a. $L=0.5e^2+1.5$

b. $\kappa(t)=\frac{2t}{\left(t^2+2\right)^2}$

Tangential and Normal Components of Acceleration and Velocity and Acceleration in Polar Coordinates

Questions

1) The acceleration of a body is given by $\vec{a}(t) = 3t\vec{i} - 4e^{-t}\vec{j} + 12t^2\vec{k}$.

The body's initial velocity is $\vec{v}(0) = \vec{j} - 3\vec{k}$ and its initial position is $\vec{r}(0) = -5\vec{i} + 2\vec{j} - 3\vec{k}$.

Find the body's velocity and position functions.

2) The position of a body is given by $\vec{r}(t) = \langle \sin 3t, -\cos 3t, 5 \rangle$.

Determine the tangential and normal components of its acceleration.

3) An alien spaceship is flying through our galaxy, following a trajectory parametrized by $\vec{r}(t) = \langle \sin 2t, 3, \cos 2t \rangle$. Compute its tangential and normal acceleration at $t = \pi$.

4) Suppose that in the superball game this weekend, the Patriots Quarterback Tom Brady wants to throw the football to a receiver standing 40 yards away.
If he knows that he can throw the football with an initial speed of 30 yards per second, at what angle with the ground should he throw so that the football arrives in the receiver's hand?
What is the flight time of the football?
(Disregard air resistance effect, assume Tom and the receiver have the same height, and use $g = 11.25$ yards per square second)

Answer Key

1) Velocity function: $\vec{v}(t) = \frac{3}{2}t^2\vec{i} + \left(4e^{-t} - 3\right)\vec{j} + \left(4t^2 - 3\right)\vec{k}$.

Position function: $\vec{r}(t) = \left(0.5t^3 - 5\right)\vec{i} + \left(-4e^{-t} - 3t + 6\right)\vec{j} + \left(t^4 - 3t - 3\right)\vec{k}$.

2) $a_T = 0$, $a_N = 9$

3) $a_T(t) = 0$, $\leq \mathrm{a}_N(t) = 4$

4) $\theta = 15°$