

Workbook



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Trigonometric Integrals and Trigonometric Substitution

Trigonometric Integrals using Identities

Questions:

Compute the following integrals:

- 1) a. $\int (\sin x + \cos x) dx$ b. $\int \left(\sin 2x - 4 \cos \frac{x}{3} \right) dx$ c. $\int \sin 0.5x dx$
- 2) a. $\int \left(\frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} \right) dx$ b. $\int \frac{1}{\cos^2 4x} dx$ c. $\int \frac{1}{\sin^2 10x} dx$
- 3) a. $\int (\sin^2 x - \cos^2 x) dx$ b. $\int (\cos^4 x - \sin^4 x) dx$ c. $\int (\sin x + \cos x)^2 dx$
- 4) a. $\int (\sin x \cos x \cos 2x) dx$ b. $\int (\tan^2 x) dx$ c. $\int \frac{1}{(\sin x \cdot \cos x)^2} dx$
- 5) a. $\int (\sin 7x \cos 5x) dx$ b. $\int (\cos x \cos 2x + \sin x \sin 2x) dx$
c. $\int (\sin^4 x + \cos^4 x) dx$
- 6) a. $\int (\cos^2 x) dx$ b. $\int (\sin^4 4x) dx$ c. $\int \cos^3 x dx$
- 7) a. $\int (\sin^3 4x) dx$ b. $\int \cos^4 x dx$ c. $\int \sin^4 2x dx$
- 8) a. $\int \frac{1 + \cos 2x}{1 - \cos 2x} dx$ b. $\int \frac{\sin 5x - \sin x}{\sin 4x - \sin 2x} dx$ c. $\int \frac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} dx$
- 9) a. $\int \frac{\sin^3 x}{1 - \cos x} dx$ b. $\int \frac{1 + \cos^3 x}{\cos^2 \frac{x}{2}} dx$ c. $\int \sin^2 x \cos^4 x dx$

Answer Key:

- 1) a. $-\cos x + \sin x + C$ b. $-\frac{1}{2}\cos 2x - 3\sin \frac{x}{3} + C$ c. $-2\cos\left(\frac{1}{2}x\right) + C$
- 2) a. $-\cot x - \tan x + C$ b. $\frac{1}{4}\tan 4x + C$ c. $-\frac{1}{10}\cot 10x + C$
- 3) a. $-\frac{1}{2}\sin 2x + C$ b. $\frac{1}{2}\sin 2x + C$ c. $x - \frac{1}{2}\cos 2x + C$
- 4) a. $-\frac{1}{16}\cos 4x + C$ b. $\tan x - x + C$ c. $-2\cot(2x) + C$
- 5) a. $-\frac{1}{24}\cos 12x - \frac{1}{4}\cos 2x + C$ b. $\sin x + C$ c. $\frac{3}{4}x + \frac{1}{16}\sin 4x + C$
- 6) a. $\frac{1}{2}x + \frac{1}{4}\sin 2x + C$ b. $\frac{1}{2}x - \frac{1}{16}\sin 8x + C$ c. $\frac{3}{4}\sin x + \frac{1}{6}\sin 3x + C$
- 7) a. $-\frac{3}{16}\cos 4x + \frac{1}{48}\cos 12x + C$ b. $\frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$
c. $\frac{3}{8}x - \frac{1}{8}\sin 4x + \frac{1}{64}\sin 8x + C$
- 8) a. $-\cot x - x + C$ b. $2\sin x + C$ c. $-\ln|\cos x| + C$
- 9) a. $-\cos(x) - \frac{1}{4}\cos(2x) + C$ b. $3x - 2\sin x + \frac{1}{2}\sin 2x + C$
c. $\frac{1}{16} + \frac{1}{64}\sin 2x - \frac{1}{64}\sin 4x - \frac{1}{192}\cos 6x + C$

Trigonometric Integrals using Substitution

Questions:

Compute the following integrals:

- 1) a. $\int (\sin^2 x + \sin x + 2) \cos x dx$ b. $\int (\cos^3 x + \cos x - 2) \sin x dx$ c. $\int \cos^3 x dx$
- 2) a. $\int \sin^2 2x dx$ b. $\int \sin^4 x \cos^5 x dx$ c. $\int \sin^5 x \cos^4 x dx$
- 3) a. $\int \cos^5 x dx$ b. $\int \tan^5 x dx$ c. $\int \frac{1}{\cos x} dx$
- 4) a. $\int \frac{dx}{\sin x}$ b. $\int \sin 2x \cdot e^{\cos x} dx$ c. $\int \frac{2 \sin x}{\cos 2x + 4 \cos x + 7} dx$

Answer Key:

- 1) a. $\frac{\sin^3 x}{3} + \frac{\sin^3 x}{2} + 2 \sin x + C$ b. $-\frac{\cos^4 x}{4} - \frac{\cos^2 x}{2} + 2 \cos x + C$ c. $\sin x - \frac{\sin^3 x}{3} + C$
- 2) a. $-\frac{1}{2} \cos 2x + \frac{1}{6} \cos^3 2x + C$ b. $\frac{\sin^2 x}{5} - \frac{2 \sin^7 x}{7} + \frac{\sin^7 x}{9} + C$
- c. $\frac{\cos 5x}{5} - \frac{2 \cos^7 x}{7} + \frac{\cos^9 x}{9} + C$
- 3) a. $\sin x - \frac{2}{3} \sin^3 x + \frac{1}{3} \sin^5 x + C$ b. $\frac{1}{4} \frac{1}{\cos^4 x} - \frac{1}{\cos^2 x} - \ln |\cos(x)| + C$ c. $\frac{1}{2} \ln \left| \frac{1 - \sin x}{1 + \sin x} \right| + C$
- 4) a. $\frac{1}{2} \ln \left| \frac{\cos x - 1}{\cos x + 1} \right| + C$ b. $-2e^{\cos x} (\cos x - 1) + C$ c. $-\frac{1}{\sqrt{2}} \arctan \left(\frac{\cos x + 1}{\sqrt{2}} \right) + C$

Integration using Trigonometric Substitution

Questions:

1) $\int \frac{dx}{x^2\sqrt{4-x^2}}$

2) $\int \frac{1}{\sqrt{4+x^2}} dx$

3) $\int \frac{dx}{x^2\sqrt{x^2-1}}$

4) $\int \sqrt{4x^2-1} dx$

5) $\int \frac{x^2}{\sqrt{4-x^2}} dx$

6) $\int \sqrt{x^2+2x-3} dx$

7) $\int \sqrt{-6x-x^2} dx$

8) $\int \frac{1}{(4+x^2)^2} dx$

9) $\int \frac{1}{(x^2+2x+5)^{1.5}} dx$

10) $\int \frac{\sqrt{25x^2-4}}{x} dx$

11) $\int_{\frac{2}{5}}^{\frac{4}{5}} \frac{\sqrt{25x^2-4}}{x} dx$

12) $\int_{\frac{4}{5}}^{\frac{2}{5}} \frac{\sqrt{25x^2-4}}{x} dx$

13) $\int \frac{1}{x^4\sqrt{9-x^2}} dx$

Answer Key:

$$1) \frac{1}{4} \cot\left(\arcsin \frac{x}{2}\right) + C$$

$$2) \frac{1}{2} \ln \left| \frac{1 + \sin\left(\arctan \frac{x}{2}\right)}{1 - \sin\left(\arctan \frac{x}{2}\right)} \right| + C$$

$$3) \sin\left(\arccos \frac{1}{x}\right) + C$$

$$4) \frac{1}{4} \left[\frac{\sin\left(\arccos \frac{a}{x}\right)}{\cos^2\left(\arccos \frac{a}{x}\right)} - \ln \left| \frac{1 + \sin\left(\arccos \frac{a}{x}\right)}{\cos\left(\arccos \frac{a}{x}\right)} \right| \right] + C$$

$$5) 2 \arcsin \frac{x}{2} - \sin\left(2 \arcsin \frac{x}{2}\right) + C$$

$$6) 2 \left[\frac{\sin\left(\arccos \frac{a}{x+1}\right)}{\cos^2\left(\arccos \frac{a}{x+1}\right)} - \frac{1}{2} \ln \left| \frac{1 + \sin\left(\arccos \frac{a}{x+1}\right)}{\cos\left(\arccos \frac{a}{x+1}\right)} \right| \right] + C$$

$$7) \frac{9}{2} \left[\arcsin \frac{x+3}{2} + \frac{1}{2} \sin\left(2 \arcsin \frac{x+3}{3}\right) \right] + 3$$

$$8) \frac{1}{8} \left[\sin\left(\arctan \frac{x}{2}\right) - \frac{1}{3} \sin^3\left(\arctan \frac{x}{2}\right) \right] + C$$

$$9) \frac{1}{4} \sin\left(\arctan \frac{x+1}{2}\right) + C$$

$$10) 2 \left[\tan\left(\arccos \frac{2}{5x}\right) - \arccos \frac{2}{5x} \right] + C$$

$$11) 2 \left(\sqrt{3} - \frac{\pi}{3} \right)$$

$$12) \frac{2}{3} \pi - 2\sqrt{3}$$

$$13) -\frac{\sqrt{9-x^2}(2x^2+9)}{243x^3}$$